Calculation of the solar parallax using planetary transits.

When Edmund Halley published "A New Method of Determining the Parallax of the Sun, or His Distance from the Earth" in 1716, absolute measurements and absolute knowledge of distances within our Solar System were not known. From Newton's Third Law, published in 1687 in the *Principia Mathematica*, Halley and his contemporaries did however know the distances of the planets and the planets radii relative to each other.

In 1716, Halley discussed two methods to calculate the absolute value of the distance between the Sun and Earth, using planetary transits.

Two observers view a planetary transit from different locations on Earth. To the observer in the most northern location on Earth the planet would appear further to the south of the Sun's disc than it would appear to the observer in the most southern location on Earth. This effect is called parallax.

In the example that follows, we describe the shadow of a planet onto the solar disk as 'planet disc' and measure this value as an angle, mostly in arc seconds written as ''.
Be aware that, due to the elliptical orbits of planets around the Sun, this angle changes over an orbit.

The angular size of Venus during the transit on 5 June 2012 was 0.01198 degrees or 43''.

The angular size during Mercury at the transit on 9 May 2016 will be 0.003353 degrees or 12''.

In addition, the apparent angular size of the Sun changes when observed from the Earth. This effect is mostly caused by the elliptical orbit of the Earth around the Sun.

On 9 May 2016, the solar angular size is 0.528 degrees, or 1900''.

You can find out by yourself using the WebGeocalc Tool of the NAIF team at JPL.

Parallax or angular shift

An angular shift or parallax is the difference in the apparent position of an object against a background when viewed along two different lines of sight. You can try this for yourself, hold your thumb out, then close one eye at a time and note the position of your thumb against objects in the background.

Your thumb will appear to have changed position, Figure 1. This is because by closing one eye at a time you viewed your thumb along two different lines of sight.

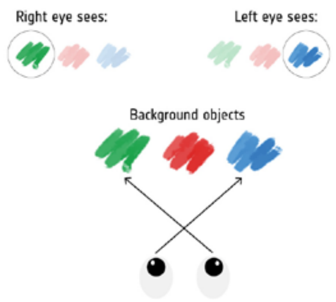

Figure 1: Diagram to show example of parallax by closing one eye at a time

Method 1: calculating the distance between the Earth and Sun

Let's assume that two observers are at the same longitude, but at different latitudes. At the same longitude they will observe the entry and exit of the planet disc onto the Sun at the same time.

Edmund Halley proposed to observe the Venus transit from London and Cape Town. So, here we use the letters L and C, accordingly.
The distance between the centres of these two discs can be described as an angle, measured in arc seconds. See Figure 2.

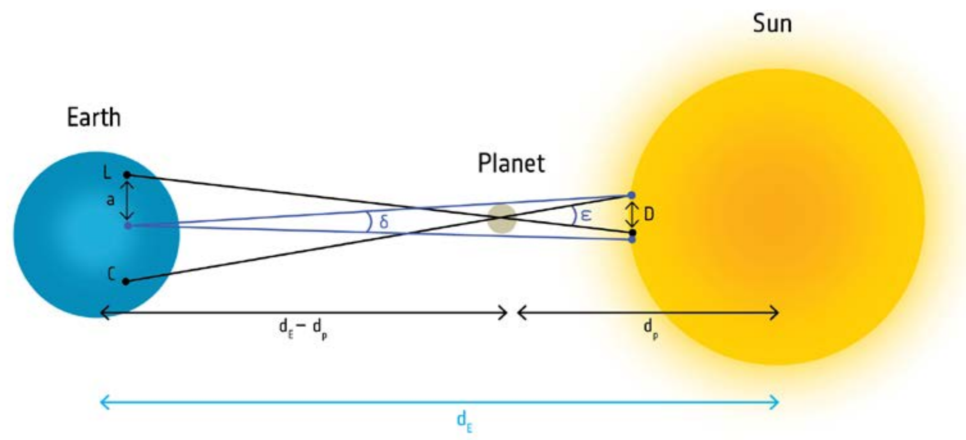

Figure 2: Sketch of two observers L and C observing a planetary transit

Figure 2 shows two triangles, a blue triangle with the angle δ and a black triangle with angle ε, where:

L = observer located in London
C = observer located in Cape Town
a = half the distance between observers
d_p = distance from planet to the Sun
d_E = distance from Earth to the Sun
ε = angular distance between the centre of the planet discs on Sun from the planet centre
δ = angular distance between the centre of planet discs on Sun from the Earth
D = distance between the centre of the planet discs on Sun as seen by each observer

From Kepler's third law of motion, it was known to Halley, that the distance of a planet (Venus or Mercury) to the Sun, d_p, and the distance from the Earth to the Sun, d_E, are related as follows

$$d_p = nd_E$$

Where, for Venus, n = 0.72, and for Mercury n = 0.39

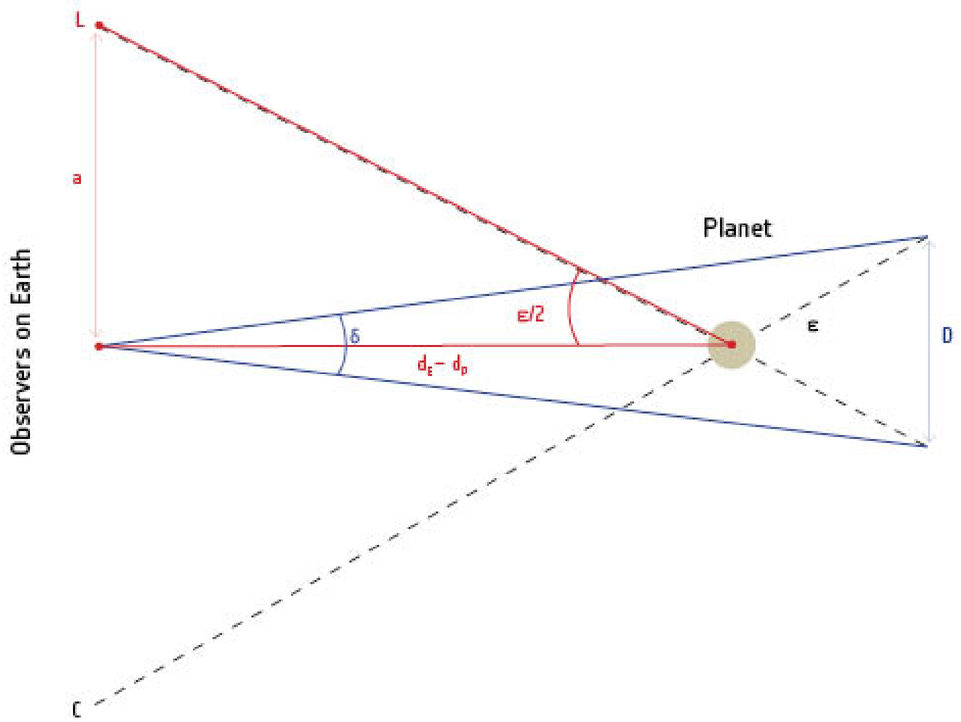

Figure 3: Triangles Earth centre to planets disc and from planet to the discs

The triangles are sketched out again in figure 3. In addition to the blue triangle with angle δ, and the black triangle with angle ε, a red triangle with the angle ε/2 is shown, where:

L = observer located in London
C = observer located in Cape Town
a = half the distance between observers
d_p = distance from planet to the Sun
d_E = distance from Earth to the Sun
ε = angular distance between the centre of the planet discs on Sun from the planet centre
δ = angular distance between the centre of planet discs on Sun from the Earth
D = distance between the centre of the planet discs on Sun as seen by each observer

Knowing angle ε, would allow the distance between the Earth and the planet to be calculated using trigonometry. With, a, being half the distance between the two observers. This distance can be calculated if the precise location of the two observers is known.

$$d_E - d_p = \frac{a}{\tan\frac{\varepsilon}{2}}$$

To calculate the angle ε, an approximation that is valid for small angles is used. The angle between the two discs of the planet, as seen by the two observes, is very small.

Looking back to Figure 2, the two triangles with the enclosing angles ε and δ (the black and the blue triangle) give

$$\tan\frac{\varepsilon}{2} = \frac{D/2}{d_p} = \frac{D/2}{nd_E}$$

and

$$\tan\frac{\delta}{2} = \frac{D/2}{d_E}$$

d_p is the distance from the planet to the Sun, as shown earlier this is nd_E, where d_E, is the distance from the Earth to the Sun.

From Kepler's third law of motion, n = 0.72 for Venus, and n = 0.39 for Mercury. Be aware, that these values are average values, as the distance changes over the planet year! Thus

$$n\tan\frac{\varepsilon}{2} = \tan\frac{\delta}{2}$$

for small angles, this can be approximated to

$$\tan\frac{n\varepsilon}{2} = \tan\frac{\delta}{2}$$

Therefore

$$\varepsilon = \frac{\delta}{n}$$

Angles ε and δ scale down in the same way as the distances from planets to the Sun.

That was the missing piece of the puzzle, as we can measure δ directly from e.g. Figure 4 . It is now possible to calculate the distance from the Earth to the Sun in kilometres.

Method 1: Example calculation using the transit of Venus 2004

Figure 4 gives an example of the Venus transit observed from different locations on Earth in 2004. The angle between the different positions of the discs of Venus is less than an arcminute.

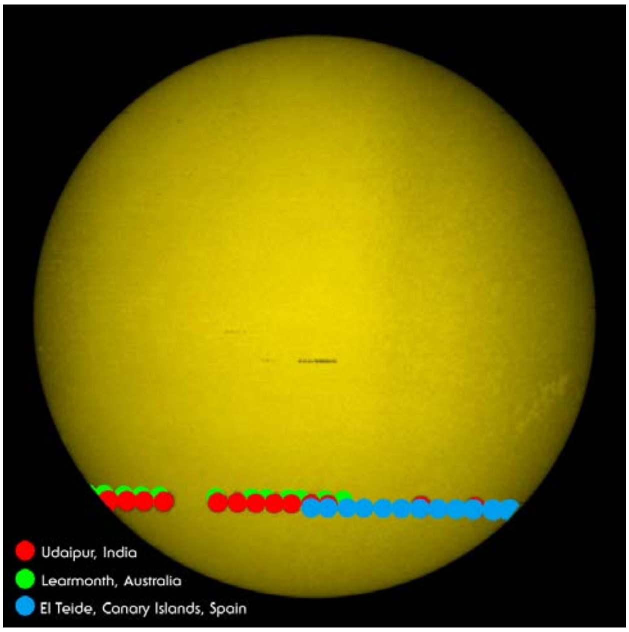

Figure 4: The transit of Venus on 8 June 2004 was recorded by three instruments in the Global Oscillation Network Group (GONG), at Learmonth (Australia), Udaipur (India), and El Teide (Spain)

For the example of the transit of Venus observed from Udaipur and Learmonth, let's apply the mathematics step by step:

Step 1

Let's assume the two coordinates for Udaipur and Learmonth are: (24.58° N, 73.68° E) and (37.25° S, 143.43° E) respectively. The radius of the Earth is 6371km. Assuming the same longitude, the distance between the two observation locations is

$$2a \quad = r_E * \sin 24.58 - \ r_E * \sin -37.25 = 6506.43\ km$$

As the difference in the longitude has not been considered, a more precise calculation, in case you consider two observers at the northern and southern hemisphere:

$$2a = \sqrt[2]{(r_E * sin24.58)^2 + (r_E * sin73.68)^2} + \sqrt[2]{(r_E * sin37.25)^2 + (r_E * sin143.43)^2}$$

$$2a = 12075\ km$$

Step 2

Suppose the two observers have projected their observations on a white piece of paper. When meeting after the transit and overlay the two drawings, they can measure the distance between the centre of the two discs of the planet at a well-defined transit time, for example, the entry.

The angular size of Venus varies between 9.6'' and 63''. During the transit in 2012, the angular size of Venus was 57'', and the measured angular distance (from Figure 4) between the centres of the discs of Venus is about a quarter of it, 57''/4, this means that δ=14.25. The angle ε is obtained as follows

$$\varepsilon = \frac{\delta}{0.7} = 20.36''$$

and further

$$d_E - d_p = \frac{\frac{6521,46}{2}}{\tan\frac{\varepsilon}{2}} = 65\ 925\ 148.69\ km$$

Step 3

As shown earlier,

$$\mathrm{d}_p = \mathrm{n}\mathrm{d}_E$$

with n=0.7 for Venus, we obtain:

$$d_E = \frac{(value\ above\ obtained\ for\ d_E - d_p)}{1-n} = 219\ 750\ 494.62\ \mathrm{km}$$

If you compare this to the current knowledge of 1 AU = 149597870 km , the obtained value is reasonably close.

Method 2: Calculating the distance using contact times

The problem that arose for Edmund Halley was of course that the separation of the discs of Venus on the Sun's surface could not be measured reasonably at his time. However, he realised that the angle ε, could be obtained differently. Halley proposed that each observer measures the transit contact points as accurately as possible. From the contact times, it is possible obtain angle ε, as follows.

The angular velocity of Venus was already known to Halley and contemporaries and is 0.06 degrees per hour or 0.06 arcseconds per second. The angular velocity of Mercury is 0.17 arcseconds per second. This information can be used to calculate the length of the path of a planets transit.

As a planet transits the Sun a possible four contact points are made (see Figure 5), it is from these contact points that the time of the transit can be measured:

1. First contact – as the disc of planet touches the Sun from the outside as it enters transit.
2. Second contact – as the disc of planet touches the Sun from the inside as it enters transit.
3. Third contact – as the disc of planet touches the Sun from the inside as it exits transit.
4. Fourth contact – as the disc of planet touches the Sun from the outside as it exits transit

Two observers measure the transit as shown in Figure 5. The transit time that an observer sees is defined by either the time between the grey planet discs or the red planet discs. D, is the distance between the centre of the planet disc on Sun as seen by each observer. L1, is the length of the transit path from observer on Earth. L2, is the length of the transit from an observer on Earth at another location.

If you make your own observations, it might be difficult to measure the time of the first point of contact exactly, as you see this point of contact only when it happens. Therefore, we propose you take the second point of contact (red disc on left side).

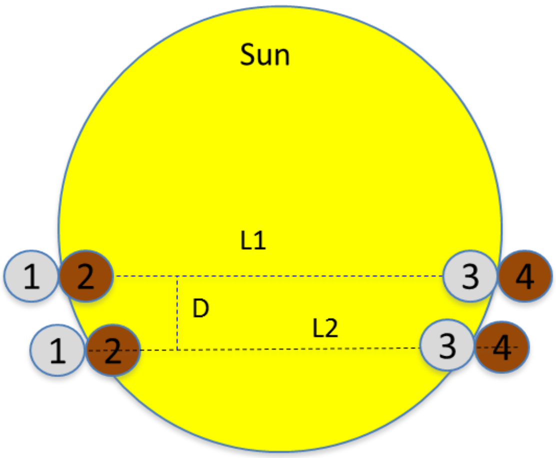

Figure 5: Transit contact points

T1 is the length of time that the planet took to transit the disc of the Sun from your observations. You can then use this, and the angular velocity of Venus to calculate the length of the transit, L1, in seconds from your location.

You will then need to select a second set of measurements observed at another location to work out T2. Repeat the same calculation to obtain the length of the transit, L2.

To work out the angular distance between the disc of the planet, as seen from the two different observing locations you can measure it by drawing the following scaled diagram of the Sun.

Using a pair of compasses, draw a circle on a piece of A4 paper. The circle will represent the Sun, which has an angular diameter of 1900''. This can be scaled so that the diameter of the circle is 19 cm, which just fits onto an A4 page.

Next, you need two set squares.

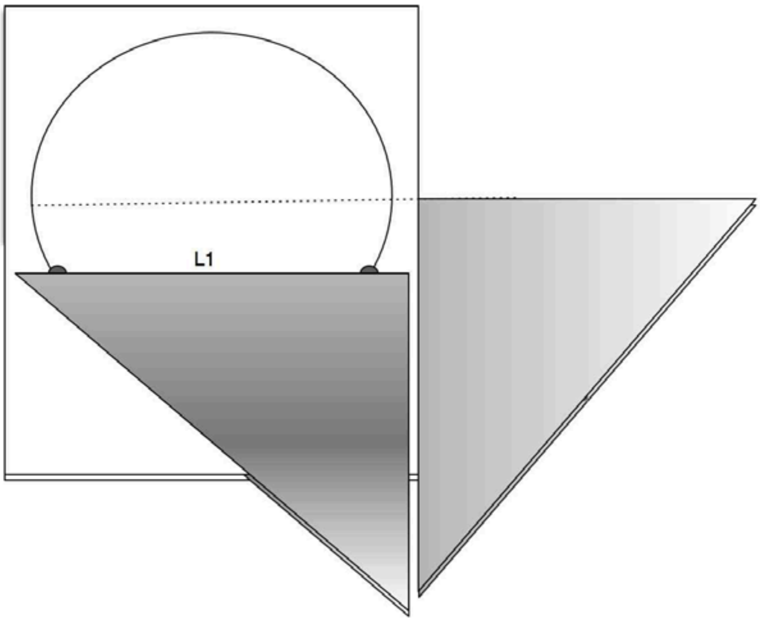

Figure 6: Measuring length of transit

The set square, shown on the right in Figure 6, needs to be held steady so that the top part is an extension of the Sun's equator. This can be done easily using graph paper.

The left side of this set square is used to slide the second protractor along. Do this until you find the line of exactly the length of L1. Draw the line L1 and do the same for the line L2 as shown in Figure 7. Both lines should be parallel to one another.

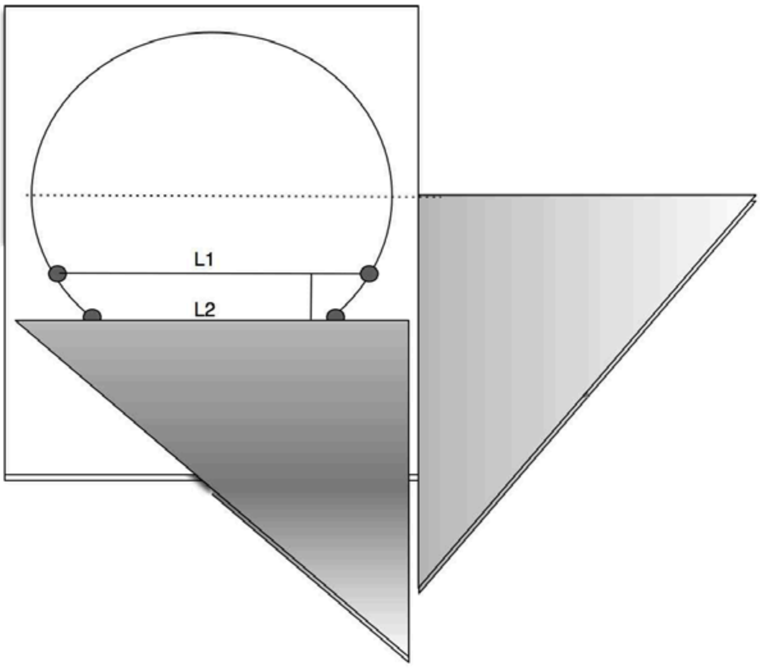

Figure 7: Measuring length of transit

The distance between the two parallel lines is angle ε, in arcseconds.

Once this angle is measured the following steps can be used to calculate the distance between the Earth and the Sun, as shown in Method 1 (see Figures 2 and 3).

$$d_E - d_p = \frac{a}{\tan\frac{\varepsilon}{2}}$$

$$d_E = \frac{(value\ above\ obtained\ for\ d_E -\ d_{p)}}{1 - n}$$

Method 2: Example calculation using the transit of Venus 2004

Following the example of the transit of Venus 2004, as used in Method 1, the transit times from the two locations are: T1 = 20055 seconds, and T2 = 19526 seconds.

Given the Venus angular speed in its orbit as 0.06 arcseonds per second, the length of the transit can be calculated as L1 = 1203.3, and L2 = 1171.56.

Plotted on a circle with diameter of 19cm, the distance between L1 and L2 is 0.22 cm, which is equal to 22'' (Figure 8).

Compared to the calculations earlier on the page, the result is similar: 203 340 555 km – even slightly nearer to the known distance Sun – Earth than the figure calculated in Method 1.

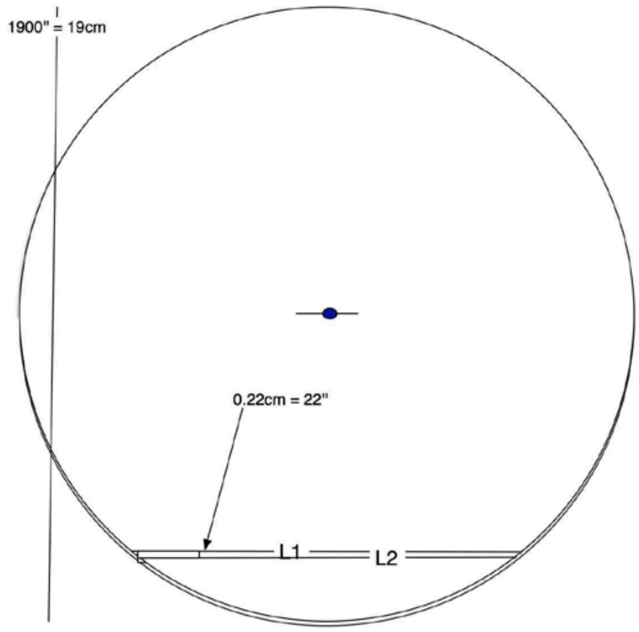

Figure 8: Complete diagram drawn to measure distance between path of transit from two observing locations

If you have a good drawing software, you might consider drawing these diagrams on the computer.

Be aware that because measurements will not be made in a controlled environment results will vary. In addition, there are many other methods of calculating the distance between the Earth and the Sun that give similar or even better results. Check out, for example, the NASA's stargaze side or Prof. Udo Backhaus pages.

If you have any questions or suggestions, please contact us.

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