The k-filtering Applied to Wave Electric and Magnetic Field Measurements from Cluster

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OUTLINES

● The k-filtering technique
  - The basics
  - Examples
● 3D characterization of the magnetosheath ULF turbulence
  - Using magnetic field measurements only
  - Combining electric and magnetic field measurements
● Conclusion
The k-Filtering technique

A multi-spacecraft data analysis technique allowing to estimate the wave-field energy distribution as a function of ω and k.

\[ A(t, r) = \begin{pmatrix} A_1(t, r) \\ A_2(t, r) \\ \vdots \\ A_L(t, r) \end{pmatrix} \]

Examples:
1) \( A(t, r) = B(t, r) \)
2) \( A(t, r) = E(t, r) \)
3) \( A(t, r) = \begin{pmatrix} E(t, r) \\ cB(t, r) \end{pmatrix} \)

\[ A(t, r) = \int \int A(\omega, k) e^{i(\omega - k \cdot r)} d\omega dk \]

\[ P_A(\omega, k) = \text{trace} \langle A(\omega, k) A^T(\omega, k) \rangle \]

Notations:

\[ A(t) = \begin{pmatrix} A(t, r_1) \\ A(t, r_2) \\ A(t, r_3) \\ A(t, r_4) \end{pmatrix} \]

\[ A(\omega) = \begin{pmatrix} A(\omega, r_1) \\ A(\omega, r_2) \\ A(\omega, r_3) \\ A(\omega, r_4) \end{pmatrix} \]

\[ M_A(\omega) = \langle A(\omega) A^T(\omega) \rangle \]

Filter bank approach

\[
A(\omega) = \begin{pmatrix}
A(\omega, r_1) \\
A(\omega, r_2) \\
A(\omega, r_3) \\
A(\omega, r_4)
\end{pmatrix}
\]

\[
A(\omega, k_1) = F^T(\omega, k_1)A(\omega)
\]

\[
P_A(\omega, k_0) = 0
\]

\[
P_A(\omega, k_1) \neq 0
\]

\[
P_A(\omega, k_2) = 0
\]
P(ω,k) Estimation

\[ A(ω, r) = \int_k A(ω, k) e^{-i \mathbf{k} \cdot \mathbf{r}} \, dk \]

\[ H(k) = \begin{pmatrix} \mathbf{I}_L e^{-i \mathbf{k} \cdot \mathbf{r}_1} \\ \mathbf{I}_L e^{-i \mathbf{k} \cdot \mathbf{r}_2} \\ \mathbf{I}_L e^{-i \mathbf{k} \cdot \mathbf{r}_3} \\ \mathbf{I}_L e^{-i \mathbf{k} \cdot \mathbf{r}_4} \end{pmatrix} \]

\[ A(ω) = \int_k H(k) A(ω, k) \, dk \]

\[ P(ω, \mathbf{k}) = \text{trace} \left\{ F^T(ω, \mathbf{k}) M_A(ω) F(ω, \mathbf{k}) \right\} = \text{minimum} \]

with \[ F^T(ω, \mathbf{k}) H(k) A(ω, \mathbf{k}) = A(ω, \mathbf{k}) \]

$P(\omega,k)$ Estimation

Cluster configuration

Cluster data

$P_A(\omega,k) = \text{trace}[(H_T(k) (M_A(\omega))^{-1} H(k))^{-1}]$

$A(\omega) = \begin{pmatrix}
A(\omega, r_1) \\
A(\omega, r_2) \\
A(\omega, r_3) \\
A(\omega, r_4)
\end{pmatrix}$

$M_A(\omega) = \langle A(\omega) A^T(\omega) \rangle$
P(ω,k) Estimation

The constraining matrix C_A(ω,k)

The extra information we have from physical laws can be included to further enhanced the k-filtering

Example: \[ A(t, r) = \begin{bmatrix} E(t, r) \\ cB(t, r) \end{bmatrix} \]

\[ \nabla \times \mathbf{E}(t, r) = -\partial_t \mathbf{B}(t, r) \Rightarrow k \times \mathbf{E}(\omega, k) = \omega \mathbf{B}(\omega, k) \]

\[ \mathbf{A}(\omega, k) = \begin{pmatrix} E_x(\omega, k) \\ E_y(\omega, k) \\ E_z(\omega, k) \\ cB_x(\omega, k) \\ cB_y(\omega, k) \\ cB_z(\omega, k) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -ck_z/\omega & ck_y/\omega \\ ck_z/\omega & 0 & -ck_x/\omega \\ -ck_y/\omega & ck_x/\omega & 0 \end{pmatrix} \begin{pmatrix} E_x(\omega, k) \\ E_y(\omega, k) \\ E_z(\omega, k) \end{pmatrix} = C_A(\omega, k) \begin{pmatrix} E_x(\omega, k) \\ E_y(\omega, k) \\ E_z(\omega, k) \end{pmatrix} \]

\[ P_A(\omega, k) = \text{trace} \left[ C_A(\omega, k) \left( C_A^{-1}(\omega, k) \mathbf{H}^T(k) C_A(\omega, k) \right) \right] \]

(Pinçon and Lefevre, JGR, 1991; Pinçon et al., ISSI SR-001, 1998)

Validity of $P(w,k)$

**Wave-field requirements**

- The wave-field is stationary
- The wave-field is homogeneous
- No aliasing

To avoid spatial aliasing the wave-field has to be free of wavelengths smaller than the minimum inter-spacecraft distance

**Geometric requirements**

- 3D analysis $\Leftrightarrow$ 3D geometry

Tetrahedron Geometric Factors

(P. Robert et al., ISSI Scientific Report SR-001, 1998)

Spatial Aliasing

Two satellites cannot distinguish between \( k_A \) and \( k_B \) if:

\[
\Delta k \cdot r_{21} = 2\pi n
\]

\( (\Delta k = k_B - k_A \text{ and } n \text{ is an integer}) \)
Spatial Aliasing

Spatial aliasing if:
\[ \Delta \mathbf{k} \cdot r_{\alpha \beta} = 2 \pi n \]

(n is an integer; \( \alpha \) and \( \beta \) \( \in \{1, 2, 3, 4\} \))

\[ \Delta \mathbf{k} = n_1 \Delta \mathbf{k}_1 + n_2 \Delta \mathbf{k}_2 + n_3 \Delta \mathbf{k}_3 \]

\[ \Delta \mathbf{k}_1 = \frac{2 \pi}{V} (\mathbf{r}_{24} \times \mathbf{r}_{34}) \]
\[ \Delta \mathbf{k}_2 = \frac{2 \pi}{V} (\mathbf{r}_{34} \times \mathbf{r}_{14}) \]
\[ \Delta \mathbf{k}_3 = \frac{2 \pi}{V} (\mathbf{r}_{14} \times \mathbf{r}_{24}) \]

\[ V = \mathbf{r}_{14} \cdot (\mathbf{r}_{24} \times \mathbf{r}_{34}) \]

Neubauer and Glassmeier, JGR, 1990
Representation of the wave-field energy distribution

\[ P(\omega, k_x, k_y, k_z) : 4 \text{ variables!} \]

- For a given \( \omega \) we split the 3D-k domain in slices corresponding to different values of \( k_z \).

- For each slice we adopt a contour line representation.

Simple 1D examples

Two satellites (at $x_1$ and $x_2$) measuring one field quantity $\Phi(t,x)$

A) The wave field is given by: $\Phi(t,x) = \Phi_o \exp[i(\omega_o t - k_o x)] + \text{noise}$

\[ P(\omega, k) = |\phi_o|^2 \frac{\varepsilon(2 + \varepsilon)}{2(1 + \varepsilon - \cos[(k - k_o)(x_1 - x_2)])} \]

$P(\omega, k)$ is periodic in $k$

$k_o = 0.2$
$x_1 - x_2 = 1$
$\Phi_o = 1$
B) The wave field is given by: $\Phi(t,x) = \Phi_1 \exp[i(\omega_0 t - k_1 x)] + \Phi_2 \exp[i(\omega_0 t - k_2 x)]$

The two waves are assumed to not be phase coherent

$$P(\omega_o, k) = \frac{|\phi_1(\omega_o)|^2 |\phi_2(\omega_o)|^2 (1 - \cos[(k_1 - k_2)\Delta x])}{|\phi_1(\omega_o)|^2 (1 - \cos[(k_1 - k)\Delta x]) + |\phi_2(\omega_o)|^2 (1 - \cos[(k_2 - k)\Delta x])}$$

$P(\omega_o,k)$: solution with only one peak.

$k_1 = 0.2$

$k_2 = 1.1$

$\Delta x = x_1 - x_2 = 1$

$\Phi_1 = \Phi_2 = 1$
Simple 1D examples

C) Same as B), one field quantity but using three satellites

The more satellites we use, the better resolution we get

D) Same as B), two satellites but using two field quantities

The more field quantities we use, the better resolution we get
Application:
Study of ULF wave fluctuations in the magnetosheath

Theoretical argument
- Intense ULF wave fluctuations are observed in the magnetosheath near the magnetopause.
- These fluctuations are expected to play an important role in the transfers between the solar wind and the magnetosphere.

Experimental issues
Interpretation of the fluctuations:
- MHD waves, mirror mode, low hybrid waves.
- Weak turbulence.
- Strong turbulence.

Properties of the fluctuations:
- No monochromatic waves.
- Continuous spectra.
- No clear polarization.

Cluster data
E and B Cluster measurements + K-filtering technique → 3D-characterization of ULF wave-field fluctuations
FGM data related to the selected event

2002-02-18 [05:34:01 – 05:36:45] UT

- The wave-field is stationary
- The wave-field is homogeneous

$B_z \parallel B_0$

$B_y$

$B_x$

$x_{GSE} = 5.6 \, R_E$

$y_{GSE} = 4.6 \, R_E$

$z_{GSE} = 8.4 \, R_E$
Geometrical shape of the tetrahedron during the event

Interspacecraft distance ~ 100 km

GSE coordinate system (km)

35959  35990  35926  36024
\( r_1 = 29220 \quad r_2 = 29147 \quad r_3 = 29205 \quad r_4 = 29233 \)
53688  53636  53596  53617

Elongation ≤ 0.1
Planarity ≤ 0.1

Magnetosheath plasma parameters during the selected event:

- \( n = 36 \text{ cm}^{-3} \)
- \( T_{\parallel} = 140 \text{ eV}, \ T_{\perp} = 170 \text{ eV} \)
- \( V_A = 78 \text{ km/s}, \ f_{ci} = 0.33 \text{ Hz}, \ \rho = 79 \text{ km} \)
- \( \beta_{\parallel} = 4.5, \ \beta_{\perp} = 5.4 \)

From CIS, FGM, and WHISPER experiments:

Power spectra of the ULF magnetic fluctuations:

\[ P_B(f) \]

\[ B^2(\text{nT}^2\text{Hz}^{-1}) \]

\[ y = f^{-7/3} \]

\[ f_{2\lambda} \]

\[ \parallel B_o \]

\[ P_B(f_{2\lambda},k) \]

3D characterization of ULF magnetic fluctuations

(F. Sahraoui et al., submitted PRL, 2005)
3D characterization of ULF magnetic + electric fluctuations

Problems:

- Normalisation of \( A(\omega) \)

\[
A(\omega, \mathbf{r}) = \begin{bmatrix}
E_x(\omega, \mathbf{r}) \\
E_y(\omega, \mathbf{r}) \\
R(\omega)B(\omega, \mathbf{r})
\end{bmatrix}
\]

with

\[
R(\omega) = \sqrt{\frac{\left\langle |E_x(\omega, \mathbf{r})|^2 + |E_y(\omega, \mathbf{r})|^2 \right\rangle_r}{\left( |B_x(\omega, \mathbf{r})|^2 + |B_y(\omega, \mathbf{r})|^2 + |B_z(\omega, \mathbf{r})|^2 \right)_r}}
\]

- EFW data: only two electric components

Constraining matrix \( k \cdot B(\omega, \mathbf{k}) = 0 \)

Constraining matrix derived from:

\( \omega B_z = k_x E_y - k_y E_x \)

- Technical problems on the EFW instruments:
  2001-07-25: 10 Hz filter problem on S/C2, probe 3
  2001-12-28: probe failure on S/C1, probe 1
  2002-07-29: probe failure on S/C3, probe 1

- Special spin effect cleaning for EFW data

Diagram showing the phase space with points labeled p1, p2, p3, and p4, and vectors indicating the EFW data points.
3D characterization of ULF magnetic + electric fluctuations

Magnetic field only

\[ P_B(f_2,k) \]

Magnetic and electric fields

\[ P_{EB}(f_2,k) \]

Advantages:
- Drastic reduction of aliasing effects outside the validity domain
- No more aliased peak (B) inside the validity domain
- Higher resolution: two local maxima within (A)

(A. Tjulin et al., accepted JGR, 2005)

3D characterization of ULF magnetic + electric fluctuations

<table>
<thead>
<tr>
<th></th>
<th>f1=0.37</th>
<th>f2=0.61</th>
<th>f3=1.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>B:</td>
<td>K₁=-0.0110 0.0024 0.0053</td>
<td>K₂=-0.0167 0.0040 0.0068</td>
<td>K₃=-0.0307 0.0094 0.0144</td>
</tr>
<tr>
<td>E+B:</td>
<td>K₁=-0.0099 0.0023 0.0064</td>
<td>K₂a=-0.0137 0.0016 0.0159</td>
<td>K₂b=-0.0230 0.0069 0.0014</td>
</tr>
</tbody>
</table>

\[ K₂ \approx \langle K₂a + K₂b \rangle \]

Additional information: ratio between magnetic and electric field energy

Ratio for k₂a=97 (nT/(mV/m))²
Ratio for k₂b=52 (nT/(mV/m))²

K₂b is more "electrostatic" than K₂a

Conclusion

The k-filtering technique is a method based on simultaneous multi-point measurements to characterize the wave-field fluctuations in space plasmas in terms of the wave-field energy distribution in the frequency and k vector space.

- Application to 3D characterization of the magnetosheath ULF turbulence
  - Wave field energy is dominated by mirror modes at all frequencies
  - Wave field energy is cascading from large scale to smaller scales along the plasma flow.
- Combining E and B measurements:
  - Less aliasing effects
  - Better resolution
  - Additional interesting information

The K-filtering technique is a rather complex tool and cannot be used routinely.