DISCONTINUITY ANALYSIS WITH CLUSTER

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ABSTRACT

We present an overview of Cluster's ability to determine the orientation, motion and thickness of boundary layers, and try to identify where and when a particular method is more suitable than another. With Cluster, three different principles can be used for discontinuity analysis; four-spacecraft timing methods, four-spacecraft gradient methods and single-spacecraft residue methods based on conservation laws. Timing methods make use of (and require) data from all four Cluster satellites. Boundary layer orientation, motion and thickness are determined from time differences between the crossings by the four spacecraft and the crossing duration. Variations of the timing methods can be used to take into account non-uniform thickness or non-constant motion of boundaries. Cluster's ability to determine gradients can also be used for discontinuity analysis. The orientation of a discontinuity can in some cases be obtained by examining the gradient of the field or plasma data directly. Alternatively, the gradient operator can be used to determine the electric current via Amperes law, or the time variation of the magnetic field via Faraday's law of induction. Variance analysis of these quantities thereafter give the orientation and integration across the discontinuity can be used to determine the velocity and thickness of the layer. Discontinuity analysis methods based on conservation laws and residue analysis may utilize data from one or more of the Cluster satellites, and can be combined with multi-spacecraft methods to improve these or for consistency checks. Constraints, for example by requiring the magnetic field to be tangential to the discontinuity or by requiring zero plasma flow respectively Alfvénic plasma flow across the discontinuity, can be used to improve the stability of the results.

Key words: Discontinuity analysis, Magnetopause, Techniques and methods.

1. INTRODUCTION

The determination of orientation, thickness and motion of boundaries was one of the prime objectives of the Cluster mission. These parameters are of vital importance for the analysis of many physical processes taking place in a plasma region.

In this paper, we give an overview of the different methods, and try to identify where and when one method may be better suited than another. We also show some examples of application of Cluster data.

1.1. Definitions

In this paper, we use the term *discontinuity* to a region of space where an abrupt spatial change in field or plasma occurs. With the Cluster orbit in mind, four regions, illustrated in Figure 1, stand out as particular interesting: Ⓐ the solar wind, including the bow shock and convected solar wind discontinuities; Ⓑ; the magnetopause; and Ⓒ the polar cusp. Many of the techniques and methods described in this paper are also applicable for structures and layers in the current sheet and plasma sheet of the magnetotail Ⓓ. Since the parameters and physical processes are quite different in these regions, different methods and approaches have been used to investigate them. We will argue that the magnetopause and polar cusp put the greatest challenge to the application of discontinuity analysis;

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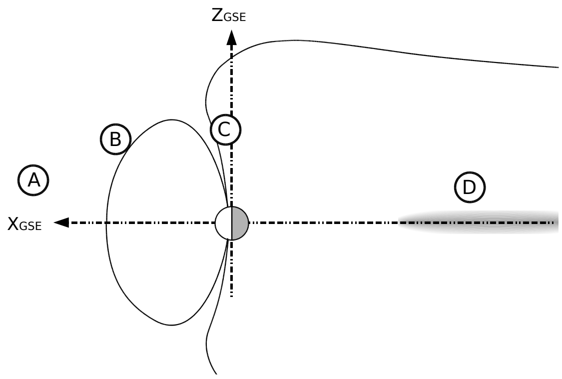

Figure 1. The four main discontinuity regions traversed by Cluster. Ⓐ *solar wind, including the bow shock and convected solar wind discontinuities,* Ⓑ *the magnetopause,* Ⓒ *the polar cusp, and* Ⓓ *the magnetotail.*

Both are highly dynamical, and often have small scale compared to the Cluster tetrahedron.

1.2. Cluster advantages

A unique feature of the Cluster mission is that it provides the first four-spacecraft measurement in and near the Earth's magnetosphere. With such an constellation, it is in principle possible to unambiguously determine both orientation, motion and any thickness of a discontinuity such as the bow shock, the magnetopause or the tail plasma sheet. Some of the most frequently used methods and their underlying assumptions will be discussed in Section 2.1. Another useful property of the tetrahedron-like constellation is that full three dimensional gradients in the measurements can be determined. Discontinuity analysis based on gradient calculations will be discussed in Section 2.3.

Each Cluster spacecraft carries a comprehensive set of instruments for both field and particle measurements. The quality of these measurements have allowed testing of new single-spacecraft methods, not possible with earlier missions. Section 2.4 gives an overview of single spacecraft methods for discontinuity analysis.

2. TECHNIQUES

Table 1 gives an overview of techniques that have been used or tested with Cluster data. In this section, we give a brief description of each method.

2.1. Four spacecraft timing methods

Four spacecraft timing methods require that a single common feature of a discontinuity can be identified at four different locations and usually from different times when crossing the discontinuity. Identifying the common feature and time tagging this at each of the four spacecraft is therefore an essential element of timing methods. Additional information, for example the duration of a crossing can give information about thickness and evolution or acceleration of the discontinuity. A method, based on timing alone, for determination of the orientation, speed and thickness of a discontinuity moving past four observing spacecraft was first presented by Russell et al. (1983), who applied it to interplanetary shocks.

2.1.1. Crossing times and duration

Crossing times and durations can in principle be derived from any measured quantity provided the time resolution is sufficient. Experience with Cluster suggest that the magnetic field or density proxies from the EFW instruments provide the best results. Timing and duration from the magnetic field are often easier to estimate if the measurements are rotated into a maximum-variance coordinate system. A common coordinate system for all four spacecraft may be used (see Section 2.4.2), but this is not critical since the direction of the maximum variance component is typically well determined.

For many discontinuities, the maximum variance component, B_{max} of the magnetic field resembles a Harris-sheet-like profile; $B_{max}(t) = f(t) \propto tanh(t/2\,\tau)$ (see e.g., Haaland et al., 2004a; Paschmann et al., 2005; Thompson et al., 2005). A similar fit can often be done with plasma density measurements (e.g., Bale et al., 2003). For such cases, a natural definition of crossing time, denoted t_0 in Figure 2, is the time where $f(t)$ has reached 50% of its extremal value. Also, the hyperbolic tangent curve representing a Harris sheet has the property that 76% of the total change, $\Delta f(t)$ occurs within the characteristic time interval $2\,\tau$.

Crossing times, t_c (c=0,1,2,3), for each crossing is defined here as the time where the measured quantity f(t) crosses the zero line. This definition of crossing time is not unique: Since the multi-spacecraft methods rely on relative timing only, any distinct feature observed by all spacecraft can be used for timing.

2.1.2. Velocity and orientation

At this stage, the unknown parametres are the orientation and velocity of the discontinuity. The orientation is assumed to be the same at all four spacecraft, i.e., the discontinuity has to be planar on the scale size of the spacecraft separation distance. The velocity may be constant or variable, depending on a-priori knowledge about the

Table 1. Overview of some techniques used to obtain orientation, $\mathbf{n}$*, velocity,* V_n *and acceleration,* a_n*, of a discontinuity. The input quantities and their symbols are :* $\mathbf{R}$ *- spacecraft separation vector;* $\mathbf{B}$ *- magnetic field;* $\mathbf{V}$ *- plasma velocity;* $\mathbf{E}$ *- electric field;* $\mathbf{J}$ *- current density; n - plasma number density;* $\mathbf{n}$ *- a single spacecraft normal (DA only);* ρ *- mass density;* p_B *- magnetic pressure; P, p - particle pressure as tensor or scalar; Q - heat flux tensor;* t_c*,* τ *- crossing time and duration (see text and Figure 2).*

Method	Abbreviation	Inputs	Parameters returned		
			$\mathbf{n}$	V_n	a_n
Timing methods:					
Constant Velocity Approach	CVA	$\mathbf{R}$, t_c	√	√	-
Constant Thickness Approach	CTA	$\mathbf{R}$, t_c, τ	√	√	√
Minimum Thickness Variation	MTV	$\mathbf{R}$, t_c, τ	√	√	√
Minimum Velocity Variation	MVV	$\mathbf{R}$, t_c, τ	√	√	√
Discontinuity Analyser	DA	$\mathbf{R}$, t_c, $\mathbf{n}^+$	√	√	(√)
Gradient methods:					
Orientation from gradient of a quantity	GRAD	$\mathbf{R}$, e.g., ρ, p_B	√	√	-
Minimum variance analysis of $\nabla \times \mathbf{E}$	MVAcE	$\mathbf{R}$, $\mathbf{E}$	√	√	-
Minimum variance analysis of current density	MVAJ	$\mathbf{R}$, $\mathbf{J}$	√	√	-
Minimum Directional Derivative	MDD	$\mathbf{R}$, $\mathbf{B}$	√	-	-
Spatio-temporal Difference	STD	$\mathbf{R}$, $\mathbf{B}$	-	√	√
Single spacecraft methods:					
Minimum variance analysis of B-field	MVAB	$\mathbf{B}$	√	-	-
Maximum variance analysis of E-field	MVAE	$\mathbf{E}$	√	-	-
Minimum variance analysis of plasma velocity	MVAV	$\mathbf{V}$	√	-	-
Minimum variance analysis of current density	MVAJ	$\mathbf{J}$	√	√	-
Minimum Faraday residue analysis	MFR	$\mathbf{B}$, $\mathbf{E}^*$	√	√	-
Minimum mass flux residue analysis	MMR	$\mathbf{V}$, n	√	√	-
Minimum variance of $\partial B/\partial t$	MVADB	$\mathbf{B}$	√	√	-
Minimum linear momentum residue analysis	MLMR	$\mathbf{V}$, $\mathbf{B}$, n, P	√	√	-
Minimum total energy residue analysis	MTER	$\mathbf{V}$, $\mathbf{E}$, $\mathbf{B}$, n, P, $Q^\#$	√	√	-
Minimum entropy residue analysis	MER	$\mathbf{V}$, $\mathbf{B}$, n, $p^\dagger$,	√	√	-
deHoffmann-Teller analysis	HT	$\mathbf{V}$, $\mathbf{B}$	-	√	√

$^+$ The Discontinuity Analyzer (DA) in its original form does not explicitly provide a numerical value for acceleration.
* $\mathbf{E}$ typically derived from $-\mathbf{V} \times \mathbf{B}$.
$^\#$ The heat flux vector, Q, is often small and can be ignored.
† Method only tested with isotropic pressure.

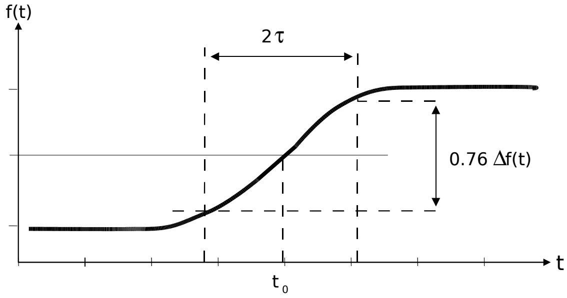

Figure 2. Idealized magnetic field (or density) profile across a discontinuity, and one possible definition of crossing time, t_0, and crossing duration, 2τ. With a known velocity, V_d, of the discontinuity, the thickness, d_d is given by $2\tau{*}V_{MP}$. After Paschmann et al. (2005)

discontinuity and the initial assumptions. The thickness of the discontinuity is also of interest. It may be identical at the four spacecraft or varying from one spacecraft to another.

A generic approach to find these quantities is as follows: Assume that the instantaneous velocity of a discontinuity is expressed by:

$$V(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 \qquad (1)$$

where A_c (crossing number c=0,1,2,3), are constants to be determined from the crossing times and durations. With the above expression for V(t), we find the discontinuity thicknesses, d_c (c=0,1,2,3), to be

$$\begin{aligned} d_c &= \int_{t_c-\tau_c}^{t_c+\tau_c} V(t)dt \\ &= 2\tau_c\left[V(t_c) + (A_2\tau_c^2)/3 + A_3 t_c \tau_c^2\right] \qquad (2) \end{aligned}$$

The center crossing times, t_c, and crossing durations, τ_c are considered as known quantities. The distance traveled by the discontinuity, between crossing CRi and crossing CR0 (not to be confused with the Cluster spacecraft naming convention; C1 ... C4) along a fixed normal direction, $\mathbf{n}$, is then

$$\begin{aligned} \mathbf{R}_c \cdot \mathbf{n} &= \int_{t=0}^{t=t_c} V(t)dt \\ &= A_0 t_c + \frac{A_1 t_c^2}{2} + \frac{A_2 t_c^3}{3} + \frac{A_3 t_c^4}{4} \qquad (3) \end{aligned}$$

where $\mathbf{R}_c$ (c=1,2,3) is the relative position of the spacecraft having the c'th crossing relative to the position of that having the first crossing (c=0).

Depending on the initial assumptions, these generic expression have to be treated differently hereafter :

2.1.3. Constant Velocity Approach

In the **Constant Velocity Approach (CVA)** (Russell et al., 1983), the coefficients A_1, A_2, and A_3 in expression (1) are put to zero so that A_0 becomes the constant, but unknown, velocity. The three equations (3) can then be solved for the vector $\mathbf{m} = \mathbf{n}/A_0$ and the coefficient A_0, which is then the velocity of the discontinuity, can be obtained from the normalization $|\mathbf{n}|^2 = 1$.

Note that the crossing duration is not needed to determine velocity or orientation with this method. Since the velocity is assumed constant, any differences in duration is here attributed to variations in the thickness of the discontinuity.

2.1.4. Acceleration

For discontinuities where a significant acceleration is present, e.g., the magnetopause or a flapping tail current sheet, the above method will not give correct answers. A better alternative in such cases may be the **Constant Thickness Approach (CTA)**, where the thicknesses d_c, rather than the velocity, are assumed the same at all spacecraft (Haaland et al., 2004a). The four equations (2) can then be solved for the four quantities $A_0/d, A_1/d, A_2/d$ and A_3/d and, subsequently, the three equations (3) for the vector $\mathbf{m} = \mathbf{n}/d$. Finally, the thickness d is obtained from the normalization of $\mathbf{n}$. An explicit expression for the acceleration is obtained by taking the time derivative of equation (1).

The **Discontinuity Analyzer (DA)**, first applied to Cluster data by Dunlop et al. (2002), also addresses acceleration of the discontinuity. In contrast to CTA, however, the normal direction is taken from one of the single spacecraft methods described in Section 2.4, typically minimum variance of the B-field. Equations (3) can then be solved for the coefficients A_0, A_1, and A_2 but A_3 must be put to zero. The DA approach is not a pure multi-spacecraft timing method because it makes use of a normal vector obtained from single-spacecraft data analysis. It has the advantage that it permits both velocity and thickness of the magnetopause to vary from crossing to crossing in an event. Its disadvantage is that the velocity polynomial in equation (1) becomes quadratic rather than cubic, which is less flexible and can easily lead to unreasonable results.

2.1.5. Combined approaches

The **Minimum Thickness variation (MTV)** method is used to obtain a single multi-spacecraft answer to the orientation and to the velocity and thickness variations during an encounter of the four Cluster spacecraft with the magnetopause. It is a combination of CVA, CTA, and DA but uses no single-spacecraft method and produces a cubic velocity polynomial.

In the MTV method, the orientation is a normalized average of the normal vectors obtained from CVA and CTA, which are usually not the same (if they are the same then the crossing has both constant velocity and constant thickness). Once the combined normal is known, the MTV method uses a scheme similar to DA, i.e., equations (3), to provide three of the four equations needed to determine the velocity coefficients $A_0 \ldots A_3$. Rather than putting A_3 = 0, a fourth equation is obtained from the subsidiary condition that the variance of the thicknesses seen at the four spacecraft should be a minimum. This condition is again not unique, but may be appropriate for the magnetopause where thickness variations are expected to be much smaller than the velocity variations during a typical event. The algebraic details of the MTV method are outlined in Paschmann et al. (2005)

Analogue to the MTV method, one could also envisage a **Minimum Velocity Variation (MVV)** method, where variations in the normal velocity rather than the thickness were minimized.

Both the MTV and MVV methods are algebraically complicated, but the advantage of producing a single answer, rather than separate answers from CVA and CTA. They are based entirely on multi-spacecraft timing and avoids the pitfalls associated with the use of single-spacecraft methods for obtaining the vector normal to the magnetopause. For individual events, it may not always be optimal but for statistical studies it is desirable to use a single method.

2.1.6. Caveats - Timing methods

There are some pitfalls that may influence the quality of timing methods :

Partial crossings. Timing methods are only useful if timing is available from all four spacecraft. For studies of periodic discontinuity crossings, e.g., waves in the magnetotail or on the magnetopause, the wave amplitude must thus be larger then the spacecraft separation distance in order to be detected on all four spacecraft.

Stationarity and planarity. The discontinuity is assumed to be stationary and planar. Quantities measured at different locations and at different times at each spacecraft are assumed to correspond to the same structure. This item is discussed in e.g., Dunlop and Woodward (1998); Schwartz (1998).

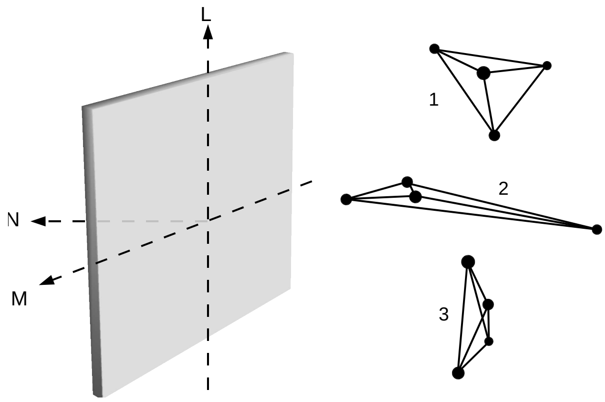

Figure 3. Illustration of different Cluster spacecraft tetrahedron shapes relative to a discontinuity. The various configurations give various success rates; The regular tetrahedron shape in configuration 1 is the best compromise to infer orientation and motion. In the elongated configuration in 2 and the nearly planar configuration in 3, the orientation, respectively the velocity is very sensitive to timing accuracy.

Timing accuracy. Although a discontinuity is planar over the spacecraft separation distance, the individual time series profiles used for timing may usually contain wiggles due to small scale structures that make exact timing difficult. Cross correlation or similar methods may be used where visual inspections or the method illustrated in Figure 2 fail. Timing errors become more important when spacecraft separation is small or the discontinuity moves very fast across the spacecraft.

Spacecraft separation and configuration. The geometrical shape of the tetrahedron formed by the four Cluster satellites changes continuously along the spacecraft orbit. Figure 3 shows a discontinuity and 3 possible spacecraft tetrahedron shapes. For the discontinuity shown, the various tetrahedron configurations will give different success rates. The nearly regular tetrahedron marked 1 is a good compromise to determine both orientation and speed of the discontinuity. In the elongated configuration 2, the normal determination will be very sensitive to the timing accuracy, whereas in configuration 3, the velocity determination will be very sensitive to timing accuracy. In extreme cases with four satellites aligned along a line, or in a plane, none of the timing methods can be used. For a discussion of tetrahedron geometry, see e.g., Robert et al. (1998) or Chanteur (1998).

2.2. Examples - Timing methods

Figure 4 shows an example of results based on timing methods. The top panel shows the thickness distributions of solar wind discontinuities (Knetter et al., 2004). They used the CVA method combined with magnetic field measurements to study the thickness and classification of discontinuities in the solar wind. Based on 129 events where data from all four spacecraft were available, they found that most of the solar wind discontinuities were tangential discontinuities (see also 2.5) less than 3000 km thickness, though there were cases with thicknesses above 15000 km.

The second and third panel show similar results from the magnetopause. Paschmann et al. (2005) used data from 96 crossings from the dawnside magnetopause. They used the MTV method and found an average thickness of $\sim$ 750 km and an average velocity of $\sim$ 50 km$\,s^{-1}$.

2.3. Gradient methods

A discontinuity involves a spatial change in magnitude or direction of a field or plasma quantity. A unique property of the Cluster mission is the ability to determine full three dimensional gradients. This can be utilized to determine properties of the discontinuity.

2.3.1. GRAD - Gradient of a scalar quantity

The simplest gradient method is to assume that the local discontinuity normal points in the direction of the gradient of a field or plasma quantity :

$$\mathbf{n} = \nabla f(t)/|\nabla f(t)| \qquad (4)$$

where $f(t)$ can be any scalar quantity, e.g., density or magnetic pressure (Shen et al., 2003).

2.3.2. MVAJ - Minimum Variance of Current Density

The MVAJ method (Haaland et al., 2004b) uses magnetic field measurement from four spacecraft to estimate the current density, followed by minimum variance analysis to establish the current sheet orientation. Velocity and thickness of the current sheet are obtained by integrating the current density across the current sheet.

The current density can be determined from the curlometer technique (e.g., Robert et al., 1998), which utilizes Ampères law and magnetic field, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. Thereafter, the current sheet orientation is established by performing minimum variance analysis of the current density (MVAJ). The underlying physics is that $\nabla \cdot \mathbf{J} = 0$ (just as the basis for minimum variance of the magnetic field is that $\nabla \cdot \mathbf{B} = 0$). This step provides a current-sheet aligned coordinate system where the three orthogonal axes are the eigenvectors from the variance analysis. In the new coordinate system, the two components of Ampères law can now be written as:

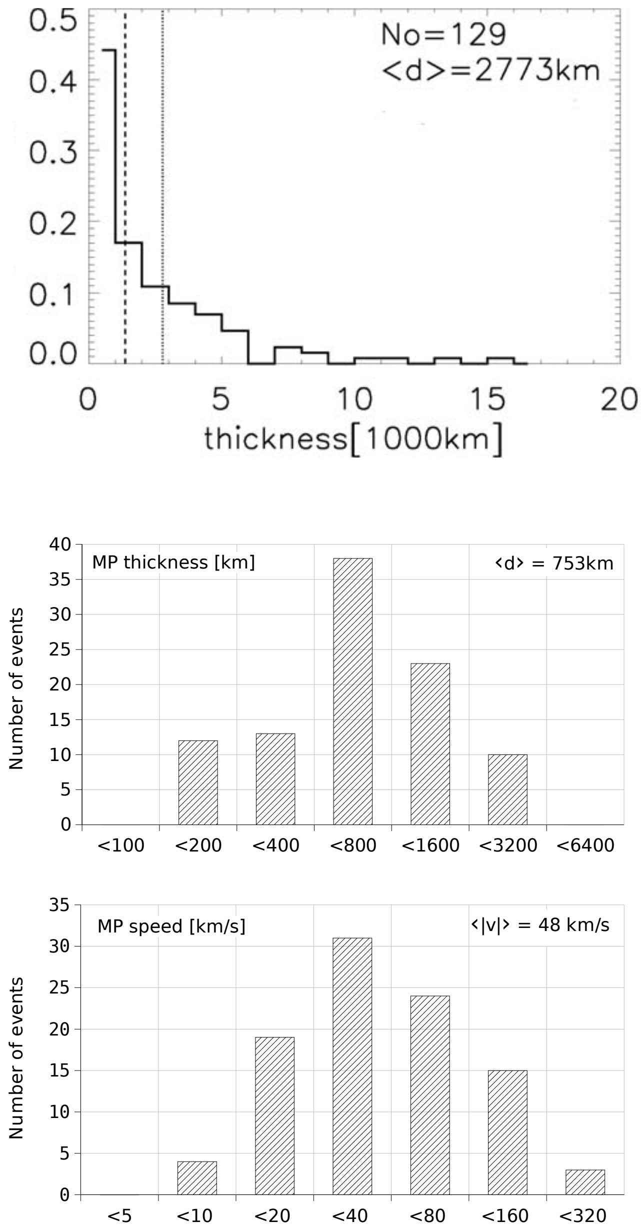

Figure 4. Top panel: Normalized distribution of solar wind discontinuity thickness. The two vertical lines denote average (2773 km) and median (1373 km) thicknesses of the 129 events studied.(Knetter et al., 2004). Middle panel and lower panel: Statistical distribution of magnetopause thickness and velocity respectively. Note that a few of the crossings have velocities above 300 km$\,s^{-1}$ *(Paschmann et al., 2005).*

$$\mu_0 J_1 = (\nabla \times \mathbf{B})_1 = -\frac{\partial B_2}{\partial x_3} = -\frac{dB_2}{dt}\frac{1}{v_3}$$
$$\mu_0 J_2 = (\nabla \times \mathbf{B})_2 = \frac{\partial B_1}{\partial x_3} = \frac{dB_1}{dt}\frac{1}{v_3} \quad (5)$$

where v_3 is the velocity, assumed constant, of the magnetopause along the the normal direction $\hat{\mathbf{x}}_3$. The conversion from spatial differentials to time differentials is made by use of v_3. Integrated across the magnetopause, these equations give:

$$\Delta B_2 = -\mu_0 v_3 \int_0^{t_1} J_1 dt$$
$$\Delta B_1 = \mu_0 v_3 \int_0^{t_1} J_2 dt \quad (6)$$

where t_1 on the right hand side is the time it takes to cross the current layer. Ideally, the two equations should give the same value for v_3 but, when applied to experimental data, uncertainties and deviations from model assumptions will almost always produce slightly different values.

The expression (6) holds for any time segment of the discontinuity; acceleration can then be addressed by considering a set of shorter time segments within the total crossing time.

MVAJ thus provide both orientation and state of motion of the discontinuity without use plasma data.

2.3.3. MDD - Minimum Directional Derivative

The timing methods mentioned in Section 2.1, all assume that the orientation of the discontinuity does not change during the time interval it takes for all four spacecraft to cross. Without using sliding segments, there is thus no way to check whether the discontinuity changes its orientation during this time period. The **Minimum Directional Derivative (MDD)** method by Shi et al. (2005b) addresses this issue by calculating a normal direction at every moment.

Assume that **n**(t) is the (yet unknown) normal of the discontinuity at time t, and $\nabla\mathbf{B}(t)$ is the magnetic field gradient tensor at that moment. A vector **D**(t), which represents the directional derivative along the normal of the magnetic field gradient, is then given by :

$$\mathbf{D}(t) = \mathbf{n}(t)\cdot\nabla\mathbf{B}(t) = \frac{\partial \mathbf{B}(t)}{\partial n(t)}$$
$$= \left[\frac{\partial B_x(t)}{\partial n(t)}, \frac{\partial B_y(t)}{\partial n(t)}, \frac{\partial B_z(t)}{\partial n(t)}\right] \quad (7)$$

In analogy with variance analysis (see Section 2.4), the normal direction $\mathbf{n}(t)$, can now be found by minimizing $\mathrm{D}^2(t)$. Algebraically, this consists of finding the eigenvalues and eigenvectors of a symmetric covariance matrix given by :

$$L_{ij} = \sum_{k=1}^{3} \frac{\partial B_k}{\partial x_i}\frac{\partial B_k}{\partial x_j}, \ \ (i,j = 1..3) \quad (8)$$

Provided that the eigenvalues are sufficiently different (see also discussion in Section 2.5.1), the eigenvector corresponding to the *largest* eigenvalue provides an estimation of the normal direction **n** of a nearly one-dimensional discontinuity. Degenerate solutions, where the three eigenvalues, $\lambda_1, \lambda_2, \lambda_3$ are nearly identical indicate a 3D structure; whereas cases with $\lambda_1, \lambda_2 \gg \lambda_3$, indicate a two-dimensional structure.

2.3.4. STD - Spatio Temporal Difference

The **Spatio Temporal Difference method (STD)** (Shi et al., 2005a) is an extension of the MDD method, and uses the magnetic gradient tensor to find the velocity of a structure. The time derivative of the magnetic field at any moment can be written

$$\frac{d\mathbf{B}(t)}{dt} = \frac{\partial \mathbf{B}(t)}{\partial t} + \mathbf{U}(\mathbf{t})\cdot\nabla\mathbf{B}(t) \quad (9)$$

where $\mathbf{U}(t)$ is the sought convective velocity of the structure. For stationary structures, left hand side of Equation (9) is zero, and one ends up with :

$$\frac{\partial \mathbf{B}(t)}{\partial t} + \mathbf{U}(\mathbf{t})\cdot\nabla\mathbf{B}(t) = 0 \quad (10)$$

which can be solved for $\mathbf{U}(t)$.

The combination of MDD and STD is thus, in principle, able to provide both orientation and velocity of a structure at any instant.

2.3.5. Caveats - Gradient methods

Many of the caveats from timing methods apply here too. Also, since all gradient methods are based on a linear approximation of gradients, the minimum requirement is that the spacecraft separation distance is smaller than the scale sizes of the structure. The ratio $\nabla\cdot\mathbf{B}/|\nabla\times\mathbf{B}|$ may be used as a rough quality estimate of gradient methods. Values $\ll 1$ are desirable, but no guarantee for a correct answer. The shape of the Cluster tetrahedron will also influence the accuracy of gradient methods - see e.g., Chanteur (1998).

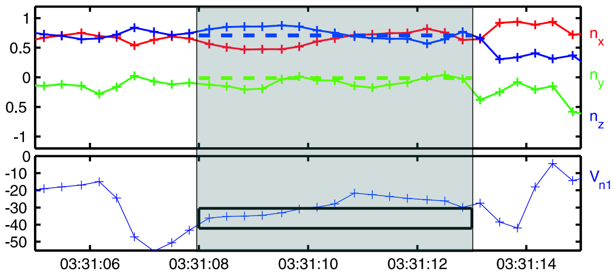

Figure 5. Magnetopause orientation and velocity during a Cluster crossing of the dayside magnetopause around 03:30 UT on 2 March, 2002. Top panel : Normal components from the MDD (solid lines) and MVAJ (dashed lines - X and Y components almost falls on top of each other) methods. Bottom panel : Normal velocity from the STD method (solid line). The black square shows the range of velocities from (Haaland et al., 2004b). Within the time interval 03:31:08 - 03:31:13 (shaded area), there is a good agreement between the methods. From Shi et al. (2005a).

2.3.6. Example - Gradient methods

Figure 5 shows an example of application of the MVAJ, MDD and STD methods to a Cluster magnetopause crossing around 03:30 UT on 2 March, 2002. Solid lines in the upper panel shows the three components of the boundary normal derived from MDD; dashed lines are the corresponding MVAJ based normal components. The velocities (lower panel) of the magnetopause in this case ranges from $\sim$ -42 km s^{-1} to -30 km s^{-1}, and there is a good overall agreement between the various methods.

2.4. Single-spacecraft methods

Each of the four Cluster spacecraft carries a comprehensive set of instruments for both plasma and field measurements. This can be utilized in a number of different ways for discontinuity analysis. In particular, the quality of some of the higher order moments from the Cluster plasma instruments allowed, for the first time, the use of plasma data alone for discontinuity analysis (see e.g. Sonnerup et al., 2004). The lower part of Table 1 lists some of the single spacecraft methods to determine motion and orientation of a discontinuity that have been tested with Cluster data.

2.4.1. A generic approach

The basic mathematical procedure of single-spacecraft methods are similar for nearly all methods. This lead Sonnerup et al. (2005) to suggest a generic approach based on residue analysis of conservation laws. This procedure incorporates the magnetohydrodynamic (MHD) conservation laws for mass, energy, momentum and entropy. The same scheme can also be adapted for conservation laws derived from Maxwell's equations; magnetic flux conservation, conservation of magnetic poles, and conservation of electric charge.

In the approach of Sonnerup et al. (2005), and employing the Einstein notation (summation over repeated indices), the generic conservation law can be written as

$$\frac{\partial \eta_i}{\partial t} + \frac{\partial q_{ij}}{\partial x_j} = 0 \qquad (11)$$

where η_i is the density of the conserved quantity and q_{ij} is the corresponding transport tensor. If the conserved quantity is a scalar, e.g., mass ($\eta_i \equiv \eta = \rho$), the index i is simply dropped, and $q_{ij} \equiv q_j = \rho v_i$ is then a transport vector.

Assume now that the one dimensional, time invariant discontinuity moves with a constant speed u_n along the yet unknown normal $\mathbf{n}$. In this co-moving frame, the time dependence disappears and Equation (11) can be written

$$-u_n \frac{d\eta_i}{dx'} + \frac{d(n_j q_{ij})}{dx'} = 0 \qquad (12)$$

where x' are the coordinates of the co-moving system. Integrated across the discontinuity, this gives

$$-\eta_i u_n + n_j q_{ij} = C \qquad (13)$$

where C is an integration constant.

For real discontinuities, there will always be deviations from the ideal one-dimensional, time invariant model above; Equations (11) to (12) will therefore not be perfectly fulfilled for any measurement across the layer, but the optimal result can be obtained by minimizing the residue :

$$\begin{aligned} R &= \frac{1}{K}\sum_{k=1}^{k=K}\left|-\eta_i^{(k)}u_n + n_j q_{ij}^{(k)} C_i\right|^2 \\ &= \left\langle \left|-\eta_i^{(k)}u_n + n_j q_{ij}^{(k)} C_i\right|^2 \right\rangle \qquad (14) \end{aligned}$$

This expression can be solved for the optimal values of $\mathbf{C}^*$ and $\mathbf{u}^*$, where $\mathbf{u}^* \cdot \mathbf{n}$ is the optimal velocity of the discontinuity. The resulting matrix, has the form $n_i Q_{ij} n_j$, where Q_{ij} is a symmetric matrix similar to covariance matrices known from minimum variance analysis. The eigenvectors of this matrix determine the orientation of the discontinuity; the eigenvector $\mathbf{x_3}$ corresponding to the smallest eigenvalue, λ_3, gives the normal of the discontinuity. Similarly, the eigenvalue ratio provides information on how well the eigenvectors are resolved.

A number of specific conservation laws can be formulated, and treated according to the above scheme. A complete description is given in Sonnerup et al. (2005); here we briefly mention some of the variants where the method has been tested with Cluster data.

Minimum Variance Analysis of the magnetic field (MVAB), first applied by Sonnerup and Cahill Jr. (1967) for discontinuity analysis of magnetopause traversals, has become a standard method to determine the boundary normal of a discontinuity. The underlying physics is the conservation of magnetic solenoidality (expressed as $\nabla \cdot \mathbf{B} = 0$), which can be cast into the generic form of Equation (11). The plasma flow along this normal gives a rough idea about the velocity, at least in cases with no reconnection and thus no plasma flow across the magnetopause. A better estimate of the speed of a discontinuity is often obtained from **deHoffmann-Teller analysis (HT)**, in which one tries to find a frame of reference where the electric field disappears, i.e., a frame co-moving with the discontinuity. A detailed discussion about HT analysis and MVAB can also be found in Sonnerup and Scheible (1998) and Khrabrov and Sonnerup (1998b).

Minimum variance of current density (MVAJ), is very similar to MVAB, and is based on the fact that the total charge is conserved across the boundary, i.e., $\nabla \cdot \mathbf{J} = 0$ The current density can, in principle, be determined from the relative ion- and electron velocities. However, MVAJ has so far only been applied to Cluster for cases with $\mathbf{J}$ derived from the four-spacecraft curlometer technique (Haaland et al., 2004b). As shown in Section 2.3.2, MVAJ provides both orientation and speed of the discontinuity.

Minimum massflow residue analysis (MMR) is based on conservation of mass across the discontinuity, and does not require any information about the magnetic field across the boundary. MMR can provide both orientation and velocity of a discontinuity. Although use of mass conservation for these purposes was first described in Sonnerup and Scheible (1998), Cluster was the first mission where the plasma moments were of sufficient quality to make use of the MMR method (Sonnerup et al., 2004).

Minimum Faraday residue analysis (MFR) (Terasawa et al., 1996) is based on conservation of magnetic flux across a boundary. It utilizes Faraday's law across the magnetopause current layer to find a moving frame and orientation such that the tangential component of the electric field is as constant as the data permit. For practical purposes, the convection electric field is often used and is calculated from the plasma velocity via $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$. MFR returns both a normal and a velocity of the discontinuity. A convenient approach to MFR is shown in Khrabrov and Sonnerup (1998a).

Minimum linear momentum residue analysis (MLMR) is based on momentum conservation across a discontinuity. It uses the MHD momentum balance equation and includes the pressure gradient and the Lorentz force. The plasma moment products from Cluster contain the full pressure tensor, so effects of non-isotropic pressure can be incorporated. In Sonnerup et al. (2005), however, MLMR was applied with a isotropic pressure. MLMR returns both a normal vector and a velocity of the discontinuity.

Minimum total energy flux residue analysis (MTER) also provides both orientation and speed of a discontinuity, and is based on energy conservation. The total energy includes both kinetic energy, heat flux and Poynting flux. Input to this method thus ideally have to include higher order plasma moments like the heat flux tensor and pressure tensor. Also, to incorporate Ohmic dissipation, electron pressure terms and Hall effects, the Poynting flux should ideally come from the measured total electric field. However, benchmarks by Sonnerup et al. (2005) suggest that isotropic pressure, omitting the heat flux, and using $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$ as a proxy for the electric field gives reasonable results.

Minimum entropy residue analysis (MER) is derived from MTER, but the viscous dissipation terms, heat conduction and electrical resistivity has been dropped. MER is applicable for rotational discontinuities (RDs) and also for cases where there is no plasma flow across the discontinuity. In MHD these are tangential discontinuities (TDs - see also Section 2.5) and contact discontinuities (CDs).

2.4.2. Combining several methods

Results from two or more of the above residue methods can be combined to produce a single estimation of the orientation and velocity. This is done by adding a set of suitable weighted and normalized covariance matrices (Q - matrices), and then calculate the eigenvalues and eigenvectors of the combined matrix. The weighting and normalization of the individual Q-matrix is not unique, but it is desirable to put more emphasis on results from individual methods for which the eigenvalues are well separated.

A composite matrix with weights, w_k for each method, can thus be expressed

$$Q_{COM_{ij}} = \sum_{k=1}^{k=K} w^{(k)} Q_{ij}^{(k)} \qquad (15)$$

The composite normal of the discontinuity is then the eigenvector $\mathbf{x_3}$ corresponding to the smallest eigenvalue, λ_3, of Q_{COM}.

Similarly, a composite velocity of the discontinuity can be obtained as

$$\mathbf{U}^*{}_{COM} = \sum_{k=1}^{k=K} w^{(k)} \mathbf{U}^*_{\mathbf{k}} \qquad (16)$$

so that the velocity of the discontinuity becomes $\mathbf{U}^*{}_{COM} \cdot \mathbf{n}$. The composite method thus utilizes all available data measured within a discontinuity. It may improve accuracy and reveal properties not immediately seen if only one of the methods were used.

As a variant of the composite method, one may also add Q-matrices from different spacecraft, or combine data vectors from more spacecraft in the above methods. For small spacecraft separations, this may be useful when the discontinuity is thin, and only a handful of measurements are available from each spacecraft. In the example in Figure 6, this method has been used to construct a composite MVAB normal.

2.4.3. Caveats and error sources

For methods using plasma data, time resolution and/or the quality of higher order moments may be insufficient for some cases. For Cluster, some of the higher order moments are only available at $\sim$12 second resolution. Also, since all of the above methods are based on variance analysis, the eigenvalue ratio from the eigen analysis of the covariance matrix provides a rough estimate about the quality of the normal determination. Well separated eigenvalues are desirable, but no guarantee for a correct determination of orientation - see e.g., Haaland et al. (2004a). It is recommended that the analysis is performed on nested sets of data intervals centered around the discontinuity being studied. Statistical error analysis for MVAB (and related methods) are described in Sonnerup and Scheible (1998).

2.4.4. Examples - Single spacecraft methods

Figure 6 shows a polar plot of normal directions obtained with various methods during a Cluster magnetopause crossing on 5 July 2001 (this event is described in detail in e.g., Haaland et al., 2004a). Each symbol represent a projection of a normal into a the plane perpendicular to a reference normal (“bulls eye normal”) constructed from a combination of the individual MVAB results from each spacecraft. The concentric circles represent deviations (cone angle) from this reference normal. Despite being based on widely different data and different conservation laws, all methods give a normal within 6 degrees of the reference normal. The orientations from some of the multi-spacecraft methods described in 2.1 are shown for comparison.

2.5. Classification of discontinuity type

While four-spacecraft methods may be better suited for robust determination of speed, orientation and thickness, some properties of the discontinuity are better estimated

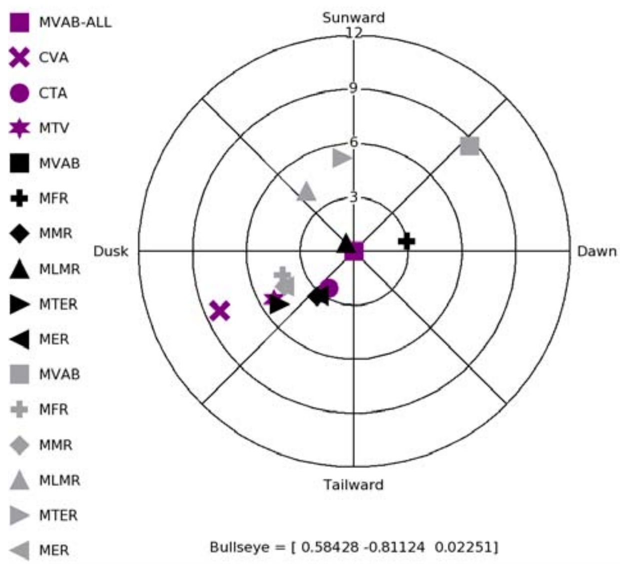

Figure 6. Polar plots of normal orientations obtained from various methods during a magnetopause crossing on 5 July, 2001. Each symbol represents one method to determine the orientation of a discontinuity (see text for abbreviations). The center of the plot (the “bullseye” normal) is here the $\mathbf{n}_{COM}$ normal based on MVAB results from each spacecraft.

from analysis of individual spacecraft crossings. This applies in particular to the classification of the crossings into tangential (TD) or rotational (RD) discontinuities.

A tangential discontinuity separates two different plasma regions. In the absence of diffusion, there is no mixing of the two plasma regimes, and they may have two different compositions. A TD is thus classified by zero plasma flow across (no flow component along the normal), zero normal magnetic field component, and constant total pressure, $p = p_B + p_p$ across the discontinuity. A rotational discontinuity (RD), on the other hand, have a finite normal component, but since RDs are Alfvénic structures, the magnetic field changes should be correlated with changes in the plasma flow.

There have been attempts to classify solar wind discontinuities as RDs or TDs based on the magnetic field alone (Neugebauer et al., 1984; Horbury et al., 2001; Knetter et al., 2004), but the success of such methods inherently depends on the ability to establish the true normal of the discontinuity. In practice, there are many ambiguous cases where no classifications can be done.

A more reliable method, which utilizes plasma data, is the so-called Walén test (e.g., Khrabrov and Sonnerup, 1998b, and references therein). This consists of plotting the plasma bulk velocity components measured during a discontinuity crossing (after transformation into a suitable frame, co-moving with the discontinuity - typically the deHoffmann-Teller (HT) frame), against the corresponding components of the measured Alfvén velocities. The results are characterized in terms of the slope, α and correlation coefficients between $(\mathbf{V} - \mathbf{V_{HT}})$ and $\mathbf{V_A}$. A poor correlation or slopes close to 0 indicate tangential discontinuity, whereas RDs should have a slope close to ± 1. However, even some TDs may have Alfvénic nature; In a statistical study of the magnetopause, Paschmann et al. (2005) therefore adapted a threshold of 0.5 to distinguish between RDs and TDs. The presence of reconnection signatures, e.g., plasma jetting, provided support for the classification of cases with Walén slopes of only 0.5 as RDs.

2.5.1. Use of constraints

If one has a-priori knowledge about the nature of a discontinuity, it may be desirable to impose constraints to the minimum variance analysis. For example, an ideal TD has zero magnetic field along the normal. One might then do the analysis so that the predicted normal, $\mathbf{n}$, is perpendicular to the direction of the average magnetic field $\mathbf{e} = \langle \mathbf{B} \rangle / |\langle \mathbf{B} \rangle|$. As suggested by A. V. Khrabrov (see Sonnerup and Scheible, 1998), such a constraint can easily be imposed to the variance analysis by replacing the covariance matrix, Q, by the projection $\mathrm{Q}'_{\mathrm{kn}} = \mathrm{P}_{\mathrm{ik}}\,\mathrm{Q}_{\mathrm{kn}}\,\mathrm{P}_{\mathrm{nj}}$, where the projection matrix is given by

$$P_{ij} = \delta_{ij} - e_i e_j \qquad (17)$$

δ_{ij} being the Kronecker delta symbol. The eigenvectors of Q' now have a different meaning; since we introduce a known quantity, the vector $\mathbf{e}$, the lowest eigenvalue will be zero, and its corresponding eigenvector, $\mathbf{X_3} = \mathbf{e}$. The eigenvector $\mathbf{X_2}$, corresponding to the lowest, non-zero eigenvalue will now be the normal predictor. Typically, constrained variance analysis will give mores stable results.

The above formalism can also be used to impose other constraints on one-dimensional discontinuities. An overview of such constraints are given in Sonnerup et al. (2005).

3. SUMMARY

We have given a brief overview of Cluster's ability to determine macroscopic features like orientation, velocity and dimensions of discontinuities. With a four spacecraft mission like Cluster, a number of different methods for discontinuity analysis are possible. No single method stands out as superior, and the choice of method depends on the initial assumptions. To avoid particular caveats and weaknesses of a single method, it is always recommended to try alternative methods. When the Cluster spacecraft separation is increased, four-spacecraft methods may not work, and one has to rely on single spacecraft methods for determining orientation and motion of discontinuities.

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