Dynamics of Saturn’s small moons in coupled first order planar resonances

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26 juin 2012
Saturn system
Very small moons
New satellites: Anthe, Methone and Aegaeon

(Cooper et al., 2008; Hedman et al., 2009, 2010; Porco et al., 2005)

Very small (0.5 km to 2 km)
Vicinity of the Mimas orbit (outside and inside)

The aims of the work
A better understanding:
- of the dynamics of this population of new satellites
- of the scenario of capture into mean motion resonances
Dynamical structure of the system

We consider only:
- The resonant terms
- The secular terms causing the precessions of the orbit

When $\mu \to 0 \Rightarrow$ The symmetry is broken $\Rightarrow$ different kinds of resonances:
- Lindblad Resonance
- Corotation Resonance

D’Alembert rules:

$$\psi_c = (m + 1)\lambda' - m\lambda - \omega'$$

$$\psi_I = (m + 1)\lambda' - m\lambda - \omega$$
Corotation Resonance

- Aegaeon (7/6) : $\psi_c = 7\lambda_{Mimas} - 6\lambda_{Aegaeon} - \varpi_{Mimas}$
- Methone (14/15) : $\psi_c = 15\lambda_{Methone} - 14\lambda_{Mimas} - \varpi_{Mimas}$
- Anthe (10/11) : $\psi_c = 11\lambda_{Anthe} - 10\lambda_{Mimas} - \varpi_{Mimas}$
Corotation resonances

Mean motion resonance: $\frac{n_1}{n_2} = \frac{m+q}{m}$

Particular case: Lagrangian Equilibrium Points
Adam’s ring and Galatea

Corotation resonances: Pendular motion

\[
\begin{align*}
\dot{\chi}_c &= -\epsilon_c \sin(\psi_c) \\
\dot{\psi}_c &= \chi_c
\end{align*}
\]

where:

\[
\begin{align*}
\chi_c &= \frac{3}{2} m \frac{a-a_c}{a} \\
\epsilon_c &= 3m^2 \frac{M_s}{M_p} \frac{a}{a_c} E_e_s
\end{align*}
\]
Lindblad Resonance

\[
\begin{align*}
\dot{h} &= + \frac{\partial H}{\partial k} = -(J_C - J_L + D)k \\
\dot{k} &= - \frac{\partial H}{\partial h} = +(J_C - J_L + D)h + \varepsilon_L 
\end{align*}
\]

\[e = (h, k): \text{the vector eccentricity}\]

\[J_L \propto \frac{e^2}{2}\]

\[H_{And} = J_L^2 - (J_C + D)J_L - \varepsilon_L h\]
Dynamical study

Equations of motions ⇒ The Coralin (CORotation And LINdblad) Model

\[ \psi_c = (m + 1)\lambda' - m\lambda - \varpi' \text{ near } \dot{\psi}_c = 0 \]
\[ \psi_L = (m + 1)\lambda' - m\lambda - \varpi \text{ near } \dot{\psi}_L = 0 \]

The coupling leads to a chaotic motion
Asymptotic cases
General solutions based on adiabatic invariance arguments
Hamiltonian formalism : The Coralin Model

\[
\begin{align*}
\dot{J}_c &= -\frac{\partial H}{\partial \psi_c} = -\sin(\psi_c) \\
\dot{\psi}_c &= +\frac{\partial H}{\partial J_c} = J_C - [J_L] \\
\dot{h} &= +\frac{\partial H}{\partial k} = -([J_C] - J_L + D)k \\
\dot{k} &= -\frac{\partial H}{\partial h} = +([J_C] - J_L + D)h + \varepsilon_L
\end{align*}
\]

where the Hamiltonian $H$ is given by :

\[
H(J_c, \psi_C, J_L, \psi_L) = \frac{1}{2}(J_c - J_L)^2 - DJ_L - \cos(\psi_C) - \varepsilon_L \sqrt{3}m|e\cos(\psi_L)
\]

where :

$e = (h,k)$ : the vector eccentricity

\[
\begin{align*}
J_L &\propto \frac{e^2}{2} \\
h &\propto e \cos(\psi_L) \\
k &\propto e \sin(\psi_L)
\end{align*}
\]
To simplify

This study depends essentially of $D$ which gives the distance between the two resonances, and the forcing value $\varepsilon_L$ for a given value of the width of CER.

<table>
<thead>
<tr>
<th></th>
<th>$D$</th>
<th>$\varepsilon_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anthe</td>
<td>0.256</td>
<td>-0.133</td>
</tr>
<tr>
<td>Aegaeon</td>
<td>-1.34</td>
<td>0.132</td>
</tr>
<tr>
<td>Methone</td>
<td>0.122</td>
<td>-0.115</td>
</tr>
</tbody>
</table>
Case Anthe

$e_c = 1, \quad e_L = -0.123, \quad j = 0.27, \quad H = -1. \quad \text{section } k = 0$

$e_c = 1, \quad e_L = -0.123, \quad j = 0.27, \quad H = -1. \quad \text{section } \psi_c = 0$
Probability of capturing into the Corotation Eccentric Resonance (CER)

- add migration in the CoraLin model to estimate the capture probabilities into CER perturbed by LER
Hamiltonian formalism with migration

\[
\begin{align*}
\dot{J}_c &= - \sin(\psi_c) + \dot{a}/2a \\
\dot{\psi}_c &= J_C - J_L = \chi \\
\dot{h} &= -(J_C - J_L + D)k \\
\dot{k} &= +(J_C - J_L + D)h + \varepsilon_L
\end{align*}
\]

\[T_{av} = \dot{a}/2a = \alpha + \beta \chi\]
constant torque : No capture

\[ T_{av} = \alpha \]
torque with gradient: possible capture

\[ T_{av} = \alpha + \beta \chi \]
Torque with gradient

\[ T_{av} = \alpha + \beta \chi \]

\[ a = 197655.4 \text{ km} \]

\[ e = 0.003 \]
Case Anthe

$$\epsilon_c = \tanh[(t-5d4)/2d3] \quad \epsilon_L = -0.123 \quad j_p = 0.27 \quad dJc/dt = 7d - 5$$
Case Anthe

\[ \epsilon_c = \tanh(\frac{(t-5d4)/2d3}{2d3}) \]

\[ \epsilon_c = -0.123 \]

\[ \frac{dp}{dt} = 0.27 \]

\[ \frac{d\xi}{dt} = 7d-5 \]
Conclusions and prospects
- CoraLin : simple model to investigate the dynamics of a satellite on CER/LER
- Capture probabilities into CER + applications to our satellites
- Anthe crosses a chaotic region favors the capture
- The capture is sure in the second scenario
- For Aegaeon : need a migration in the other direction.