WAVE SOURCE LOCATIONS IN THE BOWSHOCK AND ADJACENT REGIONS

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ABSTRACT

Using a wave telescope method based on a spherical waves representation we are able to determine the location of plasma wave sources from Cluster magnetic field data. In virtue of Huygens principle, our method also allows to determine the shape of the source region. We apply the method to a typical Cluster crossing of the bowshock and we are able to locate wave sources both in the foreshock and in the magnetosheath. The foreshock wave source is close to the Cluster tetrahedron indicating that waves are locally generated in the foreshock. The magnetosheath wave originates from an elongated source region aligned with the background magnetic field.

Key words: Cluster, Wave telescope, Source location.

1. INTRODUCTION

The four Cluster spacecraft form an array of sensors providing multi-point measurements in space. To take advantage on the available information powerful methods like the wave telescope (Motschmann et al., 1995; Glassmeier et al., 2001) or k-filtering technique (Pinçon and Motschmann, 1998; Sahraoui et al., 2003) have been developed. These methods are inspired from fields with tradition in using sensor arrays like seismology and oceanography. They decompose the measured wave field in a sum of plane waves corresponding to different wave vectors and frequencies. For each such elementary wave a power corresponding to its contribution to the field is computed. This allows for the identification of the propagation direction and wave length of the dominant wave.

While wave telescope/k-filtering proves to be a robust and valuable tool in analyzing magnetic and electric field data they cannot directly answer questions such as: Are the measured waves coming from a distant place or are they locally generated in a close by region? If they are generated locally what are the dimensions of the source region? Is it a point source or a spatially extended source? What is the shape of the source region? Is the wave source moving, and if it does, in what direction and with which velocity? Assuming a plane wave representation inherently implies that these questions cannot be answered. However, if we adapt these methods to decompose the wave field into spherical rather than plane waves, the above questions can be answered.

2. THEORETICAL BACKGROUND

Let us assume in the general case that the measured field $B(\mathbf{r}, t)$ is a superposition of elementary waves $w(\mathbf{q}, \mathbf{r}, t)$ where $\mathbf{q} = (q_1, \ldots, q_M)^T$ are the parameters of the elementary wave. A sensor positioned at $\mathbf{r}_{\text{sensor}}$ in a wave field generated by N sources with parameters $\mathbf{q}_1 \ldots \mathbf{q}_N$ will measure the field

$$B(\boldsymbol{r}_{\text{sensor}}, t) = \sum_{\substack{n=1\\\text{sources}}}^{N} c_n w(\boldsymbol{q}_n, \boldsymbol{r}_{\text{sensor}}, t) \qquad (1)$$

We introduce the vector $\boldsymbol{B}(t)$ with elements $B_i = B(\boldsymbol{r}_{\text{sensor i}}, t)$ and the vector $\boldsymbol{w}(\boldsymbol{q}, t)$ with elements $w_i = w(\boldsymbol{q}, \boldsymbol{r}_{\text{sensor i}}, t)$. The dimension of these vectors is equal to the number of sensors in the array. Note that B is a scalar field. The theory can easily be expanded to vector fields.

Following Pillai (1989) we define the array power associated with the measured field as:

$$P = \left(\boldsymbol{w}^{\dagger} \underline{\boldsymbol{B}} \underline{\boldsymbol{B}}^{-1} \boldsymbol{w}\right)^{-1} \tag{2}$$

where \boldsymbol{w}^{\dagger} is the hermitian adjoint of \boldsymbol{w} and the matrix element $(\underline{\boldsymbol{B}}\underline{\boldsymbol{B}})_{ij}$ is equal to $B_i^{\star}B_j$. The * operator denotes complex conjugation.

The array power P(q) measures the relative contribution of the elementary wave w(q) to the field. By scanning the $\{q\}$ space we can identify the dominant wave parameters q_0 :

$$P(\boldsymbol{q})\big|_{\boldsymbol{q}=\boldsymbol{q}_0} = \text{maximum} \tag{3}$$

The wave telescope technique uses plane waves as elementary waves:

$$w(\boldsymbol{k},\omega,t) = Ce^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \tag{4}$$

Since in this case the parameter space is $\{k\}$, the wave telescope technique decomposes the measured field in elementary waves corresponding to different wave vectors. The power maximum will correspond to the wave vector of the dominant wave.

In order to determine the sources locations we have to choose a system of elementary waves which contains information about the distance to the source. The most natural choice is a spherical waves representation,

$$w(\rho, k, \omega, t) = C \frac{1}{\rho} e^{i(k\rho - \omega t)}$$
(5)

The source position is hidden in the distance to the source $\rho = \|\mathbf{r}_{sensor} - \mathbf{r}_{source}\|$.

In contrast to the three dimensional parameters space of the wave telescope $\{k\}$, the parameters space of the source locator $\{q\} = \{r_{\text{source}}, k\}$ is four dimensional. As for the wave telescope in order to find the sources parameters we have to scan the parameter space computing the array power in each point of a grid and identify the peaks.

3. CASE STUDY

To illustrate the potential of the source locator we choose a bowshock inbound crossing which occurred on February 26, 2002 around 21:30 UT. During this time the shock regime was quasi-parallel and the spacecraft formation was close to a regular tetrahedron with the minimum spacecraft separation of 87 km and the maximum separation of 135 km. For a location analysis we use the magnetic field magnitude measured by the FGM sensors on board the Cluster spacecraft (Fig. 1). We are going to analyze two events: one in the foreshock and the second in the magnetosheath. Both data intervals are 512 s in length with a resolution of 1 s.

We compute the array power for each point on a $30 \times 30 \times 30 \times 30$ grid in the scan domain (source latitude,



Figure 1. The magnetic field magnitude measurements. The two red rectangles mark the time intervals to be analyzed.

source longitude, source distance, wave length) and identify the power maximum. In order to represent this four dimensional array we plot two dimensional sections which contain the point of maximum power.

The results of location analysis applied on the foreshock interval for a frequency of 31 mHz are shown in Fig. 2 and Fig. 3. The maximum array power indicates a wave source located upstream (latitude= 15° , longitude = -4°), having a wave length of 1200 km, at a distance of 823 km from the Cluster configuration center. This shows that waves are locally generated in the foreshock region. The squares in Fig. 3 represent the spacecraft positions. The continuous line is the projection of the magnetic field line passing through the source and the dashed line represents the same for the plasma flow direction. The triangles on those lines are the points of closest approach to the Cluster configuration center. The power maximum in Fig. 3 does not have a symmetric shape as we would expect for a stationary point source. It has a rather elongated shape, but since it is not aligned either with the magnetic field nor with the plasma flow direction, it cannot unambiguously be interpreted.

For the magnetosheath interval we show the array power at a frequency of 181 mHz in Fig. 4 and Fig. 5. This time the wave source is at a much greater distance (more than 2600 km) having a latitude of 10° and a longitude of -46° . The measured wave length was 550 km. The latitude – longitude plot in Fig. 5 shows that the power maximum is in this case clearly aligned with the average magnetic field line direction. Since the distance to the source is large we interpret this rather as an indication of the source region geometry than as a point source moving along the magnetic field direction.



Figure 2. Array power for the foreshock interval, slice (distance, wave number). The power maximizes at 823 km from the s/c configuration center indicating a local wave source



Figure 3. Array power for the foreshock interval, slice (longitude, latitude). The continuous line is representing the average magnetic field direction and the dashed line the plasma flow direction. The triangles on the lines mark the points of closest approach to the configuration center. The squares represent the s/c positions.

4. CONCLUSIONS

The source locator is the generalization of the wave telescope technique to spherical waves. In addition to the wave vector which is given by the wave telescope, the source locator provides for the distance to the source. This allows the identification of active regions where waves are generated. If the wave source have a spatial extent, the source locator can give information about the shape of the source region.

The location analysis performed in the foreshock interval has revealed a wave source close to the Cluster configuration center, confirming that waves are locally generated in the foreshock. For the magne-



Figure 4. Array power for the magnetosheath interval, slice (distance, wave number). The power maximum lies outside of the scanning domain indicating a distant wave source.



Figure 5. Array power for the magnetosheath interval, slice (longitude, latitude). The symbols have the same signification as those in Fig. 3. The power maximum is elongated in a direction parallel to the average magnetic field.

tosheath interval we have identified a distant wave source which appears to be strongly elongated in the magnetic field direction.

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