

HIGH TIME RESOLUTION AMBIENT ELECTRON DENSITY MEASUREMENTS BY CLUSTER NEAR THE EARTH'S BOW SHOCK: FEBRUARY-MAY 2001

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ABSTRACT

Spacecraft-to-probe potential difference (ΔV) performed by an electric field double probe experiment is strongly correlated to the ambient electron density, in most plasma regimes. Directly relating these two quantities provides access to high time resolution (< 1 s) estimate of the ambient electron density. High time resolution is of particular interest to study sharp electron density gradients, commonly observed at the boundary of magnetospheric regions but often under sampled by other types of instruments. Empirical laws expressing the bulk electron density as a function of ΔV , for data collected by the Cluster mission in the Earth's bow shock vicinity, are presented. These relationships are valid for measurements performed from February to June 2001. During this time period, two different bias current values have been used. For each time period, a relationship between thermal electron density and spacecraft-to-probe potential difference measurements with error bar is proposed. These relationships have been derived thanks to the accurate measure of the ambient electron density derived from the WHISPER relaxation sounder measurements, over multiple bow shock crossings by one Cluster spacecraft.

1. INTRODUCTION

The electron density is one of the fundamental physical parameters in plasmas. Its measurement is of key importance to study the dynamics of planetary environments such as the Earth's. Understanding of physical processes at all spatial scales require an accurate measure of this physical quantity. Moreover, high time resolution (< 1 s) is required to investigate, in particular, magnetospheric boundaries with sharp density gradients, commonly observed in the Earth's environment (see top panel Fig. 1).

The first formation flying multi-spacecraft mission dedicated to the study of the Earth's environment, Cluster, carry an identical set of 11 scientific instruments. On each spacecraft, this set is able to probe all key parameters governing charged particles and natural waves in plasmas, including the electron density.

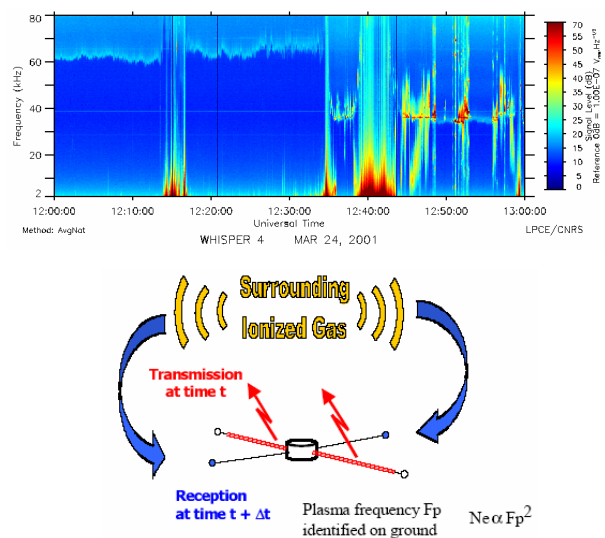


Fig. 1. [Top panel] Electric field spectrogram measured by WHISPER (2 kHz -80 kHz range) onboard Cluster 4, on 24 March 2001, during an outbound crossing of the Earth's bow shock. Most of the time, clear lower cut-off of natural emissions (thin light blue curve) in the magnetosheath and in the foreshock enables to derive accurate estimates of the bulk density. However, density values can not be derived during short time intervals, in particular at the bow shock crossing ($\sim 12:35$), due to natural broad band electrostatic emissions; [Bottom panel] Sketch of the WHISPER relaxation sounder principle of measurements [5].

As we will see in this study, inter-calibration between two instruments enables to derive thermal electron density estimates with 0.2s time resolution.

In most of the regions crossed by the Cluster mission, accurate thermal electron density data on all four spacecraft can be derived from the active sounding of the WHISPER experiment (Waves of HIGH frequency and Sounder for Probing of Electron density by Relaxation) [1]. From February to June 2001, these measurements were performed every 28 or 104 seconds in normal mode (every 52 or 104 s after 2001).

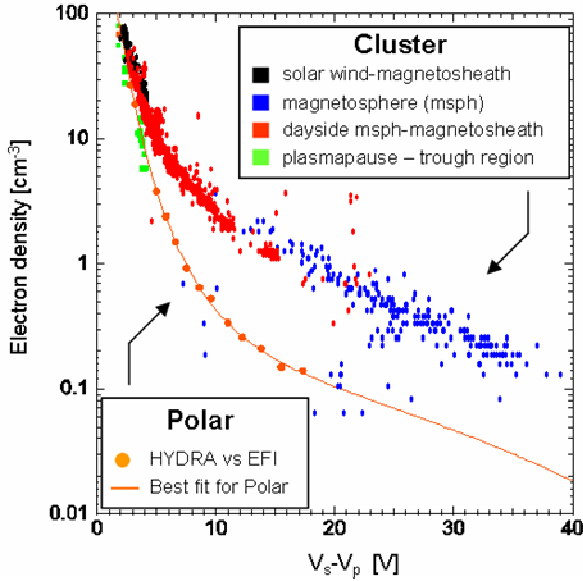


Fig. 2. Electron densities measured by WHISPER on Cluster and HYDRA on POLAR as a function of the spacecraft-to-probe potential difference, $V_s - V_p$ [8].

When a clear plasma frequency cut-off is observed, the electric field spectra obtained by the WHISPER wave receiver can also be used to extract the total density of thermal electrons. The time resolution of these measurements is either 1.7 s, 2.15 s or 3.4 s in normal mode.

The potential difference between each spherical probe of the Electric Field and Waves (EFW) experiment and the spacecraft can also be used to estimate the ambient electron density. It is measured every 0.2 s in normal mode. Its variation is strongly dependent on the electron density but also on other plasma and experimental parameters (see section 2 and section 3).

Thus, calibrating such measurements on Cluster enables to derive electron density datasets with a time resolution up to 260 times higher than the WHISPER active sounding data in normal mode.

The EFW and WHISPER instruments will be now briefly presented.

2. EFW AND WHISPER

The EFW instrument on Cluster is an electric field spherical double probe experiment. It consists of 4 spherical probes, 8 cm in diameter, at the end of long wire booms in the spin plane (bottom panel of Fig. 1), with a separation of 88 m between opposite probes [2].

Accurate electric field measurements require that the probe-plasma impedance, $Z = dV/dI$, be as small as possible to ensure a good coupling to the plasma and insensitivity to spurious currents [3]. However, in tenuous plasmas, the potential of a probe (V_p) can reach large positive values, resulting in large Z . By using a current source to force an electron flux from the spacecraft to each probe, each probe potential is kept at

a few volts positive with respect to the ambient plasma. This current, called bias current or I_b , is applied to each probe on all Cluster spacecraft. For a given ambient electron density, the intensity of this current affects the value of the probe potential as, by definition, a probe immersed in a plasma reaches an equilibrium potential for which the sum of all the currents to the probe is zero. Consequently, the bias current intensity also affects the spacecraft-to-probe potential difference measurements by EFW and has to be taken into account when calibrating these measurements against ambient electron density (see section 4).

The WHISPER experiment mainly consists of a pulse transmitter, a wave receiver in the 2 kHz - 80 kHz range and a wave spectrum analyser [4]. Electric signals are acquired by the EFW sensors.

In its sounding mode, the transmitter sends, through the conductive outer braids of one double sphere electric antennae (see bottom panel of Fig. 1), a wave train at a given frequency during a very short time interval (1 ms or less). A few milliseconds later, the receiver connected to the two other spherical probes, via high impedance preamplifiers, records the signal. The working frequency is then shifted for a new sounding until the whole frequency range is covered. When the transmitted pulse frequency is close to a plasma characteristic frequency, such as the plasma frequency F_p , a very intense echo is received. Let us remind that the ambient electron density N_e , number of electrons per cubic centimetre, is simply related to the plasma frequency, in kHz, as follows

$$N_e = F_p^2 / 81 \quad (1)$$

In the solar wind and the magnetosheath, only one strong resonance is usually observed close to the plasma frequency, which enables to monitor the ambient electron density (see section 4).

When a clear lower frequency cut-off is observed, the electric field spectra obtained by the WHISPER passive wave receiver can also be used to extract the bulk electron density.

3. STATISTICAL APPROACH

3.1 Any lessons learned from past missions?

Electric field spherical double probe experiments are operated in space, for almost 30 years, onboard various magnetospheric missions including: S3-3, GEOS-1, GEOS-2, ISEE-1, CRRES, Viking, Geotail, POLAR, Interball and Cluster. Each time, spacecraft-to-probe potential difference ($V_s - V_p$) measurements with these double probe experiments have been used to derive estimates of the ambient electron density (see e.g. [6]). So why is there still a need to calibrate these measurements on Cluster?

Fig. 2 shows an example of such measurements together with the simultaneous measure of the thermal

electron density by WHISPER for Cluster and the HYDRA electron experiment for POLAR. As one can see, there is a clear departure between Cluster measurements and the POLAR electron density versus V_s - V_p curve derived by Scudder et al. [7]. But Cluster measurements are not only different from POLAR measurements. As discussed in Pedersen et al. [8], the corresponding Cluster curve has higher current densities and higher e-folding energies for the escaping photoelectrons than for previous missions. The difference is more pronounced for lower electron densities corresponding to more positive spacecraft-to-probe potentials.

A possible explanation for the discrepancy between different magnetospheric missions is the fact that the design of double probe experiment has constantly evolved from one mission to the next. The concept of the experiment remained the same but key elements such as the location of the spherical probes, material of the probes or the location of the preamplifiers have been modified from one mission to another. The design of the spacecraft, including the location of the solar panels (which plays an obvious role in the photoemission behaviour) is also mission dependent. Other possible factors are proposed in Pedersen et al. [8]. In particular, it is underlined that the photoemission is more pronounced at solar maximum, when Cluster data were collected, than for the POLAR data collected closer to solar minimum. As Cluster is still in operation after 5 years, data recorded under similar solar activity can be compared to check this hypothesis (subject of a future publication).

However, numerical simulations and in-flight investigations have allowed identifying some key parameters governing the spacecraft-to-probe difference potential behaviour for specific type of plasmas. For example, Escoubet et al. [9] has shown that for low energy electron media (plasmaphere and solar wind), V_s - V_p is a function of the electron density and a weak function of the electron energy. Numerical analysis conducted by Laakso and Pedersen [10] clearly showed that the N_e versus V_s - V_p is strongly affected by the photoelectron temperature and the bias current in tenuous plasmas (up to tens of electrons per cm^{-3}).

3.2 Current balance equations and statistical approach

In tenuous plasmas, the close relationship between the bulk electron density and a satellite-to-biased probe potential difference is an experimental fact known for decades (e.g. [6] and references therein). However, this relationship is also affected by a number of other plasma and experimental parameters including: the photoelectron temperature, the ambient electron temperature, the bias current, the saturation photoelectron current density and the radius of the probe. Let us now briefly detail how the spacecraft body potential, V_s , and the potential of an individual spherical

probe, V_p , immersed in a plasma are related to these parameters, for plasma conditions encountered by Cluster.

The value of V_s is determined by the balance of currents associated with ambient electrons and ions impacting on the spacecraft body surface and photoelectrons emitted from it (I_b can be ignored in the current balance equation for the satellite potential). For positive potentials (plasma ion current(s) negligible) and outside eclipse period, the balance of currents can be simply expressed as [11]

$$I_{\text{phs}} - I_{\text{es}} + I_{\text{se}} = 0 \quad (2)$$

with I_{phs} the current of photoelectrons rejected from the spacecraft's sunlit surface, I_{es} the current of incident ambient electrons and I_{se} the current of secondary electrons due to incident ambient electrons. Outside eclipse, the latter current is usually neglected. However, secondary electrons seem to play a non-negligible role on Cluster in the magnetosheath [8].

I_{es} is mainly dependent on V_s and the characteristics of the electron distribution, which is function of the electron density N_e and temperature T_e . Analytical solutions of $I_{\text{es}}(N_e, T_e, V_s)$ can be derived for a conducting spherical body immersed in an unmagnetised collisionless maxwellian plasma [12]. Are these hypotheses (spherical shape, unmagnetised plasma, maxwellian distribution of the ambient electrons) valid for Cluster?

Each Cluster spacecraft is of cylindrical shape with conductive surfaces but it attracts ambient electrons like a sphere as long as the Debye length λ_D ($\lambda_D = 6.9 \sqrt{(T_e/N_e)}$) is larger than the size of the body [14]. This condition is fulfilled by the Cluster mission all along its orbit, except near perigee, within the plasmopause boundary layer ($T_e \sim \text{few eV}$), when the electron density exceeds 50 cm^{-3} [8].

Moreover, as shown by Bouhram et al. [15] for the Interball-2 magnetospheric spacecraft, the Earth's magnetic field magnitude affects I_{es} as soon as the electron gyroradius is comparable to the satellite body size. On Cluster, the Earth's magnetic field \mathbf{B}_0 might affect I_{es} but only very close to perigee (~ 4 Earth radii altitude) where \mathbf{B}_0 is of the order of 500 nT, hence an electron gyroradius ~ 7 m for $T_e \sim 1$ eV (diameter of a Cluster spacecraft ~ 3 m).

However, a maxwellian distribution of the ambient electrons remains an assumption of these analytical solutions.

The photoelectron current I_{phs} depends on V_s , the velocity distribution function of the photoelectrons (function of the photoelectron temperature T_{ph}) and the saturation photoelectron current density. I_{phs} is usually derived on the basis of laboratory measurements [16] and in-flight calibration work (e.g. [6, 7], for Cluster [8]). The velocity distribution function of the photoelectrons (such as maxwellian or bi-maxwellian) is usually assumed.

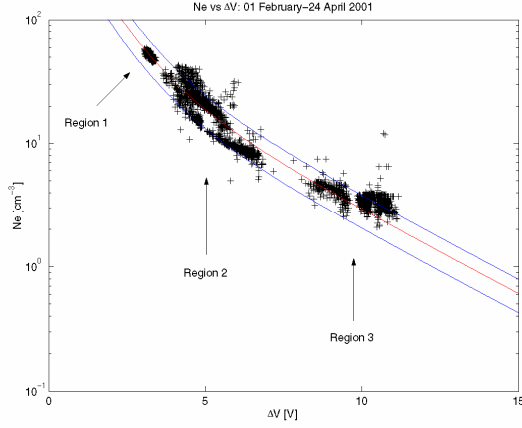


Fig. 3. Bulk electron density data versus spacecraft-to-probe potential difference measurements, collected between 1 February and 24 April 2001 in the vicinity of the Earth's bow shock by Cluster 4 (black crosses). The red curve corresponds to the best fit obtained. Blue curves correspond to $\pm 30\%$ discrepancy of this fit.

Similarly, when a conducting probe is immersed in a plasma, it acquires an equilibrium potential for which the sum of all the currents to the probe is zero. This time the bias current is no more negligible and the (positive) probe potential is obtained by solving

$$I_{ep} + I_b - I_{php} = 0 \quad (3)$$

with I_{php} the current of escaping photoelectrons, I_{es} the current of collected ambient electrons and I_b the bias current. Any ion current (I_i) is again neglected as, for positive potentials, I_i is smaller than I_e due to the ion to electron mass ratio. Let us remind that when V_p is near zero or negative (N_e greater than a few hundreds of electrons per cm^3), $V_s - V_p$ is no more a reliable proxy of the bulk electron density.

This brief overview on the current balance equations show that deriving a relationship between N_e and $V_s - V_p$ rely on a set of assumptions and in-flight investigations (e.g. the determination of the probe saturation photoelectron current density). Moreover, analytical solutions do not exist for all plasma regimes crossed by the Cluster mission in the Earth's environment.

A related issue is the fact that physical parameters, such as the characteristics of the ambient electron distribution and photoelectron distribution, are not always accessible experimentally in all plasma regimes crossed by the Cluster mission. Therefore, assumptions can not always be experimentally checked and therefore, sometimes, further assumptions (e.g. the photoelectron temperature) need to be made.

A different approach to derive a N_e versus $V_s - V_p$ relationship will now be presented. It is based on in-flight investigations only. The basic idea is to derive an average empirical law (or profile), with error bar, of N_e versus $V_s - V_p$, from a collection of accurate bulk density

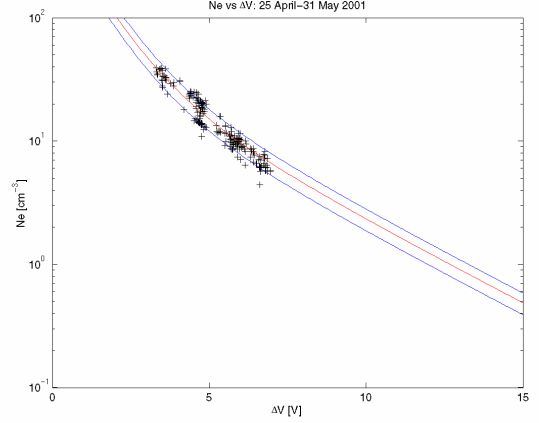


Fig. 4. Bulk electron density data versus spacecraft-to-probe potential difference measurements, collected in the vicinity of the Earth's bow shock by Cluster 4, between 01 February and 24 April 2001 (black crosses). The red curve corresponds to the best fit obtained. Blue curves correspond to $\pm 20\%$ discrepancy of this fit.

data derived from WHISPER measurements and $V_s - V_p$ data from EFW, collected in the same region of space by one Cluster spacecraft.

In the following section, such a statistical approach is used to derive two empirical laws between N_e and $V_s - V_p$ on the basis of data collected in the vicinity of the Earth's bow shock, from February to May 2001.

4. ELECTRON DENSITY VS S/C POTENTIAL NEAR THE BOW SHOCK (FEB.-MAY 2001)

Different values of the bias current intensity have been used since the beginning of the science operations of the Cluster mission (February 2001): 180 nA on opposite probes 1 and 2 (or P12), and 220 nA on P34 from 1 February to 24 April 2001; 180 nA on all probes from 25 April 2001 to 31 May 2001; 140 nA on all probes from 1 June 2001 onwards.

Moreover, from 1 January 2002 onwards, probe 1 on Cluster 1 (or C1) has been used in Langmuir mode (voltage bias instead of current bias). For this spacecraft, $V_s - V_p$ can only be calculated from P34. The same is true for C3 since 6 August 2002 (more info on EFW operations: <http://www.cluster.irfu.se/efw/ops/>). Therefore, for the sake of consistency with future publications focused on EFW measurements after 1 June 2001, only measurements from P34 have been used.

According to Laakso and Pedersen [10], for electron temperature in the 1-100 eV range, the ambient electron density (up to tens of electrons cm^{-3}) versus $V_s - V_p$ is most affected by the photoelectron temperature and the bias current. Therefore, two separate N_e versus $V_s - V_p$ relationships are presented below: one corresponding to EFW measurements performed on P34 from 1 February

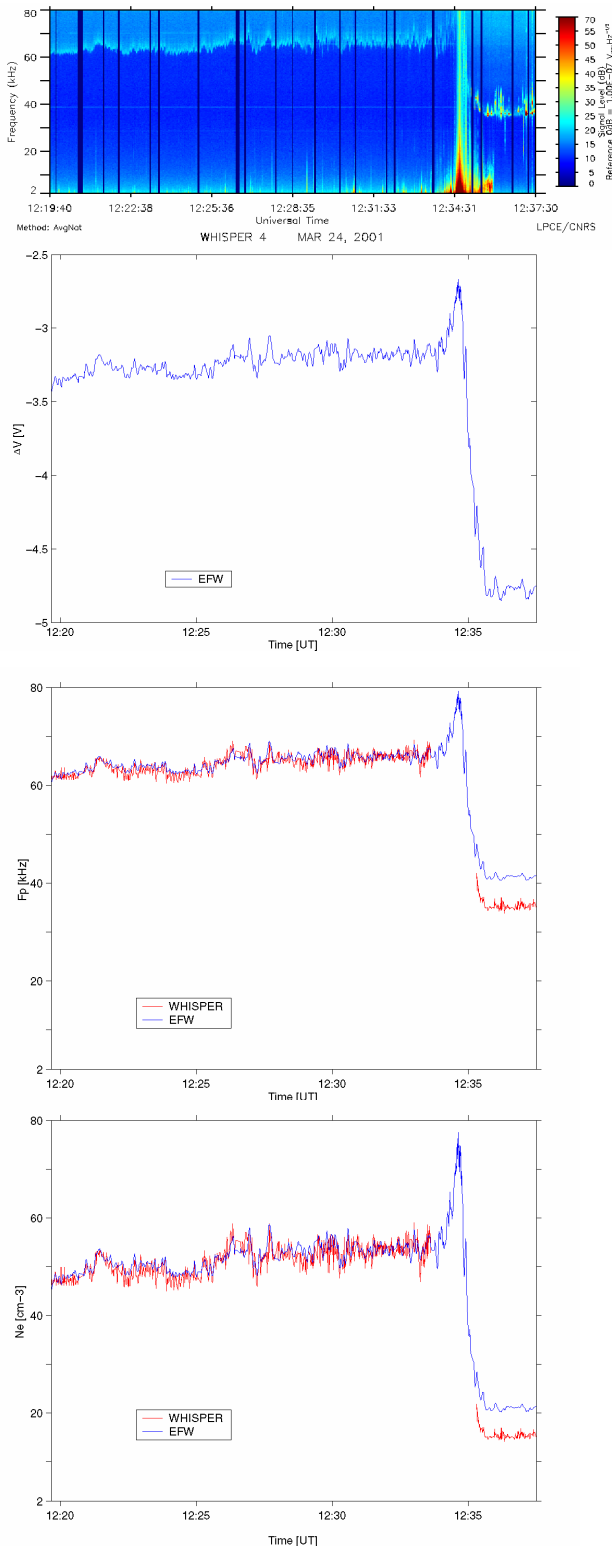


Fig. 5. [Top panel] Electric field spectrogram measured by WHISPER onboard C4, on 24 March 2001; [Second panel from top] Simultaneous $V_s - V_p$ measurements by EFW using P34; [Third panel from top] Bulk electron densities versus time derived from both WHISPER and EFW, turned to F_p ; [Bottom panel] Bulk electron densities versus time, derived from both EFW and WHISPER.

to 24 April 2001 ($I_b=220$ nA); the other one corresponding to EFW measurements performed on P34 from 25 April 2001 to 31 May 2001 ($I_b=180$ nA). The time resolution of the EFW measurements used is 0.2s. As each Cluster spacecraft is a spinning platform (spin period: 4s), all EFW data used have been first despun, however by a very simple method.

4.1 01 February-24 April 2001 period

Bulk density and spacecraft-to-probe potential difference data, $\Delta V = V_s - V_p$, collected over fifteen bow shock crossings by Cluster 4, from 01 February to 24 April 2001, are gathered on Fig. 3. Based on these data, the following best fit (quasi-newton method) has been derived

$$N_e = 744 \times 10^{-\Delta V/2.1} + 58.4 \times 10^{-\Delta V/4.3} + 52.4 \times 10^{-\Delta V/7.7} \quad (4)$$

with N_e the bulk electron density in cm^{-3} and ΔV in Volts. This empirical law is plotted in red. Blue lines correspond to $\pm 30\%$ of this profile. More than 84 % of the data points are described by the best fit with a 30 % accuracy. It can not be excluded that some data points among the remaining 15% are spurious data points due to data analysis only. This could be attributed either to the simple despun procedure used and/or errors in the extraction of electron density values from wave measurements (semi-automatic procedure).

Three clouds of data points can be distinguished in Fig. 3. The first one (Region 1) corresponds to few datasets with $\Delta V < 4\text{V}$ together with $N_e \sim 50$ e-/cc ($F_p \sim 60\text{kHz}$). These data have been acquired in the magnetosheath. Region 2, around 5V, is a mix of data from the magnetosheath and the foreshock collected under various conditions of electron temperature, which explains this spreading in density. Region 3, around $\Delta V = 10\text{V}$, corresponding to $N_e \sim 3-5$ e-/cc ($F_p \sim 20\text{kHz}$), have been collected by Cluster in the solar wind. The high values of ΔV with respect to densities is due to the high level of bias current used ($I_b = 220$ nA).

As an example, on 24 March 2001, Cluster 4 spacecraft crossed the Earth's bow shock at around 12:35 UT, coming from the magnetosheath and entering the foreshock region (Fig. 5, top panel). In the magnetosheath (from 12:19 to 12:34 UT), the lower frequency cut-off of natural emissions around 60 kHz corresponds to the plasma frequency, providing an accurate mean to derive the bulk electron density against time. The Earth's bow shock, crossed at 12:35 is characterised by broadband electrostatic emissions covering the whole WHISPER frequency range. As no sounding was performed during this crossing, no density value can be derived from these measurements. After the bow shock crossing, the foreshock region is characterised by bursty oscillations around the plasma frequency ([4] and references therein). All thin black stripes correspond to data gaps.

As one can see in the second panel of Fig. 5, a sharp gradient (within a minute) is observed on EFW

measurements also at around 12:35 UT. By applying equation (4) to these EFW data, one gets an estimation of the bulk electron density (hence the plasma frequency) with a 0.2 s time resolution, displayed in blue in the two remaining panels of Fig. 5. Electron density data derived from WHISPER measurements are superimposed (in red) on the EFW data converted to either plasma frequencies or electron densities. As one can see, in this case, the agreement between both experiments is good in the magnetosheath (mean discrepancy $\sim 4\%$) but rather poor in the foreshock region (mean discrepancy $\sim 30\%$).

The discrepancy between EFW and WHISPER might be entirely due to the simple way EFW measurements have been turned into electron densities. However, as underlined in Décréau et al. [4], past observations have revealed that the characteristic frequency of the bursty oscillations, observed in the foreshock region, are sometimes well below the local plasma frequency. Spectra obtained by the WHISPER sounder allows to check if the characteristic frequency of these bursty oscillations corresponds or not to the local plasma frequency [4]. In our case, only one sounding is available after the bow shock crossing and is in agreement with passive measurements. This may indicate that the discrepancy is due to the method used to derive densities from EFW measurements. This issue will be addressed in a future study.

4.2 25 April – 31 May 2001 period

Similar data collected during three bow shock crossings by Cluster 4, from 25 April to 31 May 2001, are gathered on Fig. 4. More precisely, crossings on 16 May, 23 May and 28 May have been considered. This represents of course only a minor subset of all data collected in the bow shock vicinity during this time period. Based on these data, the following empirical law has been derived (quasi-newton fit with the same exponents found for the previous period)

$$N_e = 595 \times 10^{-\Delta V/2.1} + 46.7 \times 10^{-\Delta V/4.3} + 41.9 \times 10^{-\Delta V/7.7} \quad (5)$$

with N_e the bulk electron density in cm^{-3} and ΔV in Volts.

This density profile is plotted in red on Fig. 4. Blue curves correspond to $\pm 20\%$ of this profile. More than 80% of the data points collected are described by equation (5) with a 20% accuracy.

The impact of the bias current change (220 nA to 180 nA) is $\sim 20\%$ in the density range and plasma regimes related to these measurements. This result confirms the significant influence of the bias current value on the spacecraft-to-probe potential difference measurements by a spherical double probe electric field experiment.

This empirical law is however a preliminary result considering the few number of crossings taken into account.

5. CONCLUSION

This study is a first attempt to derive an empirical law explicitly relating the spacecraft-to-probe potential difference measurements on Cluster to bulk electron density estimates, in the vicinity of the bow shock, for two time periods: 01 February-24 April 2001 and 25 April-31 May 2001 (different bias current used during these two time periods). These empirical laws have been derived by comparing EFW data, collected over multiple bow shock crossings, with simultaneous accurate electron density measurements performed by WHISPER.

Using probes 3 and 4 on Cluster 4, the following empirical laws have been obtained

01 February - 24 April 2001

$$N_e = 744 \times 10^{-\Delta V/2.1} + 58.4 \times 10^{-\Delta V/4.3} + 52.4 \times 10^{-\Delta V/7.7} \pm 30\%$$

25 April 2001-31 May 2001

$$N_e = 595 \times 10^{-\Delta V/2.1} + 46.7 \times 10^{-\Delta V/4.3} + 41.9 \times 10^{-\Delta V/7.7} \pm 20\%$$

The difference between these two laws is around 20% due to the different bias current values used during these two time periods (220 nA then 180 nA).

It is worth underlining that the empirical law derived for the 25 April - 31 May 2001 time period is a preliminary result. A future study of this time period shall include much more crossings to gain confidence in the empirical law found.

The derivation of a similar empirical law for the main bulk of Cluster data, collected since June 2001 (a 140 nA bias current value has been kept since), is under study.

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