THE ESA/ESO ASTRONOMY EXERCISE SERIES

Student exercises in astronomy using observations from the NASA/ESA Hubble Space Telescope and the ESO telescopes

Toolkits

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$\omega = \frac{(F-1)d}{}$

Magnitudes: a concept first developed in 120 BC

When we look at the sky on a clear night we see stars. Some appear bright and others very faint as seen from Earth. Some of the faint stars are intrinsically very bright, but are very distant. Some of the brightest stars in the sky are very faint stars that just happen to lie very close to us. When observing, we are forced to stay on Earth or nearby and can only measure the intensity of the light that reaches us.
Unfortunately this does not immediately tell us anything about a star's *internal properties*. If we want to know more about a star, its size or physical/internal brightness, for example, we need to know its *distance* from Earth.

Historically, the stars visible to the naked eye were put into six different brightness classes, called magnitudes. This system was originally devised by the Greek astronomer Hipparchus about 120 BC and is still in use today in a slightly revised form. Hipparchus chose to categorise the brightest stars as magnitude 1, and the faintest as magnitude 6.

Astronomy has changed a lot since Hipparchus lived! Instead of using only the naked eye, light is now collected by large mirrors in either ground-based telescopes such as the VLT in the Atacama Desert in Chile or the Hubble Space Telescope above the Earth's atmosphere. The collected light is then analysed by instruments able to detect objects billions of times fainter than any human eye can see.

However, even today astronomers still use a slightly revised form of Hipparchus' magnitude scheme called *apparent magnitudes*. The modern definition of magnitudes was chosen so that the magnitude measurements already in use did not have to be changed.
Astronomers use two different types of magnitudes: *apparent magnitudes* and *absolute magnitudes*.

Apparent magnitude

The apparent magnitude, m, of a star is a measure of how bright a star appears as observed on or near Earth.
Instead of defining the apparent magnitude from the number of light photons we observe, it is defined relative to the magnitude and intensity of a reference star. This means that an astronomer can measure the magnitudes of stars by comparing the measurements with some standard stars that have already been measured in an absolute (as opposed to relative) way.

The apparent magnitude, m, is given by:

$m = m_{ref} - 2.5 \log_{10} (I/I_{ref})$

where mref is the apparent magnitude of the

Figure 1: Hipparchus of Nicaea (c.190 – c.120 BC) at work
Hipparchus, a Greek astronomer, invented the first scale to rate the brightness of the stars.

reference star, I is the measured intensity of the light from the star, and I_{ref} is the intensity of the light from the reference star. The scale factor 2.5 brings the modern definition into line with the older, more subjective apparent magnitudes.

It is interesting to note that the scale that Hipparchus selected on an intuitive basis, using just the naked eye, is already logarithmic as a result of the way our eyes respond to light.

For comparison, the apparent magnitude of the full Moon is about –12.7, the magnitude of Venus can be as high as –4 and the Sun has a magnitude of about –26.5.

Absolute magnitude

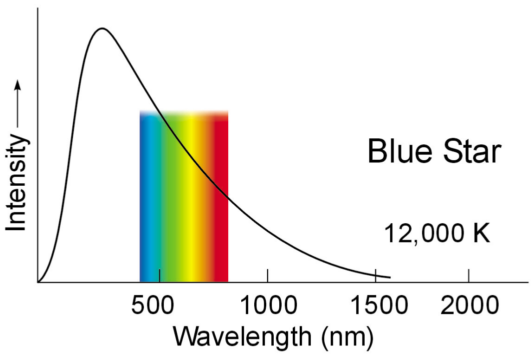
We now have a proper definition for the apparent magnitude. It is a useful tool for astronomers, but does not tell us anything about the intrinsic properties of a star. We need to establish a common property that we can use to compare different stars and use in statistical analysis. This property is the absolute magnitude.

The absolute magnitude, M, of a star is defined as the relative magnitude a star would have if it were placed 10 parsecs (read about parsecs in the Mathematical Toolkit if needed) from the Sun.

Since only a very few stars are exactly 10 parsecs away, we can use an equation that will allow us to calculate the absolute magnitude for stars at different distances: the distance equation. The equation naturally also works the other way — given the absolute magnitude the distance can be calculated.

Different colours, different magnitudes

By the late 19th century, when astronomers were using photographs to record the sky and to measure the apparent magnitudes of stars, a new problem arose. Some stars that appeared to have the same brightness when observed with the naked eye appeared to have different brightnesses on film, and vice versa. Compared to the eye, the photographic emulsions used were more sensitive to blue light and less so to red light.
Accordingly, two separate scales were devised: *visual magnitude*, or m_{vis}, describing how a star looked to the eye and *photographic magnitude*, or m_{phot}, referring to measurements made with blue-sensitive black-and-white film. These are now abbreviated to m_v and m_p.
However, different types of photographic emulsions differ in their sensitivity to different colours. And people's eyes differ too! Magnitude systems designed for different wavelength ranges had to be more firmly calibrated.



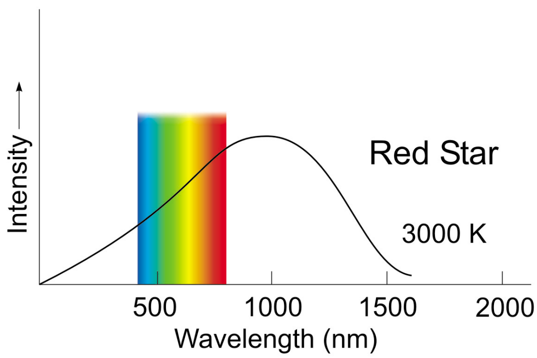


Figure 2: Temperature and colour of stars
This schematic diagram shows the relationship between the colour of a star and its surface temperature. Intensity is plotted against wavelength for two hypothetical stars. The visible part of the spectrum is indicated. The star's colour is determined by where in the visible part of the spectrum, the peak of the intensity curve lies.

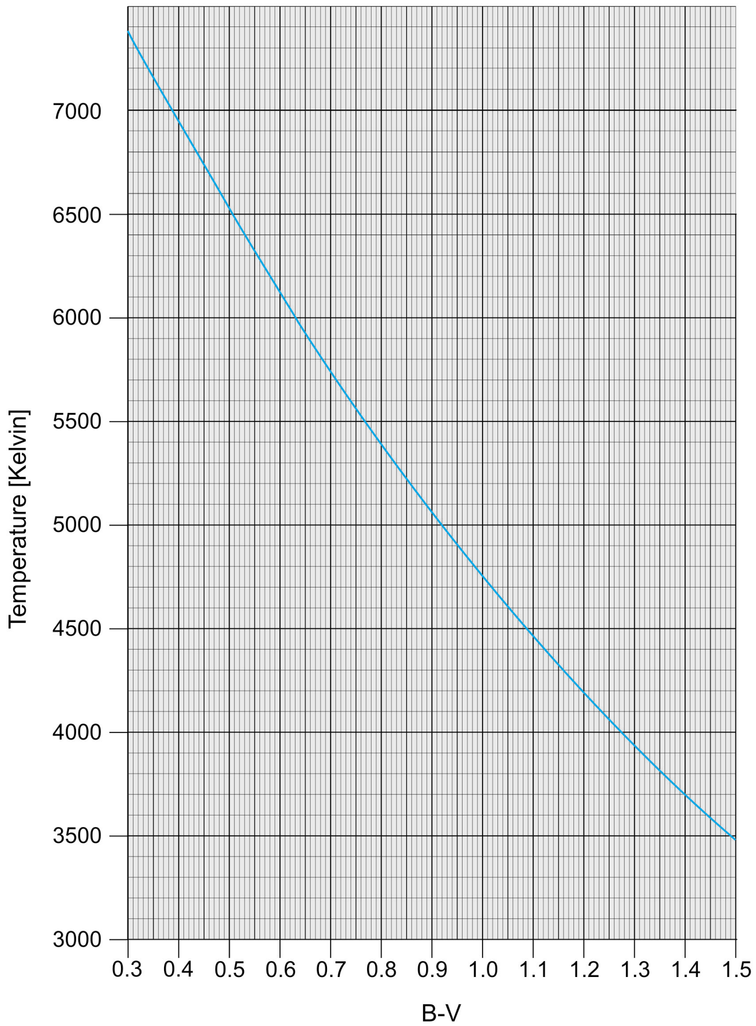


Figure 3: Surface temperature versus B-V colour index
This diagram shows the relation between the surface temperature of a star, T, and its B-V colour index. Knowing either the surface temperature or the B-V colour index you can find the other value from this diagram.

Today, precise magnitudes are specified by measurements from a standard photoelectric photometer through standard colour filters. Several photometric systems have been devised; the most familiar is called UBV after the three filters most commonly used. The U filter lets mostly near-ultraviolet light through, B mainly blue light, and V corresponds fairly closely to the old visual magnitude; its wide peak is in the yellow-green band, where the eye is most sensitive. The corresponding magnitudes in this system are called m_U, m_B and m_V.

From B-V colour index to temperature

The term *B-V colour index* (nicknamed *B–V* by astronomers) is defined as the difference in the two magnitudes, $m_B - m_V$ (as measured in the UBV system). A pure white star has a B–V colour index of about 0.2, our yellow Sun of 0.63, the orange-red Betelgeuse of 1.85 and the bluest star possible is believed to have a B–V colour index of –0.4. One way of thinking about colour index is that the bluer a star is, the more negative its B magnitude and therefore the lower the difference $m_B - m_V$ will be.

There is a clear relation between the surface temperature T of a star and its B-V colour index (see Reed, C., 1998, Journal of the Royal Society of Canada, 92, 36–37) so we can find the surface temperature of the star by using a diagram of T versus $m_B - m_V$ (see Fig. 3).

$$\log_{10}(T) = (14.551 - (m_B - m_V))/3.684$$

The distance equation

The distance equation is written as:

$$m - M = 5 \log_{10}(D/10 \text{ pc}) = 5 \log_{10}(D) - 5$$

This equation establishes the connection between the apparent magnitude, m, the absolute magnitude, M, and the distance, D, measured in parsec. The value m-M is known as the *distance modulus* and can be used to determine the distance to an object.

A little algebra will transform this equation to an equivalent form that is sometimes more convenient (feel free to test this yourselves):

$$D = 10^{(m-M+5)/5}$$

When determining distances to objects in the Universe we measure the apparent magnitude m first. Then, if we also know the intrinsic brightness of an object (its absolute magnitude M), we can calculate its distance D. Much of the hardest work in finding astronomical distances is concerned with determining the absolute magnitudes of certain types of astronomical objects. Absolute magnitudes have for instance been measured by ESA's HIPPARCOS satellite. HIPPARCOS is a satellite that, among many other things, measured accurate distances and apparent magnitudes of a large number of nearby stars.

Short training tasks

These short tasks should familiarise you with the different quantities just introduced.

Task AT1

The star α-Orionis (Betelgeuse) has an apparent magnitude of m = 0.45 and an absolute magnitude of M = –5.14.

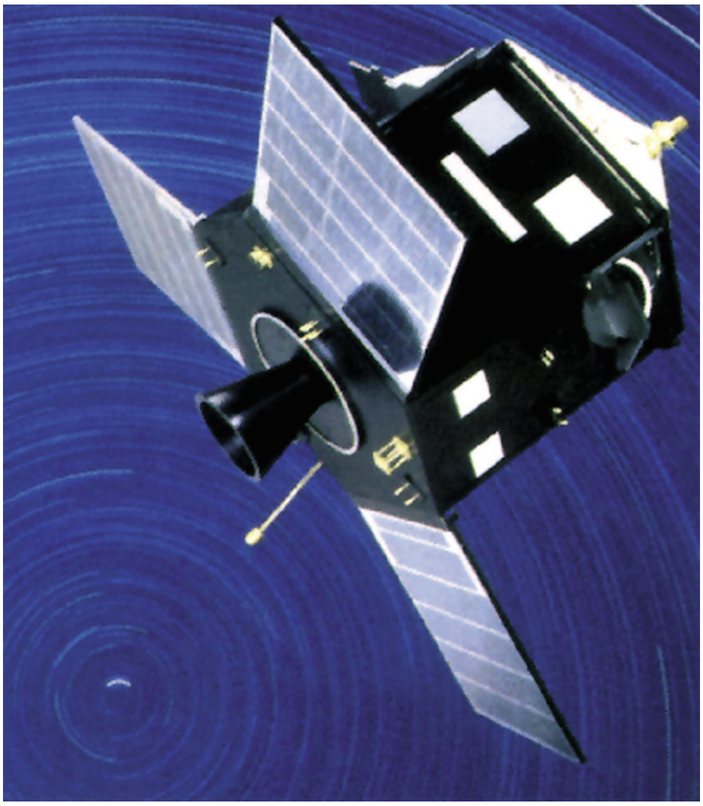


Figure 4: The ESA HIPPARCOS satellite
The HIPPARCOS satellite was launched on the night of 8 August 1989 by a European Ariane 4 launcher. The principal objective of ESA's HIPPARCOS mission was the production of a star catalogue of unprecedented precision. The positions and the distances of a set of about 120,000 preselected stars with magnitudes down to $m_B = 13$ were determined with high accuracy. The HIPPARCOS mission ended in 1993 and the final star catalogue was published in 1997.

Photo 1: Betelgeuse (Orion – The Hunter)

Photo 2: Vega (Lyra – The Lyre)

Photo 3: The Summer Triangle: (clockwise) Deneb (Cygnus – The Swan), Vega (Lyra – The Lyre), Altair (Aquila – The Eagle)

Photo 4: Sirius (Canis Major - The Greater Dog)

? Find the distance to Betelgeuse.

Betelgeuse is the red star at the left shoulder of Orion (seen from Earth) and is a red supergiant. When viewed with the naked eye, it has a clear orange-red hue.

Task AT2

α-Lyrae (Vega), with an absolute magnitude of 0.58, is at a distance of 7.76 parsec.

? Calculate Vega's apparent magnitude.

Vega is the brightest star in the constellation of Lyra (the Lyre) and the upper right star in the Summer Triangle.

Task AT3

α-Cygni (Deneb) is the upper left star in the Summer Triangle and the main star in the Swan. Its apparent magnitude is 1.25 and the distance is 993 parsec.

? Calculate the absolute magnitude.
What does this tell you about the nature of Deneb?

Task AT4

The star α-Canis Majoris (Sirius) is the brightest star in the sky. It is at a distance of 2.64 parsecs and its apparent magnitude is –1.44.

? Calculate the absolute magnitude of Sirius.
If you compare with the absolute magnitudes for the three other stars what is your judgement of Sirius' physical or intrinsic brightness?

Task AT5

? If the stars Vega, Sirius, Betelgeuse and Deneb were located 10 parsecs from the Earth (in the same region of the sky), what would we see?

Task AT6

The absolute magnitude, M, is defined as the apparent magnitude a star would have if it were placed 10 parsecs from the Sun.

? But wouldn't it be more correct to measure this distance from the Earth? Why doesn't it make a difference whether we measure this distance from the Sun or from the Earth?

Luminosity and Intensity

Up to now we have been talking about stellar magnitudes, but we have never mentioned how much light energy is really emitted by the star. The total energy emitted as light by the star each second is called its luminosity, L, and is measured in watts (W). It is equivalent to the power emitted.

Luminosity and magnitudes are related. A remote star with a high luminosity can have the same apparent magnitude as a nearby star with a low luminosity. Knowing the apparent magnitude and the distance of a star, we are able to determine its luminosity.

The star radiates light in all directions so that its emission is spread over a sphere. To find the intensity, I, of light from a star at the Earth (the intensity is the emission per unit area), we divide its luminosity by the area of a sphere, with the star at the centre and radius equal to the distance of the star from Earth, D. See Fig. 5.

$$I = L/(4\pi D^2)$$

The luminosity of a star can also be measured as a multiple of the Sun's luminosity, $L_{sun} = 3.85 \times 10^{26}$ W. As the Sun is 'our' star and the best-known star, it is nearly always taken as the reference star.

Using some algebra we find the formula for calculating the luminosity, L, of a star relative to the Sun's luminosity:

$$L/L_{sun} = (D/D_{sun})^2 \cdot I/I_{sun}$$

The ratio I/I_{sun} can be determined using the formula given in the Apparent Magnitudes section of the Astronomical Toolkit ($m_{sun} = -26.5$).

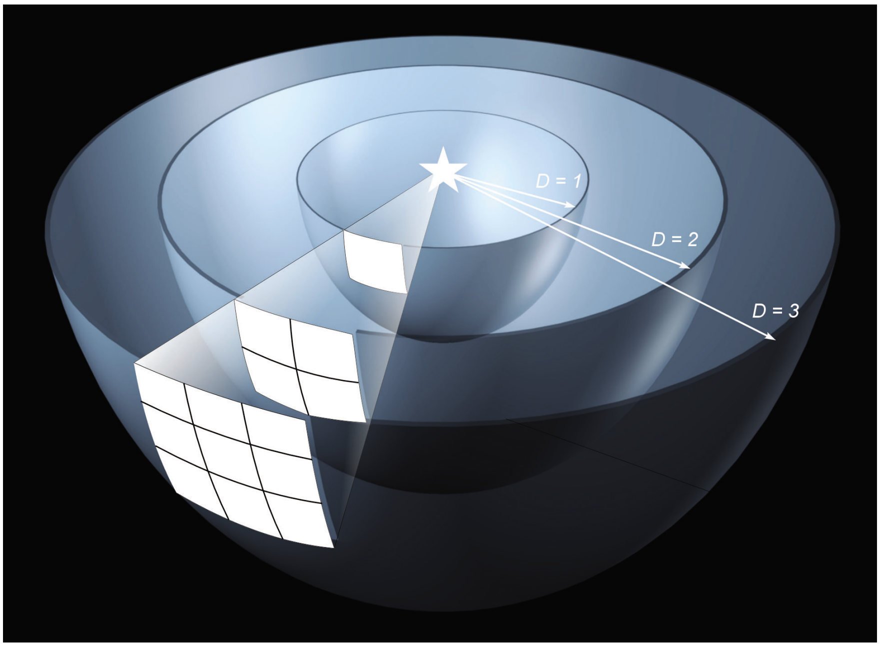


Figure 5: Intensity of light
This drawing shows how the same amount of radiation from a light source must illuminate an ever-increasing area as distance from the light source increases. The area increases as the square of the distance from the source, so the intensity decreases as the square of the distance increases.

Small angles and long distances

Have a look at Fig. 6:
If b is small compared to c, we can assume that the two longer sides of the triangle, c, have the same length as the centre line.
With the usual equations for a right-angled triangle we find:

$\sin(\beta/2) = (b/2)/c$

We can use the small-angle approximation $\sin x = x$, if we are dealing with very small angles (but only when the angle is measured in radians). This approximation may seem less justified, but it can be mathematically proven to be very good for small angles.

Task MT1

? Try this approximation yourself by calculating sin(1°), sin(1′), sin (1″). Note that you have first to convert the angles from degrees to radians.

Now you have a simple relationship between b, c, and β without the trigonometric function:

$\beta/2 = (b/2)/c$

$c = b/\beta$

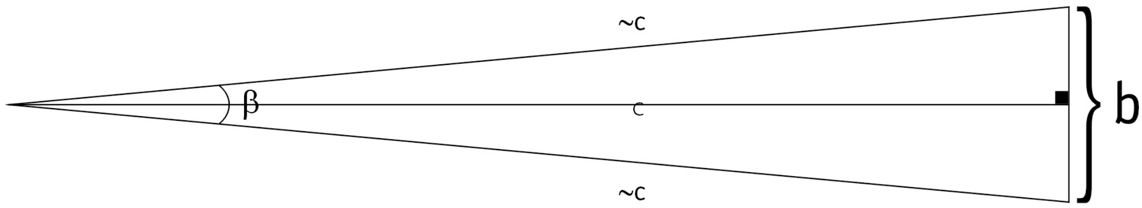


Figure 6: Dealing with small angles
If b is small compared to c, this implies that β is a small angle. We can therefore get a relationship between b, c and β without trigonometric functions.

Units and other basic data

1 arcminute = 1′ = 1/60 of a degree = 2.9089×10^{-4} radians
1 arcsecond = 1″ = 1/3600 of a degree = 4.8481×10^{-6} radians
1 milliarcsecond (mas) = 1/1000 arcsecond
Speed of light (c) = 2.997×10^{8} m/s
1 parsec (pc) = 3.086×10^{13} km = 3.26 light-years
1 kiloparsec (kpc) = 1000 parsec
1 Megaparsec (Mpc) = 10^{6} parsec
1 nanometer (nm) = 10^{-9} m

This teacher's guide contains solutions to the short training tasks.

Task AT1: **D = 131 parsecs**

Task AT2: **m = 0.03**

Task AT3: **M = −8.73**

This is an unusually bright star.

Task AT4: **M = 1.45**

Compared with Deneb (M = −8.73), Betelgeuse (M = −5.14), and Vega (M = 0.58) Sirius is actually rather a faint star. This demonstrates that our senses are not always well-equipped to detect the physical reality of the world around us.

Task AT5:

If placed at a distance of 10 pc, Vega and Sirius would be somewhat fainter, but still be among the brightest stars in the sky. However the stars Deneb and Betelgeuse would both be very much brighter than any stars we see in the night sky from Earth.

Task AT6:

There is no reason to distinguish between measuring the distance from Earth and from the Sun since the distance from Earth to the Sun is very small compared with 10 parsecs.
Calculating the difference in apparent magnitudes by using the distances from, respectively the Earth and the Sun, gives a difference of, at most, the order of 10^{-6} mag.

Task MT1:

sin(1°) = sin(0.017453293 rad) = **0.017452406**
sin(1′) = sin(0.000290888 rad) = **0.000290888**
sin(1″) = sin(4.84814×10^{-6} rad) = **4.84814×10^{-6}**

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