Quantum Mechanics and the Equivalence Principle

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Einstein's Equivalence Principle (EEP)

- **Universality of Free Fall (UFF):** "Test bodies" determine path structure on spacetime (not necessarily of Riemannian type). UFF-violations are parametrised by the Eötvös factor

$$\eta(A,B) := 2\,\frac{|a(A)-a(B)|}{|a(A)+a(B)|} \approx \sum_\alpha \eta_\alpha \left(\frac{E_\alpha(A)}{m_i(A)c^2} - \frac{E_\alpha(B)}{m_i(B)c^2}\right) \tag{1}$$

- **Local Lorentz Invariance (LLI):** Local non-gravitational experiments exhibit no preferred directions in spacetime, neither timelike nor spacelike. Possible violations of LLI concern, e.g., variations in $\Delta c/c$.
- **Universality of Gravitational Redshift (UGR):** "Standard clocks" are universally affected by the gravitational field. UGR-violations are parametrised by the α-factor

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2} \tag{2}$$

$\Rightarrow$ **Geometrisation of gravity and unification with inertial structure. Far reaching consequences. One of them is: Gravity is not a force!**

Axiomatics of space-time structure

- Characterise spacetime structure by means of **primitives**, like **clocks** and **rods** (complicated), or, alternatively, **light rays** and freely falling **particles**.
- in latter case get (Weyl 1923, Ehlers-Pirani-Schild 1972)

$$\left.\begin{array}{ccc}\text{light-rays} & \Rightarrow & \text{conf. structure}\\ \text{particles} & \Rightarrow & \text{proj. structure}\end{array}\right\} + \text{compatibility} \Rightarrow \text{Weyl} \overset{?}{\Rightarrow} \text{Riemann}$$

- Weyl geometries comprise Riemann geometries, but are more general. They suffice to characterise **standard clocks** (Perlick 1987) and allow for **second clock-effects**.
- Weyl geometries are reduced to Riemann geometries by the requirement that trajectories of the short-wave limit of classical massive fields agree with the geodesics of the Weyl connection (Audretsch 1983, Audretsch-Gähler-Straumann 1984).

UFF, UGR, and energy conservation

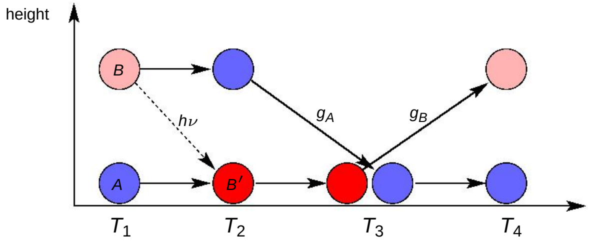
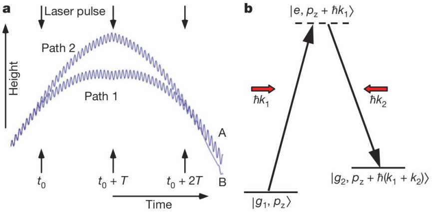


Figure: Gedankenexperiment by NORDTVEDT to show that energy conservation connects violations of UFF and UGR. Considered are two copies of a system that is capable of 3 energy states A, B, and B' (blue, pink, and red), with $E_A < E_B < E_{B'}$. Initially system 2 is in state B and placed a height h above system 1 which is in state A. At time T_1 system 2 makes a transition $B \longrightarrow A$ and sends out a photon of energy $h\nu = E_B - E_A$. At time T_2 system 1 absorbs this photon, which is now blue-shifted, and makes a transition $A \longrightarrow B'$. At T_3 system 2 has been dropped from height h with acceleration g_A, has hit system 1 inelastically, leaving one system in state A and at rest, and the other system in state B with an upward motion with kinetic energy $E_{\mathrm{kin}} = M_A g_A h + (E_{B'} - E_B)$. The latter motion is decelerated by g_B, which may differ from g_A. At T_4 the system in state B has climbed to the **same** height h by energy conservation. Hence have $E_{\mathrm{kin}} = M_B g_B h$ and therefore $M_A g_A h + M_{B'} c^2 = M_B c^2 + M_B g_B h$, from which we get

$$\frac{\delta\nu}{\nu} = \frac{(M_{B'} - M_A) - (M_B - M_A)}{M_B - M_A} = \frac{g_B h}{c^2}\left[1 + \frac{M_A}{M_B - M_A}\,\frac{g_B - g_A}{g_B}\right] \tag{3a}$$

$$\Rightarrow \alpha = \frac{M_A}{M_B - M_A}\,\frac{g_B - g_A}{g_B} =: \frac{\delta g/g}{\delta M/M} \tag{3b}$$

A brief comment on Müller *et al.* (Nature 2010)

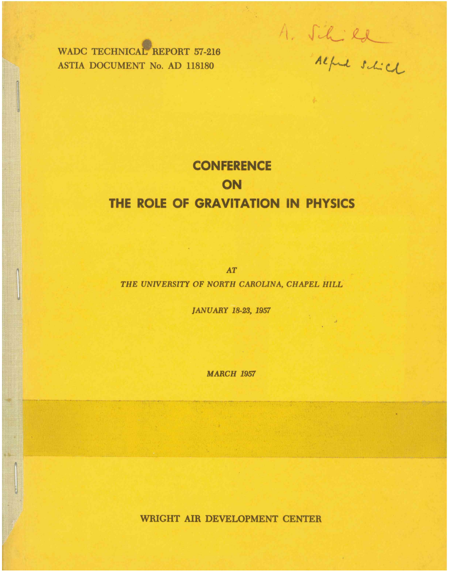


Have

$$\Delta\phi = \kappa T^2 g^{(Cs)} = \kappa T^2 \frac{m_g^{(Cs)}}{m_i^{(Cs)}} g^{(Earth)} = \kappa T^2 \frac{m_g^{(Cs)}}{m_i^{(Cs)}} \frac{m_i^{(Ref)}}{m_g^{(Ref)}} g^{(\mathrm{Ref})} \tag{4}$$

$$= \eta(Cs, Ref)\, \kappa T^2 g^{(\mathrm{Ref})}$$

- Proportional to (1+Eötvös-factor) in *UFF*-violating theories.
- $\mathbb{Q}$ How does it depend on α in *UGR*-violating theories? Müller *et al.* argue for $\propto (1+\alpha)$ by *representation dependent* interpretation of $\Delta\phi$ as a mere redshift.
- Refutation of this interpretation does not answer $\mathbb{Q}$.



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A. Schild
Alfred Schild

CONFERENCE
ON
THE ROLE OF GRAVITATION IN PHYSICS

AT
THE UNIVERSITY OF NORTH CAROLINA, CHAPEL HILL

JANUARY 18-23, 1957

MARCH 1957

WRIGHT AIR DEVELOPMENT CENTER

DE WITT called attention to the fact that there is an ambiguity in the choice of field variables in terms of which one may make an expansion about the Minkowski metric $\eta_{\mu\nu}$. For example, one might use either $\psi_{\mu\nu}$ or $\varphi_{\mu\nu}$ where $g_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu}$, $\mathfrak{g}^{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}$. To be consistent in a self-energy calculation one should expand out to the second order, and the difference in choice leads to a difference in the graviton-vacuum expectation value of the interaction term which is proportional to the trace of the non-gravitational (or matter) stress tensor. There are some fields, notably the electromagnetic field, for which this trace vanishes. In this case, you get the same result in the second order, no matter what you expand in terms of. It is a curious thing in the electromagnetic case (although, as Utiyama has pointed out, you do have the derivative coupling and therefore the divergence is of the second or third order), that if you include second order terms in the interaction, the old-fashioned self-energy will be exactly compensated. In fact, one gets no photon self-interaction up to the second order. MISNER asked if the computation has also been done for the neutrino field. DE WITT replied that although it should be done, it has not been done for two reasons: (1) The mathematics of the spinor problem up to the second order is considerably more complicated and has not yet been fully worked out. (2) Interest has in the meantime shifted to the problem of tackling the complete nonlinear gravitational field.

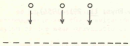
WHEELER pointed out that the linear starting point is incompatible with any topology other than a Euclidean one; that if one has curved space or "worm holes," you just can't start expanding this way. BELINFANTE asked if anyone had found it possible to have hole theory in a curved space; i.e., can one make a covariant distinction between positive and negative energy states?

ROSENFELD said that Dirac, just after he had formulated hole theory and was faced with the objection that there is an infinite energy associated with these states, had attempted unsuccessfully to overcome this difficulty by introducing a closed universe.

MISNER said that one could get a quite good qualitative idea of what happens in curved space (the metric being externally impressed) by using the Feynman prescription. Since the action is still quadratic in the interesting field variables, there is no difficulty. In a spherical space there exist states of excitation which do not scatter each other; but as soon as the space has "bumps" in it, photons in one state get scattered into other states. Hole theory is possible if the metric is static; however, a time dependent metric causes electrons to go to positive energy states.

BERGMANN pointed out that, on this account, one does not have to exclude hole theory, because if the electrons get excited, there is occasional pair production.

SALECKER introduced a thought experiment, involving a stream of particles falling on a diffraction grating. On account of the de Broglie relation for the waves associated with the stream, $\lambda = \frac{h}{mv}$, one expects that particles of different mass will scatter differently if they fall from a given height. According to general relativity, one expects the same behavior for different masses with the same initial state of motion. Therefore, we arrive at a contradiction with the principle of equivalence.



FEYNMAN asked if the grating is here allowed to exert forces on the particles which are non-gravitational. DE WITT said that one needs rather a grating (made, for example, out of planets) which acts only through its gravitational field on the stream. FEYNMAN then said that he did not believe that the principle of equivalence denies the possibility of distinguishing between two different masses. Of course, the principle of equivalence would prevent one from distinguishing between masses by means of this particular experiment if only classical laws were operative. However, the introduction of Planck's constant into the scheme of things introduces new possibilities, which are not necessarily in contradiction with the principle of equivalence. As far as this particular experiment is concerned, all that the principle of equivalence would say would be that if one performs the experiment in an elevator, he will obtain the same result as in a corresponding gravitational field. FEYNMAN also emphasized that the quantities G and c by themselves do not lead to a unit of mass, whereas such a unit exists if $\hbar$ is included.

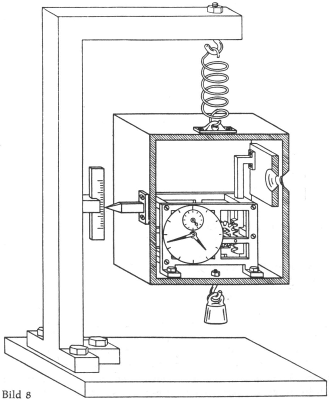
WHEELER pointed out that the principle of equivalence only denies the possibility of distinguishing between the gravitational and inertial masses of a single body, but definitely does not prevent one from distinguishing the masses of two different bodies, even when only gravitational forces are involved. For example, we know the relative sizes of the masses of the sun and the various planets solely from observation of their gravitational interactions. BERGMANN added that the principle of equivalence makes a statement about local conditions only. Therefore you can do one of two things: either (1) use a small diffraction grating that is not gravitational, or (2) use a diffraction grating made of planets. In this case, the conditions are certainly not local.

FEYNMAN characterized the point which Salecker had raised as an interesting point and a true point, but not necessarily a paradoxical one. If the falling particles are not allowed to react back on the grating, then according to the classical theory they will all follow the same paths. Whereas, in the quantum theory they will give rise to different diffraction patterns depending on their masses.

SALECKER then raised again the question why the gravitational field needs to be quantized at all. In his opinion, charged quantized particles already serve as sources for a Coulomb field which is not quantized. [Editor's Note: Salecker did not make completely clear what he meant by this. If he meant that some forces could be represented by actions-at-a-distance, then, although he was misunderstood, he was right. For the corresponding field can then be eliminated from the theory and hence remain unquantized. He may have meant to imply that one should try to build up a completely action-at-a-distance theory of gravitation, modified by the relativistic necessities of using both advanced and retarded interactions and imbedded in an "absorber theory of radiation" to preserve causality. In this case, gravitation per se could remain unquantized. However, these questions were not discussed until later in the session.]

BELINFANTE insisted that the Coulomb field is quantized through the ψ-field. He then repeated DeWitt's argument that it is not logical to allow an "expectation value" to serve as the source of the gravitational field. There are two quantities which are involved in the description of any quantized physical system. One of them gives information about the general dynamical behavior of the system, and is represented by a certain operator (or operators). The other gives information about our

Worries & hopes: QM needs UGR



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- Einstein argues to be able to violate $\Delta E \Delta T > \hbar$.
- Bohr argues that inequality holds due to UGR:

$$\text{QM:} \quad \Delta q > \frac{\hbar}{\Delta p} > \frac{\hbar}{T g \Delta m}$$

$$\text{ART:} \quad \Delta T = \frac{gT}{c^2}\,\Delta q$$

$$\Longrightarrow \quad \Delta T > \frac{\hbar}{\Delta m\, c^2} = \frac{\hbar}{\Delta E}$$

- Bohr's argument can be (and has been) criticised on various accounts, but its underlying logic (QT needs GR for consistency) seems truly remarkable.

Worries & hopes: QFT needs UGR - 1

- Consider thin mass shell of Radius R, inertial rest-mass M_0, gravitational mass M_g, and electric charge Q. Its total energy is

$$E = M_0 c^2 + \frac{Q^2}{2R} - G\frac{M_g^2}{2R} \tag{5}$$

- Now use the following two principles:

$$\begin{aligned} E &= M_i c^2 \\ M_g &= M_i \end{aligned} \tag{6}$$

- Get quadratic equation for mass $M := M_i = M_g$:

$$\Rightarrow \quad M := \frac{E}{c^2} = M_0 + \frac{Q^2}{2c^2R} - G\frac{M^2}{2c^2R} \tag{7}$$

Worries & hopes: QFT needs UGR - 2

- The solution is

$$M(R) = \frac{Rc^2}{G}\left\{-1+\sqrt{1+\frac{2G}{Rc^2}\left(M_0+\frac{Q^2}{2c^2R}\right)}\right\} \tag{8}$$

- Its $R \to 0$ limit exits

$$\lim_{R\to O} M(R) = \sqrt{\frac{2Q^2}{G}} = \sqrt{2\alpha}\cdot\frac{|Q|}{e}\cdot M_{\mathrm{Planck}} \tag{9}$$

but its small-G approximation is not uniform in R at $R = 0$:

$$\begin{aligned} M &= \left(m_0+\frac{Q^2}{2c^2R}\right) \\ &+ \sum_{n=1}^{\infty}\frac{(2n-1)!!}{(n+1)!}\cdot\left(-\frac{G}{Rc^2}\right)^n\cdot\left(m_0+\frac{Q^2}{2c^2R}\right)^{n+1} \end{aligned} \tag{10}$$

The gravitational "H-atom"

- Centrifugal force equals gravitational attraction

$$m_i\omega^2 r = G\frac{m_g M_g}{r^2}\,. \tag{11}$$

- Angular momentum ($\propto m_i$) is quantised

$$m_i\omega r^2 = n\hbar \tag{12}$$

- Bohr radii and frequencies

$$r(n) = \left(\frac{1}{m_i m_g}\right)\cdot\frac{n^2\hbar^2}{GM_g}\,, \qquad \omega(n) = (m_i m_g^2)\cdot\frac{G^2M_g^2}{n^3\hbar^3}\,, \tag{13}$$

and energies

$$E(n) = \tfrac{1}{2}m_i\omega^2(n)r^2(n) - \frac{G\,m_g M_g}{r(n)} = -(m_i m_g^2)\cdot\frac{G^2M_g^2}{2\,n^2\hbar^2}\,. \tag{14}$$

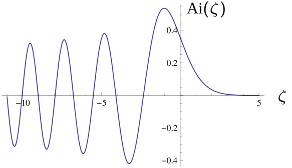
Homogeneous static gravitational field

- Time independent Schrödinger equation in linear potential $V(z) = m_g gz$ is equivalent to:

$$\left(\frac{d^2}{d\zeta^2} - \zeta\right)\psi = 0\,, \quad \zeta := \kappa z - \varepsilon \tag{15}$$

where

$$\kappa := \left[\frac{2m_i\, m_g\, g}{\hbar^2}\right]^{\frac{1}{3}}\,, \quad \varepsilon := E\cdot\left[\frac{2m_i}{m_g^2\, g^2\, \hbar^2}\right]^{\frac{1}{3}}\,. \tag{16}$$



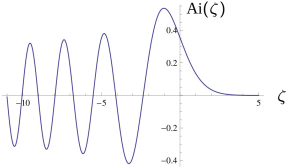
- Complement by hard (horizontal) wall $V(z) = \infty$ for $z \leq 0$ get energy eigenstates from boundary condition $\psi(z=0) = 0$, hence $\varepsilon = -z_n$ (Abele *et al.* 2002, Kajari *et al.* 2010, ...):

$$E(n) = -z_n \left[\frac{m_g^2}{m_i}\cdot\frac{g^2\hbar^2}{2}\right]^{\frac{1}{3}}\,. \tag{17}$$

Homogeneous static gravitational field

- Classical turning point z_{turn}

$$m_g g\, z_{\text{turn}} = E \;\Leftrightarrow\; z_{\text{turn}} = \frac{E}{m_g g} = \frac{\varepsilon}{\kappa} \;\Leftrightarrow\; \zeta = 0\,. \tag{18}$$



- Large $(-\zeta)$ - expansion of Airy function gives decomposition of ingoing and outgoing waves with phase delay of

$$\Delta\theta(z) = \frac{4}{3}\Big[\kappa\big(E/m_g g - z\big)\Big]^{3/2} - \pi/2 \tag{19}$$

corresponding to a "Peres time of flight" (Davies 2004)

$$T(z) := \hbar\frac{\partial\Delta\theta}{\partial E} = 2\,\frac{\hbar\kappa^{\frac{3}{2}}}{m_g g}\sqrt{z_{\text{turn}} - z} = 2\sqrt{\frac{m_i}{m_g}}\cdot\sqrt{2\cdot\frac{z_{\text{turn}} - z}{g}} \tag{20}$$

- For other than linear potential we will *not* get *classical* return time.

Proposition: One-particle Schrödinger wave in homogeneous force-field

ψ solves the Schrödinger Equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m_i}\Delta - \vec{F}(t)\cdot\vec{x}\right)\psi \tag{21}$$

iff

$$\psi = \bigl(\exp(i\alpha)\,\psi'\bigr)\circ\Phi^{-1}\,, \tag{22}$$

where ψ' solves the free Schrödinger equation (i.e. without potential).

$\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ is the following spacetime diffeomorphism (preserving time)

$$\Phi(t,\vec{x}) = \bigl(t, \vec{x} + \xi(t)\bigr)\,. \tag{23}$$

ξ is a solution to

$$\ddot{\vec{\xi}}(t) = \vec{F}(t)/m_i \tag{24}$$

with $\vec{\xi}(0) = \vec{0}$ and $\alpha : \mathbb{R}^4 \to \mathbb{R}$ is given by

$$\alpha(t,\vec{x}) = \frac{m_i}{\hbar}\left\{\dot{\vec{\xi}}(t)\cdot\bigl(\vec{x}+\vec{\xi}(t)\bigr) - \frac{1}{2}\int^t dt'\,\|\dot{\vec{\xi}}(t')\|^2\right\}\,. \tag{25}$$

Schrödinger-Newton equation

- Consider Einstein – Klein-Gordon system

$$R_{ab} - \tfrac{1}{2} g_{ab} R = \tfrac{8\pi G}{c^4}\, T_{ab}^{KG}(\phi)\,, \quad (\Box_g + m^2)\phi = 0 \tag{26}$$

- Make WKB-like ansatz

$$\phi(\vec{x},t) = \exp\left(\frac{ic^2}{\hbar} S(\vec{x},t)\right) \sum_{n=0}^{\infty} \left(\frac{\sqrt{\hbar}}{c}\right)^n a_n(\vec{x},t), \tag{27}$$

  and perform $1/c$ expansion (D.G. & A. Großardt 2012).
- Obtain

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta + mV\right)\psi \tag{28}$$

  where

$$\Delta V = 4\pi G\left(\rho + m|\psi|^2\right). \tag{29}$$

- Ignoring self-coupling, this just generalises previous results and conforms with expectations.

Schrödinger-Newton equation

- Without external sources get **"Schrödinger-Newton equation"** (Diosi 1984, Penrose 1998):

$$\boxed{i\hbar\,\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\Delta - Gm^2\int\frac{|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|}\,d^3y\right)\psi(t,\vec{x})} \qquad (30)$$

- The SNE has the same symmetries as ordinary Schrödinger equation: Full inhomogeneous Galilei group, including parity and time reversal, and global $U(1)$ phase transformations.

- Also it has the following scaling covariance: Let

$$S_\lambda[\psi](t,\vec{x}) := \lambda^{9/2}\psi(\lambda^5 t\,,\,\lambda^3\vec{x})\,, \qquad (31)$$

then $S_\lambda[\psi]$ satisfies the SNE for mass parameter λm iff ψ satisfies SNE for mass parameter m

The time-dependent SN-Equation

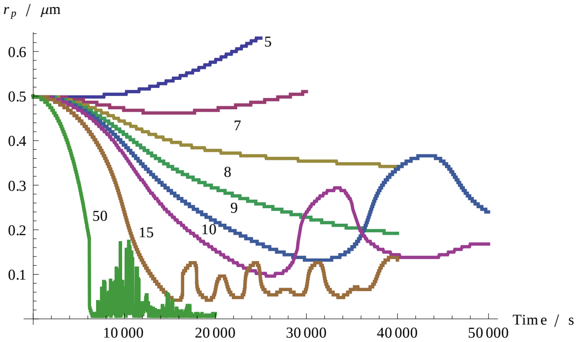


Figure: Time evolution of peak of radial probability density for increasing masses. First bounces back from minimal contraction are seen within shown interval of time above masses of 9×10^9 u. (D.G. and A. Großardt 2011)

Summary

- QM does not contradict EEP, but rather has the potential to give rise to more accurate tests of it.
- However, so-called "Quantum tests of *the* equivalence principle" need to properly state the form this principle takes in the quantum regime.
- The intended outcome of any formulation of EEP is that the interaction between matter and gravitation can be fully described by geometric structures on spacetime together with a universal coupling scheme which are *common to all forms of matter*.
- All gravitational couplings of quantum matter investigated in experiments so far concern *external* gravitational fields (Earth, Sun, Moon, Galaxy).
- Within the range of applicability of the semi-classical Einstein Equations the gravitational self-interaction is expected to give rise to effective non-linear quantum-evolutions. These can alter the c.o.m - evolution in the sense of inhibitions of dispersion for certain mass ranges and widths,
- For masses above $6.5 \times 10^9\,\mathrm{u}$ and width around $500\,\mathrm{nm}$, collapse times are still of the orders of hours. Due to scaling law, tenfold mass and 10^{-3} width results in 10^{-5} collapse time.
- All this *ignores* the possible quantum nature of the gravitational field.

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