USER REQUIREMENTS DOCUMENT FOR

`EChOSim`: A Simulator for the Exoplanet Characterization Observatory

Version 3.0
September 14, 2013

Contributors:
Marc Ollivier, Bruce Swinyard, Alexander Amaral-Rogers, Jean-Philippe Beaulieu, Céline Cavarroc, Vincent Coudé du Foresto, Carrie MacTavish, Andreas Papageorgiou, Enzo Pascale, Locke Spencer, Marcel Tessenyi, Giovanna Tinetti and Ingo Waldmann

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Version	Date	Description/change
1-1	3 Feb 2012	Issued for software definition meeting 7 Feb 2012
1-2	8 Feb 2012	Updates following meeting 7 Feb 2012 – mainly making it consistent with adopted methodology of using "per beam" Change history added Table of RD and AD added
2	17 May 2012	Re-organised for readability and to better follow software architecture in ADD
3	Updated with description of EChOSim Version 3.0	

Reference Documents

RD1	*Photometric model description*
RD2	*EChO Science Requirements Document*
RD3	*EChO Payload Definition Document*
RD4	*EChO Assesment Study Design Report*
RD5	*Mission Requirements Document*
RD6	*Ingo's note on pointing stability*

Applicable Documents

AD1	`EChOSim` *Software Requirements Document*

1 Introduction

1.1 Purpose

The purpose of this document is to ouline the user requirements for the EchO simulator ensuring an accurate scientific description of what the model does. The target audience is users and developers of the simulation tool.

1.2 Project Scope

The purpose of this project is to develop a simulation tool for the Exoplanet Characterisation Observatory. The software, `EChOSim`, simulates the astrophysical scene (star and transiting planet) as well as the stationary and dynamic characteristics of the instrument and observing strategy. The goal is determine the best overall observing strategy as well as validating and setting benchmarks for the instrument parameters. Users of the tool are members of the EChO payload consortium.

1.3 References

- `EChOSim`: More about design can be found in the `EChOSim` Software Requirements Document (SRD – AD1). This documents can be found in the *Documents* folder on the EChO wiki `http://sim.echo-spacemission.com/` and in the *docs* directory of the `EChOSim` package.

- **EChO *Instrument and Telescope***: More information about the EChO mission can be found in the following documents available on `http://echo-spacemission.eu/`*:*

  - RD1: EChO Radiometric model description
  - RD2: EChO Science Requirements Document
  - RD3: EChO Payload Definition Document
  - RD4: EChO Mission Proposal
  - RD5: Assessment Study Design Report

- **echo_transit software**: The prototype for this project was provided by an initial static simulator provided by Marc Ollivier and developed using the IDL environment (prototype available under request, please contact M. Ollivier)

2 Overall description of the EChO simulator

2.1 General philosophy of the simulator

The instrument model aims at getting a global estimation of the instrument performance both as static radiometric model and in the time domain to simulate the actual observation scenario. It takes into account the major effects and parameters of the observation and the instrument. It also allows the influence of each parameter to be studied, aiding the optimization of the instrument design and development of the observation and calibration strategies.

In order to be usable by the largest part of the community, the instrument model is easy to use and representative of the observation conditions. It reproduces the instrument modes, starting from the astrophysical scene, instrument and environment parameters as input. The output for the user are fits files representing the signal sampled by each detector over time. This output can be used by the observational pipeline, also provide for convenience in EChOSim, or by a user-provided reduction pipeline.

The general philosophy for the development of the model is a central engine that runs several modules, each dealing with aspects of the model. Outputs of the model can be computed thanks to dynamical parameters estimated within the modules or coming from data considered as inputs and computed by other ways.

2.2 Modules contents and scope

The structure of the software modules and their interaction is more fully described in AD1. In this section we give an overview of the structure and the names of the modules which are used in the rest of the document for ease of readability and cross referencing. AD1 takes precedence over the present document for actual implementation of `EChOSim`.

`EChOSim` includes the following modules:

AstroScene:

Description of the astrophysical scene (star + planet).

Foregrounds:

Description of the observation environment local zodiacal emission.

Instrument:

Description of the EChO instrument. Comprehends: i) a description of the common parts of the payload (telescope and common optics); ii) a description of the instrument transmission function for each channel as a function of wavelength (including transmission, optical throughput, and spatial modulation transfer function); iii) a description of the focal plane detector system. The module also implements a description of the self emission of the optical elements and of the detector environment.

Noise:

Description of noise components. This includes: intrinsic detector noise (e.g. dark current, readout noise), photon blip noise, telescope pointing effects.

Output:

The simulation output consists of individual detector timelines, organised into FITS files for subsequent analysis.

2.3 Observation Pipeline

An advanced data-reduction pipeline is distribute with the `EChOSim` software package, but it is not part of the `EChOSim` simulation software. The pipeline reads the `EChOSim` output and provides a reconstruction of calibrated spectra and an estimate of the associated uncertainties.

2.4 Definitions, Abbreviations, Notations, Symbols and Constants

2.4.1 Definitions

This section reminds some of the useful definitions taken from the radiometric model description document (RD1).

Channels:

The science instrument is divided into sub-instruments each covering a given wavelength range these are denoted as channels, i.e. 2 channels have at least one element which is not common (dispersive device, detector…). Implemented EChO channels are: VNIR, SWIR, MWIR1, MWIR2 and LWIR.

Effective area (A_{eff}):

True photon collecting area of the telescope. It does not contain any assumptions on the reflectivity of the mirrors (contained within the total transmission).

Instrument transmission:

the transmission of the whole instrument, from the first dichroic to the last optical element before the science detector. It is channel and wavelength dependent.

Observation efficiency:

The percentage of time during science operations phases that is actually dedicated to collecting science data. Slews, settling times, communications (if not parallel to science observations), safe mode …, all need to be deducted from the total time. Also referred to as the observatory's duty cycle.

Abbreviation	Meaning
ADC	Analog to Digital Converter
ADU	Analog to Digital Unit
EChO	Exoplanet Characterisation Observatory
FOV	Field Of View
HK	House Keeping (data)
LOS	Line Of Sight
PLM	PayLoad Module
PRNU	Pixel Response Non Uniformity
PSF	Point Spread Function
SED	Spectral Energy Density
SST	Science Study Team

Table 1: List of abbreviations.

Occultation or secondary eclipse:

When the exoplanet passes behind its host star, along the line of sight to Earth (and hence to the EChO S/C). As such the exoplanet is completely eclipsed by the star.

Total transmission:

Transmission of the complete system, including all optical elements from the telescope to the last element before the science detector: it is channel and wavelength dependent.

Transit or primary transit:

When the exoplanet passes in front of its host star, along the line of sight to Earth (and hence to the EChO S/C). As such, the exoplanet is between the EChO S/C and the star and partially eclipses it.

2.4.2 Abbreviations

The abbreviations used in this document and their meaning are given in Table 1.

2.4.3 Notation and associated symbols

The notations and symbols used in the equations are similar to those used for the radiometric model (see reference) and reminded in Table 2.

Table 2: List of notations and associated symbols.

Symbol	Name	Dimension	Comments/Definition/Standard Unit
A_{eff}	Effective area	$[m^2]$	Effective photon gathering area of telescope. Includes reduction from central hole and M2 Obscuration, excludes M1 and M2 reflectivity
a	Semi-major axis	[m]	1 A.U = 1.49598×10^{11} m
$\delta\lambda$	Spectral bandwidth	[m]	In practice: $\delta\lambda$ in μm
e	Eccentricity of the exoplanet's orbit	NA	
$\Delta F(\lambda)$	Transit depth	NA	Normalised amplitude of planetary eclipse/transit observed
ω	Argument of periastron	[radians]	Argument of periastron for eccentric orbits
$F_i(\lambda,d\lambda)$	Energy flux received from body i per surface element	$W.m^{-2}$	Between λ-dλ/2 and λ+dλ/2
g	Exoplanet surface gravity	$m.s^{-2}$	
H	Atmosphere scale height	[m]	Height of the exoplanet's atmosphere above which it is transparent to stellar flux In practice: H in km
Δz	Total height of exoplanet atmosphere	[m]	
$I_i(\lambda,T)$	Irradiance of body I at (λ,T)	$[W.m^{-2}]$	
i	Exoplanet system inclination	degrees	Relatively to the line of sight to Earth
l	Distance from observed system to telescope	[m]	Assumed as star-Sun distance In practice: l in Pc
M1	M1 geometrical area	$[m^2]$	
M2	M2 geometrical area	$[m^2]$	
N_f	Number of elementary integration per transit	[integration]	
N_t	Number of transits	[transits]	Several transits are necessary to obtain the required SNR
n	Number of pixel per spectral band	[pixel]	

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Table 2 – Continued

Symbol	Name	Dimension	Comments/Definition/Standard Unit
P	Exoplanet period around its host star	[s]	1 day = 24h = 86400 s in practice: P in day
$P_i(\lambda,d\lambda)$	Power emitted by the body i	[W]	Between λ-$d\lambda/2$ and λ+$d\lambda/2$
Δpix	Pixel dimension	[m]	In practice : Δ_{pix} is expressed in μm and is channel-dependent
$QE(\lambda)$	Quantum efficiency	[e-/photon]	Ratio of arriving photons to produced electrons at the detector level
R	Spectral resolution	NA	Ratio of a wave length over a wave range
R_i	Radius of the body i	[m]	In practice: R_i in km
r_i	Reflectivity of the body i	NA	
SNR	Signal to noise ratio	NA	
T_i	Black body temperature of body i	[K]	
z	Height	[m]	In the atmosphere of the exoplanet, z=0 at "sea level" In practice: z in km
$I_{Zodi}(\lambda)$	Zodiacal background	[W m^{-2} μm^{-1} sr^{-1}]	Local Zodiacal light spectum
α	Exoplanet bond albedo	NA	Wave length dependant
ε_i	Emissivity of the body i	NA	
λ	Wavelength	[m]	In practice: λ in μm
I_D	Dark current	[e-/pixel]	
σro	Read out noise	[e-/pixel]	
η	Instrument transmission	NA	Product of transmission/reflection of all optical elements on the optical path, excluding M1 and M2.
η_{tel}	Telescope transmission	NA	Product of M1 and M2 transmissions
$\varphi(t/\tau)$	Exoplanet day-side view factor	NA	Describes exoplanet phases: from full "Moon" to new "Moon" via crescents
ϕ	Projected Planet-Star separation	NA	Normalised separation of stellar and planetary centres in the observer's line of sight
μ	Molecular weight	[kg/mol]	Describes the relative weight of different atmospheres
τ	Transit duration	[s]	In practice: τ in hour
τ_e	Integration time	[s]	Integration time during 1 integration: $\tau_e = \tau' / N_f$ In practice : τ_e in hour
$\Omega_p(\lambda)$	Solid angle seen by one pixel of the detector	[sr]	Function of the spectral band

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Table 2 – Continued

Symbol	Name	Dimension	Comments/Definition/Standard Unit
$\Omega_{psf}(\lambda)$	Effective solid angle subtended by telescope beam	[sr]	Function of the spectral band The system étendue is given by $A_{eff}(\lambda)\Omega_{psf}(\lambda)$ m^2sr

2.4.4 Fundamental constants

The fundamental constants used in the equation are given in Table 3.

Symbol	Name	Unit	Value	Comments
c	Speed of light	[m/s]	2.99792458×10^{8}	In vacuum
G	Gravitational constant	$[m^3.kg^{-1}.s^{-2}]$	6.67300×10^{-11}	
h	Planck constant	$[m^2.kg.s^{-1}]$	6.626068×10^{-34}	
k	Boltzmann constant	$[m^2.kg.s^{-2}.K^{-1}]$	$1.3806503 \times 10^{-23}$	
N_a	Avogadro number	$[mol^{-1}]$	6.0221415×10^{23}	

Table 3: List of fundamental constants.

2.4.5 Conversion factors

Specific units can be used to describe the astrophysical quantities relative to the star or the exoplanet. They are given in Table 4 with their conversion factor to international standard units.

Unit	Description	Value in ISU
R_e	Earth radius	6.371×10^{6} m
M_e	Earth mass	5.9742×10^{24} kg
R_j	Jupiter radius	7.1492×10^{7} m
M_j	Jupiter mass	1.8986×10^{27} kg
R_{sun}	Sun radius	6.955×10^{8} m
M_{sun}	Sun mass	1.9891×10^{30} kg
au	Astronomical unit	1.49598×10^{11} m
Pc	Parsec	$3.08568025 \times 10^{16}$ m

Table 4: List of conversion factors.

3 Description of the EChO simulator modules: associated parameters and algorithms.

3.1 *Astroscene* module

The astrophysical scene description module contains an evaluation of all signal sources either considered as science signal (star, planet)

3.1.1 Overview

The 'observed' flux calculations can be summarised in a three step process:

1. Depending on the user option, the stellar emission spectrum is taken from a pre-supplied stellar SED library, or approximated by a black-body function.
2. The planetary contribution is calculated. For a primary eclipse, the area of the star occulted by the planet is calculated (this includes the wavelength dependent opacity of the planetary atmosphere). For a secondary eclipse, the planetary thermal emission and reflected flux is calculated. Either planetary contribution is expressed as fraction of the total stellar flux, $\Delta F(\lambda)$. We refer to $\Delta F(\lambda)$ as the 'transit depth' of the observed light curve.
3. We now simulate the planetary orbit using analytical equations by Mandel & Agol (2002) for a given $\Delta F(\lambda)$. This yields a time dependent fraction of the stellar flux 'observed' by `EChOSim`.

`EChOSim` requires the following astroscene related parameters: stellar radius (R_s), temperature (T_s), and distance (l), planetary radius (R_p), orbital period (P), orbital semi-major axis (a), orbital inclination (i), orbital eccentricity (e), argument of periastron (ω).

3.1.2 Star spectrum

3.1.2.1 Static description `EChOSim` will use accurate models of stellar SED for specific cases of stars. A library of stellar SEDs covering from 3000 K to 7000 K in 100K steps (log g=4.5 and solar metallicity) is provided. This has been generated using the Phoenix atmosphere models. Each file has two columns, the first is wavelength in micron and the second is flux in erg/s/cm^2/A . These values should be multiplied by 10 to get the flux in W/m^2/μm. The flux column gives the value at the stellar surface.

3.1.2.2 Limb Darkening We need to correctly account for the effect of limb darkening of the star on the primary eclipse light curve of the transiting planet. This will be done using fixed parameters that will be held in a table associated with each stellar type. The quadratic limb darkening parameters are taken from Claret (2000). An interpolated look up table will be provided versus stellar temperature and spectral pass-bands. `EChOSim` will then linearly interpolate limb-darkening coefficients for all wavelengths up to $\lambda = 3.0\mu$m and assumes no limb-darkening for redder wavelengths. We assume solar metallicities for all stars in the sample.

3.1.3 Exoplanet spectrum

The exoplanet spectrum contains 3 main elements:

- photons emitted by the exoplanet (thermal emission including the atmosphere effects).
- photons reflected by the exoplanet (assuming only the bright side contribution)
- photons transmitted through the exoplanet atmosphere at the limb, during a transit

These contributions vary, in general, with time according to the observation phase, transit / occultation, proximity to the star etc...). The three can be defined as follows:

The thermal emission of the planet can be expressed as a wavelength dependent contrast ratio provided by the user. Alternatively, this can also be represented by a black body with a temperature T_p. Using the Stefan-Boltzmann law, we can approximated the planetary temperature by

$$T_p = T_s(1-\alpha(\lambda))^{\frac{1}{4}} \times \sqrt{\frac{R_s}{2a}} \tag{1}$$

where the wavelength-dependent albedo, $\alpha(\lambda)$ is user provided.

One considers only the emission of the day side, the irradiance of the exoplanet day side is:

$$I_p(\lambda, T_p) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k T_p}}-1} \quad W/m^2/\mu m/sr \tag{2}$$

Thus, the power emitted by the exoplanet day-side in the band centred at λ with width $d\lambda$ is:

$$P_p(\lambda, d\lambda) = I_p(\lambda, T_p)d\lambda 4\pi {R_p}^2 \quad W \tag{3}$$

and the power received by the telescope from the exoplanet day-side emission is:

$$P_1(\lambda, d\lambda) = P_p(\lambda, d\lambda)\Phi(t/\tau)\frac{A_{eff}}{4\pi l^2} \quad W \tag{4}$$

where $\Phi(t/\tau)$ is the exoplanet view factor (i.e. the fraction of the planetary day-side visible at a given orbital phase).

This power is calculated for different planetary temperatures, radii and distances to the Earth. Standard test cases are given in appendix 2.

The reflected light from the planet is computed considering the bond albedo of the planet,α. The power received by the telescope from the exoplanet's reflection of star light in the band centred at λ with width $d\lambda$ is:

$$P_2(\lambda, d\lambda) = P_s(\lambda, d\lambda)\alpha\frac{\pi {R_p}^2}{4\pi a^2}\frac{A_{eff}}{4\pi l^2} \quad W \tag{5}$$

Where the quantity $P_s(\lambda, d\lambda)$ is the power emitted by the star. Standard test cases are given in Table 7 of Appendix 3.

The light transmitted through the exoplanet's atmosphere. This can be described by a user provided spectrum used as input.
Alternatively, this can be also simulated as follows. The fraction of light from the star that is transmitted by the planetary atmosphere is proportional to the atmosphere height. In other words, the apparent radius of the planet varies depending on the opacity of the exoplanetary atmosphere (at the terminator) for a given wavelength. The amplitude of this apparent radius change is dependent on the observing molecules in the atmosphere as well as the total height of the exoplanetary atmosphere, $\Delta z(\lambda)$. This height is determined by the atmospheric scale height, H:

$$H = \frac{kT_p N_a}{\mu g} \tag{6}$$

where $g = GM_p/R_p{}^2$. Assuming that the atmosphere can be probed until about one thousandth of the atmospheric pressure at the "sea level", one get the effective opaque zone of the atmosphere. It is common practice to assume an atmosphere of 5 scale-heights:

$$\Delta z(\lambda) = 5H \tag{7}$$

to a first approximation (radiometric model hypothesis), $\Delta z(\lambda)$ does not in practice depend on λ. Standard test case values for H and planetary mass are given in Table 7 of Appendix 3.

We can now express the total flux decrement for the primary eclipse case as a ratio of the areas of planetary surface plus the area of the opaque atmospheric annulus over the surface area of the host star:

$$\frac{\pi(R_p + \Delta z(\lambda))^2 - \pi R_p{}^2}{\pi R_s{}^2} \approx \frac{2R_p \Delta z(\lambda))}{R_s{}^2} \tag{8}$$

3.1.4 Derived transit depth

As described in the astroscene overview, the planetary contribution to the extrasolar system is expressed as fractional effect of the stellar SED. This is directly measurable via the transit depth ($\Delta F(\lambda)$) of the observed light curves. The transit depth definitions depend on whether we observe a primary or secondary eclipse. In the primary eclipse case, we see a diminishing of stellar flux due to the direct obscuration of the stellar surface by the planet, whilst in the secondary eclipse case we see a diminishing of flux due to the star obscuring the planetary day-side emissions. Hence the following definitions apply

Primary eclipse:

Following from equation 8, the total transit depth observed is given by

$$\Delta F(\lambda) = \frac{(R_p + \Delta z(\lambda))^2}{R_s{}^2} \tag{9}$$

and the transit depth due to the atmospheric absorption only:

$$\Delta F(\lambda) = \frac{2R_p \Delta z(\lambda))}{R_s{}^2} \tag{10}$$

Secondary eclipse:

Following from equations 4 and 5, the transit depth is given by the day-side emission of the planet plus the reflected stellar light and the planet-star surface ratio

$$\Delta F(\lambda) = \frac{4\pi l^2}{A_{eff}} \frac{P_1(\lambda, \delta\lambda) + P_2(\lambda, \delta\lambda)}{P_s(\lambda, \delta\lambda)} \tag{11}$$

3.1.5 Planetary Orbit

Given the above derive transit-depths, the orbital solutions are now computed using the analytic equations of Mandel & Agol (2002) and Seager & Mallen-Ornelas (2003).

The transit/eclipse duration can be approximated by

$$\tau = \frac{PR_s}{\pi a}\sqrt{\left(1+\frac{R_P}{R_S}\right)^2 - \left(\frac{a}{R_s}\cos i\right)^2} \tag{12}$$

and typically $> 2\tau$ are observed to ensure an adequate amount of out-of-transit baseline. The limb-darkened light curve can be calculate as follows

$$F(\Delta F^{1/2}(\lambda),\phi) = \left[\int_0^1 2\varpi I(\varpi)\mathrm{d}\varpi\right]^{-1}\int_0^1 I(\varpi)\mathrm{d}\varpi\frac{\mathrm{d}[F^u(\Delta F^{1/2}(\lambda)/\varpi,\phi/\varpi)\varpi^2]}{\mathrm{d}\varpi} \tag{13}$$

where $F(\Delta F^{1/2}(\lambda),\phi)$ is the flux decrement due to a planetary transit in front of a limb-darkened star. Here $I(\varpi)$ is the limb-darkened stellar intensity and given by $I(\varpi) = 1-\sum_{n=1}^4 c_n(1-\mu_\varpi^{n/2})$, where $\mu_\varpi = (1-\varpi^2)^{1/2}$ and c_n are limb-darkening coefficients taken from Claret (2000). `EChOSim` uses the quadratic limb-darkening coefficient approximation. F^u is the light curve of a uniform source (i.e. no limb-darkening) and is given by

$$F^u(\Delta F^{1/2}(\lambda),\phi) = 1-\lambda^u(\Delta F^{1/2}(\lambda),\phi) \tag{14}$$

and

$$\lambda^u(\Delta F^{1/2}(\lambda),\phi) = \begin{cases} 0, & 1+\Delta F^{1/2}(\lambda) < \phi, \\ \frac{1}{\pi}\left[\Delta F(\lambda)\kappa_0 + \kappa_1 - \sqrt{\frac{4\phi^2-(1+\phi^2-\Delta F(\lambda))}{4}}\right], & |1-\Delta F^{1/2}(\lambda)| < \phi \le 1+\phi, \\ \Delta F(\lambda), & \phi \le 1-\Delta F(\lambda), \\ 1, & \phi \le \Delta F(\lambda)-1, \end{cases} \tag{15}$$

where $\kappa_0 = \cos^{-1}[(\Delta F(\lambda)+\phi^2-1)/2\Delta F(\lambda)\phi]$ and $\kappa_1 = \cos^{-1}[(1-\Delta F(\lambda)+\phi^2)/2\phi]$. Here and above, ϕ is the normalised separation from the stellar and the planetary centres projected to the line of sight of the observer. For a circular orbit, this is given by

$$\phi = \frac{a}{R_S}\sqrt{1-(\cos^2(P/\tau_c)\sin^2 i)} \tag{16}$$

eccentric orbits require solving Kepler's equations, which is fully integrated into EChOsim. For more information on eccentric orbit calculations refer to Kipping (2008). For a detailed description of the complete light curve model implemented by `EChOSim`, please refer to Mandel & Agol (2002).

3.2 *Foregrounds* module

3.2.1 Local zodiacal emission

The zodiacal light is dominated at short wavelengths (< 3.5 μm) by scattered sunlight and at long wavelengths (> 3.5 μm) by thermal emission from the same dust. In agreement with the SST we will adopt a modified version of the JWST-MIRI Zodiacal "model" that is parameterised as follows:

$$I_{Zodi}(\lambda) = B_\lambda(5500K)3.5E^{-14} + B_\lambda(270K)3.58E^{-8} \quad W/m^2/sr/\mu m \tag{17}$$

Where $B_\lambda(T)$ is the Planck function for temperature T.

In order to represent the variation in Zodiacal light seen at different Ecliptic latitudes we will take the following cases:

Minimum = 0.9 x Zodi(λ)

Maximum = 8 x Zodi(λ)

Average = 2.5 x Zodi(λ)

Alternatively, the multiplicative factor can be user-defined. This is estimated from the best fit of equation 17 to the full Kelsell et al. (1998 ApJ. 508, 44) model, estimated at the ecliptic latitude and longitude of the target, and at the expected time of observation. The Kelsell et al. model is publicly available as an IDL procedure[1]

3.3 *Instrument* Module

The instrument module provides a description of the detection from the telescope to the absorption of the light by the focal plane detectors.

3.3.1 Telescope and Common Optics

The EChO telescope and common optics are made by a set of N_{tel} reflective surfaces (including M1 and M2) The telescope and each reflective surfaces can be described by:

- the collecting area $Aeff$ of the telescope
- the geometrical area of each surface
- the emission spectrum at (λ, T) assuming they are ideal grey-bodies with wavelength-dependent emissivities ε_{Mi} where 'i is the index of the reflective surface ($i = 1..N_{tel}$)
- the wavelength-dependent reflectivity of each surface : r_{Mi}

The overall telescope transmission is given as follows

$$\eta_{tel} = r_{M1} r_{M2} ... r_{MN_{tel}} \tag{18}$$

[1] http://lambda.gsfc.nasa.gov//product//cobe//dirbe_zodi_sw.cfm .

The spectral profile of the individual telescope reflective optics is given by the Planck equation

$$I_{Mi}(\lambda, T_i) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k T_i}} - 1} \quad [\text{W m}^{-2}\mu\text{m}^{-1}\ \text{sr}^{-1}] \quad , \tag{19}$$

and the emission spectra are then given by

$$I_i = \varepsilon_{Mi} I_{Mi} \tag{20}$$

The simulated telescope and common optics outputs are:

$$Q^T(t, \lambda) = \eta_{tel} A_{eff} F_{point}(t, \lambda) \ \ [\text{W}\mu\text{m}^{-1}] \tag{21}$$

$$Q^T_{zodi}(\lambda) = \eta_{tel} A_{eff} I_{zodi}(\lambda) \ \ [\text{W}\mu\text{m}^{-1}\text{sr}^{-1}] \tag{22}$$

$$Q^T_{telescope} = I_{N_{tel}} + \sum_{i=1}^{N_{tel}-1} I_i \prod_{j=i+1}^{N_{tel}} r_{Mj} \ \ [\text{W}\mu\text{m}^{-1}\text{sr}^{-1}] \tag{23}$$

where t is the time. Equation 21 is the point source (star+planet) and is time and wavelength-dependent. Equation 22 is the zodi contribution to the total signal. Equation 23 is the telescope emission contribution to the total signal, and it is supposed to be time-independent. The impact of temperature fluctuations to the noise budget can be investigated by running several simulations with different telescope temperatures.

The three data structures (Q^T, Q^T_{zodi}, $Q^T_{telescope}$) are propagated as three separate quantities for further processing.

3.3.2 Instrument

The output from the telescope is split by the EChO beam splitter and dichroic chain into the 5 EChO channels. Each element is described by a transmission, DT_k, and a reflection, DR_k wavelength-dependent response, where k is an index identifying the element number. A wavelength-dependent emissivity, ε_{Dk} is used to calculate the dichroic thermal emission.

This allows to estimate the optical input to each dispersive element. These are: $Q^C(t, \lambda)$ [Wμm^{-1}] is the power from the point source in ; $Q^C_{zodi}(\lambda)$ [Wμm^{-1} sr^{-1}] is the contribution from the zodi; $Q^C_{inst}(\lambda)$ [Wμm^{-1} sr^{-1}] is the contribution arising from the emission of all optical surface (telescope + elements in the optical path).

3.3.3 Dispersive optics

The spectral dispersion onto the focal plane is modeled by a linear dispersion law:

$$LD = \frac{\Delta x}{\Delta \lambda} = \frac{2\Delta_{pix}}{\lambda} R(\lambda) \tag{24}$$

Where $R(\lambda)$ is the spectral resolving power at a given wavelength, and a sampling of 2 pixels per spectral resolving element ($\Delta\lambda$) is assumed. The detector pixel size is Δ_{pix}. `EChOSim` assumes a constant LD estimated at the central wavelength of each channel. We have verified that this is a very good approximation for the SWIR and longer wavelength channels. For the VNIR, a constant resolving power is a more appropriate model of the dispersion law (see RD4) and so it's implemented in `EChOSim`.

3.3.4 Point Spread Function and Detector sampling

The dispersed signals are sampled by each detector by assuming a wavelength-dependent PSF, and a detector intra-pixel response function.

The PSF is approximated by a Gaussian function for the SWIR, MWIR and LWIR channels (VNIR treated differently). Optical aberrations are included. The advantage is in computational efficiency compared to a full diffraction pattern which could be accounted for using a Bessel representation of the PSF. This does not compromise the fidelity of the simulations as no background sources are currently implemented in the `EChOSim` simulations.

The PSF is then given by

$$p(x,y,\lambda) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-x_0(\lambda))^2}{2\sigma_x^2}} \times \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{y^2}{2\sigma_y^2}} \tag{25}$$

where x is the coordinate in the spectral direction, y is the coordinate in the spatial direction, and

$$\sigma_x = \frac{1}{\pi}\sqrt{2/K_x}F_{\#}\lambda, \quad \sigma_y = \frac{1}{\pi}\sqrt{2/K_y F_{\#}}\lambda. \tag{26}$$

Where $F_{\#}$ is the ratio between the channel effective focal length and the instrument effective diameter, defined as $A_{eff} = \frac{\pi}{4}\Phi_{eff}$. The two constants K_x and K_y accounts for channel-specific optical aberrations.

The PSF can be decomposed in the product of a spectral (along x) and spatial (along y) components

$$p(x,y,\lambda) = p(x,\lambda) \times p(y,\lambda).$$

The relation between the x coordinate in the array and wavelength is given by

$$x_0(\lambda) = LD(\lambda) \times (\lambda - \lambda_0)$$

where λ_0 is the central wavelength of each channel.

The VNIR channel is treated differently. Here the PSF is represented by the fibre response, given by the convolution of a top-hat function (of the size of the fibre) with the fibre diffraction pattern.

Each detector has an antenna response pattern or intra-pixel response. Following Barron et al. (2007, PASP, 119, 466) this is given by

$$F(y) = \arctan\left\{\tanh\left[\frac{1}{2l_d}\left(y + \frac{\Delta_{pix}}{2}\right)\right]\right\} - \arctan\left\{\tanh\left[\frac{1}{2l_d}\left(y - \frac{\Delta_{pix}}{2}\right)\right]\right\}$$

where l_d is the diffusion length, assumed to be 1.7μm (Barron et al., 2007).

The sampled PSF is the effective pixel response, obtained by combining the telescope PSF and the intra-pixel response. This is given by

$$p_s(x,y,\lambda) = p(x,\lambda) * F(x) \times p(y,\lambda) * F(y) = p_s(x,\lambda) \times p_s(y,\lambda)$$

where $*$ is the convolution operator.

3.3.5 Point source signal

The signal from the point source (star + planet) is sampled on the focal plane array. This is made of $N \times M$ detector pixels (N in the spectral direction and M in the spatial direction).

Each detector-pixel in the given focal plane array can be labeled by two indexes

$$i = -N/2...(N/2-1) \tag{27}$$
$$j = -M/2...(M/2-1) \tag{28}$$

The position coordinates of pixel centres are

$$(x_i, y_i) = (i\Delta_{pix}, j\Delta_{pix})$$

and the wavelength sampled at the centre of each pixel is

$$\lambda_i = i\frac{\Delta_{pix}}{LD} + \lambda_0$$

The point-source signal detected by each detector is then

$$Q^P_{ij}(t) = \int QE(\lambda)Q^C(t,\lambda)p_s[LD(\lambda-\lambda_i), y_j, \lambda]d\lambda \quad [\text{es}^{-1}]$$

This is simplified considering that the shape of the sampled PSF is a slow function of the wavelength, obtaining

$$Q^P_{ij}(t) = p_s(y_j, \lambda_i)\int QE(\lambda)Q^C(t,\lambda)p_s[LD(\lambda-\lambda_i), \lambda_i]d\lambda \quad [\text{es}^{-1}] \tag{29}$$

3.3.6 Diffuse radiation from sky (zodi)

A detector pixel subtend an angle in the sky given by Δ_p/f_{eff}, where f_{eff} is the channel effective focal length. The pixel solid angle is, assuming a square pixel,

$$\Omega_p = \left(\frac{\Delta_{pix}}{f_{eff}}\right)^2$$

If L is the width in pixel of the spectrometer slit, then each detector pixel receive radiation in the wavelength interval

$$\left[\lambda_i - \frac{1}{2}\frac{L\Delta_p}{LD}, \lambda_i + \frac{1}{2}\frac{L\Delta_p}{LD}\right].$$

Therefore the pixel response to diffuse radiation from the sky is given by the convolution of the spectrum with a top-hat function:

$$Q^Z_{ij} = \Omega_p \int_{\lambda_i - \frac{1}{2}\frac{L\Delta_p}{LD}}^{\lambda_i + \frac{1}{2}\frac{L\Delta_p}{LD}} QE(\lambda)Q^C_{zodi}(\lambda)d\lambda \quad [\text{es}^{-1}] \tag{30}$$

3.3.7 Instrument emission

The emission from the instrument before the dispersing element depends from the entendue, which is $\frac{\pi}{4}\frac{\Delta_p^2}{f_\#^2}$,where $f_\#$ is the channel working f-number. Similarly to the case of diffused sky radiation, the pixel response is

$$Q^I_{ij} = \frac{\pi}{4}\frac{\Delta_p^2}{f_\#^2}\int_{\lambda_i - \frac{1}{2}\frac{L\Delta_p}{LD}}^{\lambda_i + \frac{1}{2}\frac{L\Delta_p}{LD}} QE(\lambda)Q^C_{inst}(\lambda)d\lambda \quad [\text{es}^{-1}]$$

3.4 *Noise* Module

The output of the instrument module are noise-less timelines. The sampling cadence is chosen to be the minimum required to Nyquist sample the time-varying astronomical signal, which is the modulation represented by the light-curve.

The first task of the noise module is to re-sample the timelines (point source, diffuse emission and instrument emission) to the user-defined sampling rate.

Detector effects are added to each timeline.

Detector inter-pixel response is treated as a detector-dependent gain. Each detector gain is randomly applied to the signal timelines:

$$G_{ij} = N(1, \sigma)$$

where σ is a user defined parameter and N indicates a Gaussian random number generator.

The detector pixel dark current is estimated as a detector temperature-dependent exponential law:

$$I_{DC} = I_0 e^{-\alpha/T}.$$

when available, with obvious meaning of the symbols used. Otherwise a fixed dark current is used.

The total timeline is then given by

$$Q_{ij}^{tot} = \left[G_{ij}\left(Q_{ij}^{P} + Q_{ij}^{Z} + Q_{ij}^{I}\right) + I_{DC}\right]\Delta T \tag{31}$$

To this signal-only timeline, noise is added in the time domain.

Blip noise, accounting for both photon and dark current noise, is estimated assuming PV detectors as

$$Q_{ij}^{B} = N(0, \sqrt{Q_{ij}^{tot}})$$

which is a good approximation as the number of electrons involved is always large enough to approximate a Poisson distribution with a Gaussian.

Detector **readout noise** is estimated as a Gaussian random process

$$Q_{ij}^{ro} = N(0, \sigma_{ro})$$

This is the effective readout noise which depends from the readout method used. For Correlated double sampling $\sigma_{ro} = \sqrt{2}\sigma_r$ were σ_r is the noise on an individual read-event. When the follow-up-the-ramp method is used, then

$$\sigma_{ro} = 12N/[N(N^2-1)]\sigma_r^2$$

Both in normal mode, and in bright mode currently being considered, the readout noise is negligible compared to other sources of noise. However, the user should provide the parameter σ_{ro} accordingly to the assumed read-out strategy.

The pointing stability of the telescope is simulated as an additional source of photometric uncertainty and it is modelled as an additional noise term. The effect manifests both in the spatial domain and in the spectral domain. `EChOSim` follows the approach of Deroo and Swain 2011

(Optimising Instrument Stability for Exoplanet Spectrophotometry) and only jitter in the spatial domain is implemented. Jitter in the spectral domain is effectively de-correlated in the data reduction process by tracking the shift through the many star spectral lines.

Jitter in the spatial domain does not affect diffuse radiation, and only affect the astronomical signal from star+planet. Using equation 29 the pointing jitter in the spatial domain provides a noise term

$$Q^J_{ij}(t) = \Delta T Q^P_{ij} \frac{1}{p_s} \left(\frac{\partial p_s}{\partial y_j} < \delta y >_{\Delta T} + \frac{1}{2} \frac{\partial^2 p_s}{\partial y_j^2} < \delta y^2 >_{\Delta T} \right) \tag{32}$$

Where the operation $<>_{\Delta T}$ indicates a time average over the integration time.

The focal plane displacement δy can be expressed in terms of the angular displacement of the telescope line of sight, $\delta\theta$ as:

$$\delta y = f_{eff} \delta\theta$$

The time-dependent variable $\delta\theta$ is generated using a Gaussian random process with a user defined coloured power spectral density expressed in units of masc/$\sqrt{\text{Hz}}$, to represent the short term (RPE) and long term (PRE) pointing jitter.

The jitter information $\delta\theta$ is generated at high frequency, which is then averaged and decimated to the user-defined integration time.

The same jitter $\delta\theta$ is applied to every detector, as this is a source of correlated noise across the focal plane.

Housekeeping information is also generated in 1s integration to represent information from the FGS. This HK information is generated from the same jitter variable $\delta\theta$. Additional noise is added to this HK signal to simulate the uncertainties associated with the knowledge of the pointing reported by FGS. Therefore

$$\delta\theta_{HK}(t) =< \delta\theta >_{1\text{s}} + N(0, \sigma_{FGS})$$

Where $\delta\theta_{HK}(t)$ is the FGS HK information at a rate of 1 second, and N represents a white noise random number generator, with zero mean, and a standard deviation given by $\sigma_F GS$.

All noise sources are added to the total timeline:

$$Q^{out}_{ij} = Q^{tot}_{ij} + Q^B_{ij}(t) + Q^{ro}_{ij}(t) + Q^J_{ij}(t) \tag{33}$$

In order to simulate the co-addition of several independent transits, we assume that the noise is uncorrelated between one transit observation and an other, and therefore averages down as $\sqrt{N_t}$, therefore obtaining

$$Q^{out}_{ij} = Q^{tot}_{ij} + \left[Q^B_{ij}(t) + Q^{ro}_{ij}(t) + Q^J_{ij}(t) \right] / \sqrt{N_t} \tag{34}$$

3.5 *Output* Module

In the output module each detector timeline Q^{out}_{ij} is saved into fits files. Each fits file is a frame, i.e. data collected during one integration time. Each fits file contains a number of images extensions, each corresponding to an EChO channel. Additional housekeeping information is also stored in the fits header. At the moment this includes only the FGS HK, but can include S/C telemetry HK when required.

3.6 *Observational Output* Module

A data reduction pipeline is included in the `EChOSim` software package for convenience, although this observational module should not be considered part of `EChOSim`. The reduction pipeline currently includes the following steps:

- Read of fits files and meta data.
- The 1D spectrum is extracted using optimal PSF (from `EChOSim`). If set, the PSF is attenuated by the FGS centroiding for PRE jitter de trending.
- The spectrum is binned given specifications. This binning can be log-constant or constant in Resolution.
- The spectra are background subtracted and 1D time series are created.
- The time series are normalised (if set to MCMC, this normalisation is left as free parameter).
- Wavelength dependent stellar limb-darkening grids are calculated.
- The radiometric model output is calculated.
- Individual time series are model fitted using a Mandel & Agol (2002) analytical light curve modelling code. The model fitting can be run using a Markov Chain Monte Carlo (MCMC) or a Simplex Downhill algorithm.
- The results are collated, the Signal-to-Noise of the observation is calculated and spectra and SNR figures are created. The results are also dumped to disk in ascii format.

In the *phase II* implementation of `EChOSim` the data will be de-trended using various statical component separation techniques (PCA/ICA) to de-correlate wavelength and time-dependent trends in the data. A full statistical analysis of lightcurve parameter fits and potential data-degeneracies and limitations is envisaged.

4 APPENDIXES

4.1 Appendix 1 : Correspondence between stellar type, effective temperature and stellar radius for a dwarf star.

Spectral type	Effective temperature (K)	Stellar radius (R_{sun})
F2	7000	1.5
F5	6650	1.4
F8	6250	1.2
G0	5940	1.05
G2	5790	1.00
G5	5560	0.92
G8	5310	0.84
K0	5150	0.79
K5	4410	0.67
M0	3850	0.54
M0.5	3790	0.52
M1	3720	0.50
M1.5	3650	0.46
M2	3580	0.41
M2.5	3520	0.37
M3	3470	0.32
M3.5	3420	0.27
M4	3370	0.25
M4.5	3310	0.22
M5	3200	0.18
M5.5	3070	0.16

Table 5: Correspondance between stellar type, effective temperature and stellar radius.

4.2 Appendix 2: Stellar distance to the Earth for several type of stars as a function of their magnitude.

Type	Mstar (Msun)	Teff (K)	Rstar (Rsun)	Dist. (pc)	K	V	Tplan (K)	Rplan (Re)	P (days)	a (au)
Hot Super earths										
M4	0.25	3370	0.25	20.72	9	13.70	1500	1.8	0.11	0.0028
M2	0.41	3580	0.41	23.44	8	12.30	1500	1.8	0.21	0.0052
K7	0.61	4130	0.61	26.64	7	10.47	1500	1.8	0.48	0.0102
K0	0.79	5150	0.79	25.70	6	8.11	1500	1.8	1.21	0.0205
G0	1.05	5940	1.05	23.44	5	6.48	1500	1.8	2.47	0.0363
Warm Super earths										
M4	0.25	3370	0.25	20.72	9	13.70	600	1.8	1.39	0.0153
M2	0.41	3580	0.41	23.44	8	12.30	600	1.8	2.73	0.0284
K7	0.61	4130	0.61	26.64	7	10.47	600	1.8	6.23	0.0562
Temperate Super earths										
M4	0.25	3370	0.25	13.08	8	12.70	320	1.8	9.15	0.0539
M2	0.41	3580	0.41	14.79	7	11.30	320	1.8	17.98	0.0998
M7	0.61	4130	0.61	26.64	7	10.47	320	1.8	41.08	0.1976
Hot Neptunes										
K7	0.61	4130	0.61	42.23	8	11.47	1500	4.0	0.48	0.0102
K0	0.79	5150	0.79	40.74	7	9.11	1500	4.0	1.21	0.0205
G0	1.05	5940	1.05	37.15	6	7.48	1500	4.0	2.47	0.0363
Warm Neptunes										
K7	0.61	4130	0.61	42.23	8	11.47	600	4.0	6.23	0.0562
K0	0.79	5150	0.79	64.56	8	10.11	600	4.0	15.65	0.1132
Hot Jupiters										
K0	0.79	5150	0.79	102.32	9	11.11	1500	10.0	1.21	0.0205
G0	1.05	5940	1.05	93.32	8	9.48	1500	10.0	2.47	0.0363

Table 6: Characteristics of targets

Appendix 3 : table of planetary parameters used for the radiometric description of the planetary contributions to the planetary signal

Parameter	**Value**
T_{hot}	1500 K
T_{warm}	600 K
$T_{temperate}$	320 K
Bond albedo = optical albedo: Hot planet Warm/temperate	0.1 0.3
Radius of “Jupiter”	10 r_{earth}
Radius of “Neptune”	4 r_{earth}
Radius of SuperEarth	1.8 r_{earth}
Mass of “Jupiter”	300 M_{Earth}
Mass of “Neptune”	15 M_{Earth}
Mass of Super Earth	5 M_{Earth}
Mean molecular weight (to determine scale height of atmosphere) Jupiters Earth-like	2g/mole 18g/mole
Number of scale heights to be used	7
Dayside view factor	1.0
Orbital eccentricity	0.

Table 7: table of planetary parameters used for the radiometric description of the planetary contributions to the planetary signal