Dynamics of Saturn's small moons in coupled first order planar resonances

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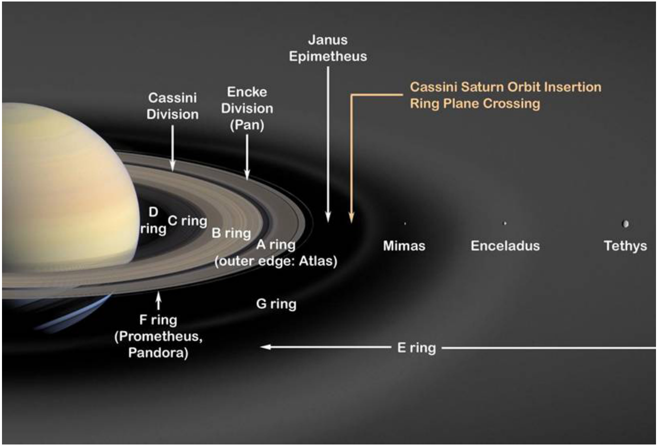
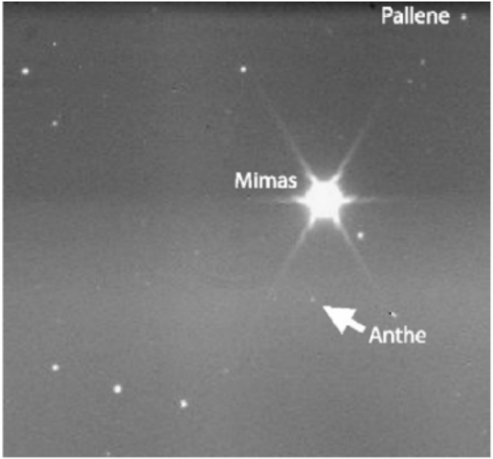
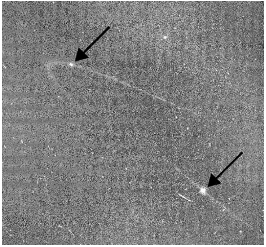
Saturn system

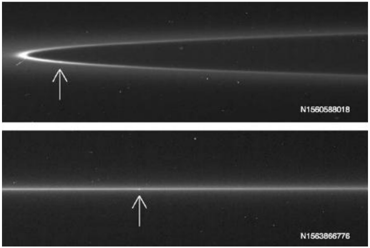


Very small moons



New satellites : Anthe, Methone and Aegaeon





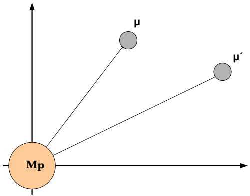
(Cooper et al., 2008 ; Hedman et al., 2009, 2010 ; Porco et al., 2005)

Very small (0.5 km to 2 km)
Vicinity of the Mimas orbit (outside and inside)

The aims of the work

A better understanding :
- of the dynamics of this population of news satellites
- of the scenario of capture into mean motion resonances

Dynamical structure of the system



We consider only :

- The resonant terms
- The secular terms causing the precessions of the orbit

When $\mu \to 0 \Rightarrow$ The symmetry is broken $\Rightarrow$ different kinds of resonances :

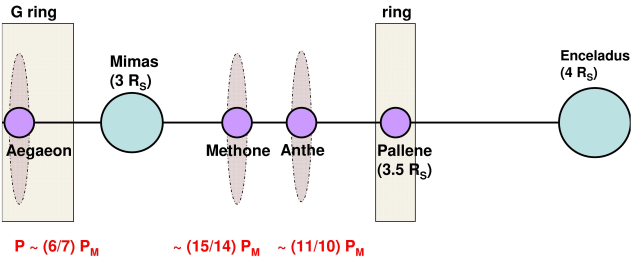
- Lindblad Resonance
- Corotation Resonance

D'Alembert rules :

$$\psi_c = (m+1)\lambda' - m\lambda - \varpi'$$
$$\psi_L = (m+1)\lambda' - m\lambda - \varpi$$

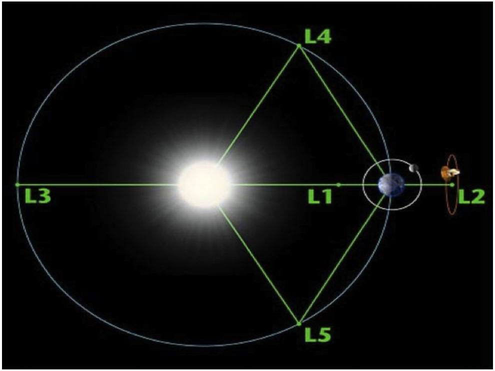
Corotation Resonance

- Aegaeon (7/6) : $\psi_c = 7\lambda_{Mimas} - 6\lambda_{Aegaeon} - \varpi_{Mimas}$
- Methone (14/15) : $\psi_c = 15\lambda_{Methone} - 14\lambda_{Mimas} - \varpi_{Mimas}$
- Anthe (10/11) : $\psi_c = 11\lambda_{Anthe} - 10\lambda_{Mimas} - \varpi_{Mimas}$

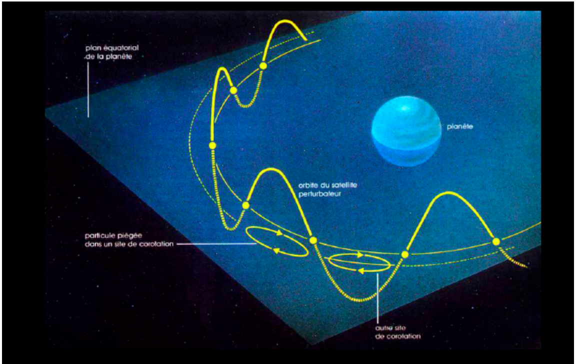


Corotation resonances

Mean motion resonance : $\frac{n_1}{n_2} = \frac{m+q}{m}$
Particular case : Lagrangian Equilibrium Points



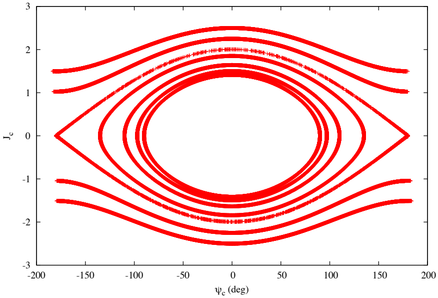
Adam's ring and Galatea



Sicardy, *La Recherche*, janvier 1994

Corotation resonances : Pendular motion

$$\begin{cases} \dot{\chi}_c = -\epsilon_c \sin(\psi_c) \\ \dot{\psi}_c = \chi_c \end{cases} \quad \text{where :} \quad \begin{cases} \chi_c = \frac{3}{2} m \frac{a-a_c}{a} \\ \epsilon_c = 3m^2 \frac{M_s}{M_p} \frac{a}{a_c} E e_s \end{cases}$$



Lindblad Resonance

$$\begin{cases} \dot{h} = +\frac{\partial H}{\partial k} = -(J_C - J_L + D)k \\ \dot{k} = -\frac{\partial H}{\partial h} = +(J_C - J_L + D)h + \varepsilon_L \end{cases} \quad (1)$$

e=(h,k) : the vector eccentricity $\begin{cases} h \propto e\cos(\psi_L) \\ k \propto e\sin(\psi_L) \end{cases}$

$J_L \propto \frac{e^2}{2}$

$H_{And} = J_L^2 - (J_c + D)J_L - \varepsilon_L h$

Dynamical study

Equations of motions $\Rightarrow$ The Coralin (CORotation And LINdblad) Model

$$\psi_c = (m+1)\lambda' - m\lambda - \varpi' \text{ near } \dot{\psi}_c = 0$$
$$\psi_L = (m+1)\lambda' - m\lambda - \varpi \text{ near } \dot{\psi}_L = 0$$

The coupling leads to a chaotic motion
Asymptotic cases
General solutions based on adiabatic invariance arguments

Hamiltonian formalism : The Coralin Model

$$\begin{cases} \dot{J}_c = -\frac{\partial H}{\partial \psi_c} = -\sin(\psi_c) \\ \dot{\psi}_c = +\frac{\partial H}{\partial J_c} = J_C - [J_L] \\ \dot{h} = +\frac{\partial H}{\partial k} = -([J_C] - J_L + D)k \\ \dot{k} = -\frac{\partial H}{\partial h} = +([J_C] - J_L + D)h + \varepsilon_L \end{cases} \quad (2)$$

where the Hamiltonian H is given by :

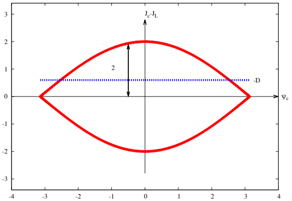
$$H(J_c, \psi_C, J_L, \psi_L) = \frac{1}{2}(J_c - J_L)^2 - DJ_L - \cos(\psi_C) - \varepsilon_L\sqrt{3}|m|e\cos(\psi_L)$$

where :

e=(h,k) : the vector eccentricity $\begin{cases} h \propto e\cos(\psi_L) \\ k \propto e\sin(\psi_L) \end{cases}$

$J_L \propto \frac{e^2}{2}$

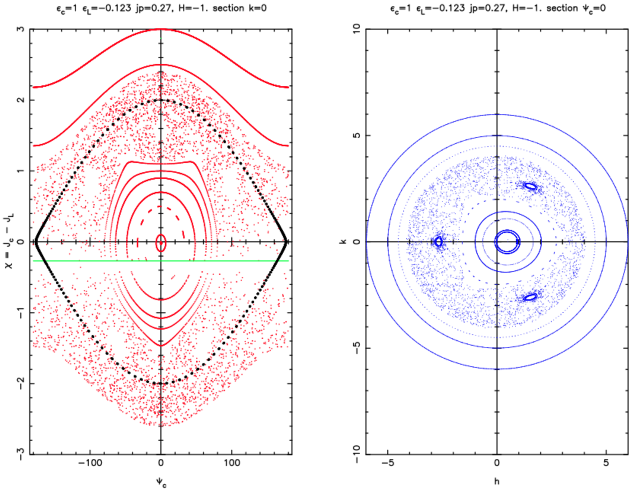
To simplify



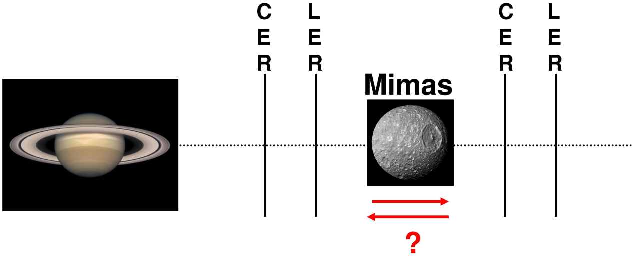
This study depends essentially of D which gives the distance between the two resonances, and the forcing value ε_L for a given value of the width of CER.

	D	ε_L
Anthe	0.256	-0.133
Aegaeon	-1.34	0.132
Methone	0.122	-0.115

Case Anthe


εc=1 εL=−0.123 jp=0.27, H=−1. section k=0
χ = Jc − JL
ψc
−100
0
100
εc=1 εL=−0.123 jp=0.27, H=−1. section ψc=0
k
h
−5
0
5
10
−10

Probability of capturing into the Corotation Eccentric Resonance (CER)



- add migration in the CoraLin model to estimate the capture probabilities into CER perturbed by LER

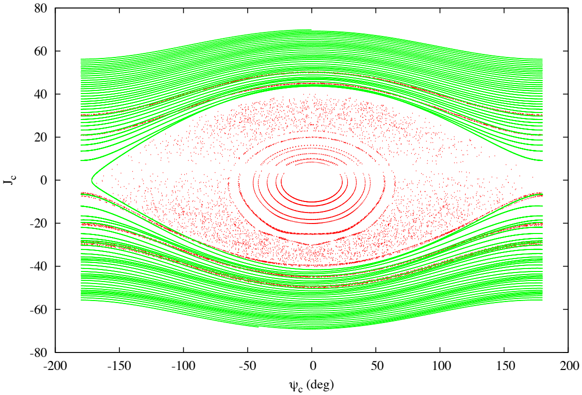
Hamiltonian formalism with migration

$$\begin{cases} \dot{J}_c = -\sin(\psi_c) + \dot{a}/2a \\ \dot{\psi}_c = J_C - J_L = \chi \\ \dot{h} = -(J_C - J_L + D)k \\ \dot{k} = +(J_C - J_L + D)h + \varepsilon_L \end{cases} \quad (3)$$

$T_{av} = \dot{a}/2a = \alpha + \beta\chi$

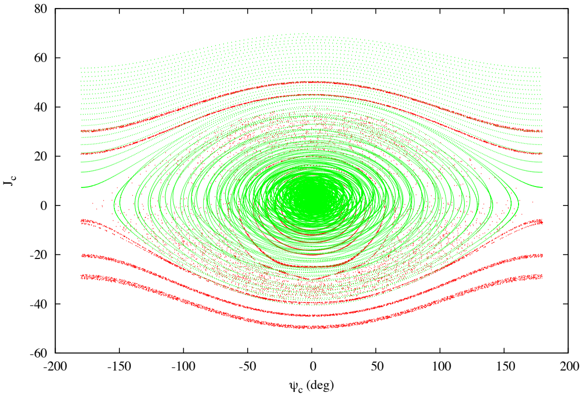
constant torque : No capture

$T_{av} = \alpha$



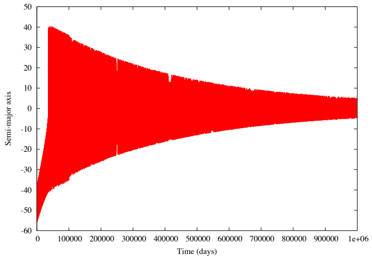
torque with gradient : possible capture

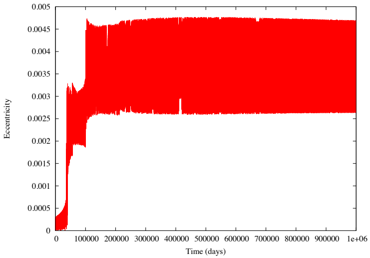
$T_{av} = \alpha + \beta\chi$



Torque with gradient

$T_{av} = \alpha + \beta\chi$

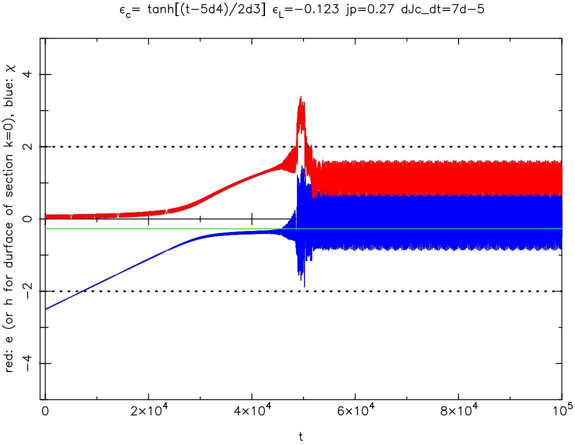




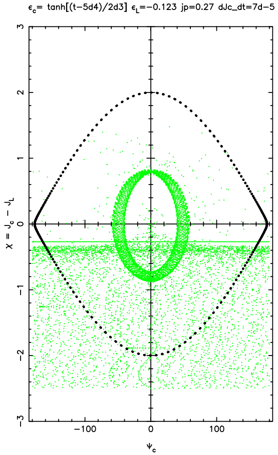
a = 197655.4 km

e = 0.003

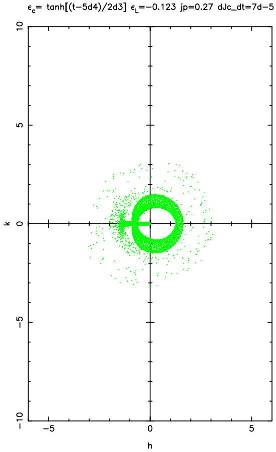
Case Anthe


ϵ_c= tanh[(t−5d4)/2d3] ϵ_L=−0.123 jp=0.27 dJc_dt=7d−5
red: e (or h for durface of section k=0), blue: χ
t
0
2×10^4
4×10^4
6×10^4
8×10^4
10^5
−4
−2
0
2
4

Case Anthe



ϵ_c= tanh[(t−5d4)/2d3] ϵ_L=−0.123 jp=0.27 dJc_dt=7d−5
$\chi = J_c - J_L$
Ψ_c
−100
0
100
3
2
1
0
−1
−2
−3



ϵ_c= tanh[(t−5d4)/2d3] ϵ_L=−0.123 jp=0.27 dJc_dt=7d−5
k
h
−5
0
5
10
−10

Conclusions and prospects

- CoraLin : simple model to investigate the dynamics of a satellite on CER/LER
- Capture probabilities into CER + applications to our satellites
- Anthe crosses a chaotic region favors the capture
- The capture is sure in the second scenario
- For Aegaeon : need a migration in the other direction.