Precision Measurement of planetary gravitomagnetic field in general relativity and laser interferometry in space

Yun Kau Lau (刘润球)
Institute of Appl. Maths and Morningside Center of Maths,
Chinese Academy of Sciences

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The CAS Gravity Consortium

**Coordinators:**
- National Microgravity Laboratory, Institute of Mechanics (Wenrui Hu)
- University of Chinese Academy of Sciences (Yueliang Wu)

**Member Institutes:**
- Academy of Mathematics and Systems Science, CAS,
- Huazhong University of Science and Technology,
- Institute of High Energy Physics, Beijing,
- Institute of Physics, Beijing,
- Institute of Theoretical Physics, Beijing,
- Nanjing Institute of Astronomical Optics and Technology,
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Scientific Background

- Mercury perihelion
- Shapiro time delay
- Precision measurement of PPN parameters in solar system
- Tests of equivalence principle
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Outstanding tests in 21st century
- Gravitational wave detection --- galactic and cosmological scale (eLISA, CAS project)
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Outstanding tests in 21st century

- Gravitational wave detection --- galactic and cosmological scale (eLISA, CAS project).
- Gravitomagnetic field detection (e.g. Lens Thirring effect)--- test on the planetary and solar system scale.
General Relativity — a generalisation of Newtonian theory of gravity

classical mechanics

general relativity

geometry of spacetime

\[ \approx \text{universal gravitation} \]

Einstein field equations

\[ R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} \]
Gravitomagnetic field in general relativity

\[ \Phi \sim \frac{GM}{r}, \quad A \sim \frac{G}{c} \frac{\mathbf{J} \times \mathbf{r}}{r^3}, \quad E = -\nabla \Phi - \frac{1}{c} \frac{\partial}{\partial t} \left( \frac{1}{2} \mathbf{A} \right), \quad B = \nabla \times \mathbf{A}, \]

Newtonian gravity field

Gravitomagnetic field predicted by Einstein for a spinning body

Einstein field equations \( \approx \) Maxwell equation \( (1PN) \)
GPB experiment

Gravity Probe B — Final Experimental Results

<table>
<thead>
<tr>
<th>Gyroscope</th>
<th>$\gamma_{\text{G}}$ (Geodetic Measurement)</th>
<th>$\gamma_{\text{E}}$ (Frame-Dragging Measurement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyroscope #1</td>
<td>$-6.586.6 \pm 31.7$ mas/yr</td>
<td>$-41.3 \pm 24.6$ mas/yr</td>
</tr>
<tr>
<td>Gyroscope #2</td>
<td>$-6.707.0 \pm 44.1$ mas/yr</td>
<td>$-16.1 \pm 29.7$ mas/yr</td>
</tr>
<tr>
<td>Gyroscope #3</td>
<td>$-6.610.5 \pm 43.2$ mas/yr</td>
<td>$-25.0 \pm 12.1$ mas/yr</td>
</tr>
<tr>
<td>Gyroscope #4</td>
<td>$-6.586.7 \pm 33.2$ mas/yr</td>
<td>$-49.3 \pm 11.4$ mas/yr</td>
</tr>
</tbody>
</table>

Weighted-Average Results for All Four Gyrosopes

| All Gyrosopes | $-6.601.8 \pm 16.3$ mas/yr | $-37.2 \pm 7.2$ mas/yr | $\sim 19\%$ error |

Schiff-Einstein Predicted Theoretical Values

| Theoretical Gyroscope | $-6.606.1$ mas/yr | $-39.2$ mas/yr |
The LAGEOS I and II and LARES Missions

Measurement in terms of Keplerian elements of spacecrafts

Highly depend on the detailed knowledge of Earth gravity field.

The Gravitomagnetic effect is now confirmed with 10% accuracy, subject to earth gravity field modelling + non-gravitational force error.

Other controversial claims of detection: Lunar Laser ranging, Mars orbiter....
Why the interest?

- Poorly tested, still remain major challenge in experimental relativity
- Impose constraints in post newtonian limit of geometric gravity theories. Provide stringent tests on low energy effective theory coming from string theory and loop quantum gravity.
- Applications in future space science such as ClockSync in space and etc....
### The scientific objectives of the proposed mission

- Direct, precision measurement of earth's gravitomagnetic field predicted by Einstein's theory of general relativity to unprecedented accuracy better than 0.1%.

- Improve the accuracy in the measurement of some post-Newtonian parameters in our solar system.

- New tests and constraints on alternative theories of gravity, low energy effective theory related to string theory and quantum gravity.

- Track global climate through the temporal variation of the Earth gravity field with geoid accuracy better than 1 cm and order of earth multipoles up to order 120........
Gradiometric measurement of gravitomagnetic field

Braginsky, Polnarev (1981), Mashhoon, Paik, Will (1989) and others

3 axis gradiometer

\[
\begin{bmatrix}
\frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} \\
\frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial y \partial z} \\
\frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z^2}
\end{bmatrix}
\]

force gradient tensor

Geodesic deviation (Jacobi) equation

\[
\frac{D^2 X^a}{D\tau^2} = -R_{bcd}^a T^b T^d X^a
\]

Analytic solution of the geodesic deviation equation at the 1PN level

\[
x(\tau) \approx \left(1 + \gamma + \frac{1}{4} \alpha_1 \right) J d \sin I \sin(\omega \tau) \frac{1}{r^3} \tau,
\]

\[
y(\tau), \ z(\tau) \ll x(\tau)
\]
Lorentz force of gravitomagnetic field

\[ \vec{F} = 2m\vec{v} \times \vec{B}_g \]

\[ |\vec{F}| = 2m|\vec{v}||\vec{B}_g| \cos \Psi \]

\[ |\vec{F}_1 - \vec{F}_2| \sim 2m|\vec{v}||\vec{B}_g|(\sin \Psi)\Delta \]
Laser interferometric readout
Fabry-Perot cavity design
The Mission Concept

- Near polar orbits.
  To increase the GM signal while suppress the noises from gravitoelectric field.

- Altitude options: 3000km to 6000km (even to 10000km).
  Higher altitude will suppress the noises from higher order gravity multipoles.

- Eccentric orbits.
  Distinguish gravitomagnetic field signal from $J_2$ signal of earth gravity.
## Precision measurement of PPN parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect</th>
<th>Limit</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma - 1$</td>
<td>(i) Time delay</td>
<td>$2.3 \times 10^{-5}$</td>
<td>Cassini tracking</td>
</tr>
<tr>
<td></td>
<td>(ii) Light deflection</td>
<td>$4 \times 10^{-4}$</td>
<td>VLBI</td>
</tr>
<tr>
<td>$\beta - 1$</td>
<td>(i) Perihelion shift</td>
<td>$3 \times 10^{-3}$</td>
<td>$J_2 = 10^{-7}$ from helioseismology</td>
</tr>
<tr>
<td></td>
<td>(ii) Nordtvedt effect</td>
<td>$2.3 \times 10^{-4}$</td>
<td>$\eta = 4\beta - \gamma - 3$ assumed</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Earth tides</td>
<td>$10^{-3}$</td>
<td>Gravimeter data</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Orbital polarization</td>
<td>$10^{-4}$</td>
<td>Lunar laser ranging</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Solar spin precession</td>
<td>$4 \times 10^{-7}$</td>
<td>Alignment of Sun and ecliptic</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>Pulsar acceleration</td>
<td>$2 \times 10^{-20}$</td>
<td>Pulsar $\dot{P}$ statistics</td>
</tr>
<tr>
<td>$\eta^a$</td>
<td>Nordtvedt effect</td>
<td>$9 \times 10^{-4}$</td>
<td>Lunar laser ranging</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>–</td>
<td>$2 \times 10^{-2}$</td>
<td>Combined PPN bounds</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Binary motion</td>
<td>$4 \times 10^{-5}$</td>
<td>$\ddot{P}$ for PSR 1913 + 16</td>
</tr>
<tr>
<td>$\xi_3$</td>
<td>Newton’s 3rd law</td>
<td>$10^{-8}$</td>
<td>Lunar acceleration</td>
</tr>
<tr>
<td>$\xi_4$</td>
<td>–</td>
<td>–</td>
<td>Not independent</td>
</tr>
</tbody>
</table>

(Will, Theory and experiments in general relativity)

- Improve the accuracy in the measurement of $\alpha_1$ to $10^{-5}$!
- $\alpha_1$ --- a measure of local Lorentz invariance of gravity theories.
- A test of quantum gravity violation of Lorentz invariance!
Chern-Simons modified gravity serves as a representative phenomenological model predicted by string theory and loop quantum gravity

\[ S_{CS} = \frac{1}{16\pi G} \int d^4x \frac{1}{4} f R^* R, \]

A characteristic and rather large signal in the in-line direction in our experiment.

\[ - dJ \chi \sin I(\tau \omega \sin(\tau \omega) + \cos(\tau \omega)) \]
\[ \frac{2r^3\omega}{} \]

The dimensionless parameter \( \chi = 2 \frac{j}{r} \) describes the magnitude of the Chern-Simons action, which can be precisely measured or constrained to \( 10^{-15} \)!
# Bases for Joint Mission

<table>
<thead>
<tr>
<th>eLISA consortium</th>
<th>CAS gravity consortium</th>
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<tbody>
<tr>
<td>- Accelerometer</td>
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<tr>
<td>micro-newton ion thrusters</td>
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</tr>
<tr>
<td>(under development)</td>
<td>(just started)</td>
</tr>
<tr>
<td>- Optics in space</td>
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</tr>
<tr>
<td>(development completed for LPF)</td>
<td>laser interferometry (100pm/√Hz),</td>
</tr>
<tr>
<td>- Charge management (UV LED)</td>
<td>phase meter (in progress)</td>
</tr>
<tr>
<td>(not started yet)</td>
<td>Charge management (UV LED)</td>
</tr>
<tr>
<td>- satellite platform</td>
<td>(not started yet)</td>
</tr>
<tr>
<td>- launcher</td>
<td>- satellite platform</td>
</tr>
<tr>
<td></td>
<td>- launcher</td>
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Thanks!!