The Curlometer and other multipoint analysis

M W Dunlop[1,2]

with contributions from:

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Four-spacecraft gradients

Accurate quantities at each point in time

Context for plasma measurements (magnetic field)

CCLRC Rutherford Appleton Laboratory

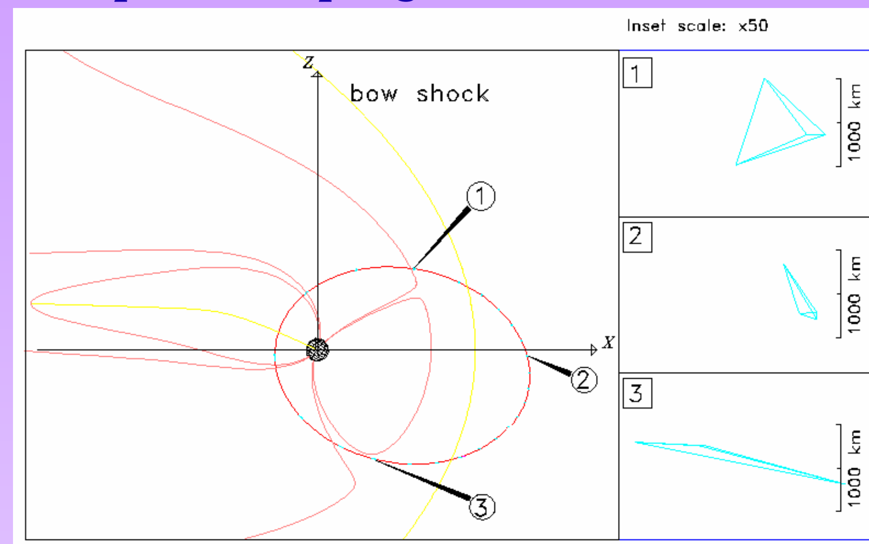
Outline

- **Curlometer**
  - MP currents and boundary thickness
  - Cusp boundaries
  - Currents in FTEs
  - Tail current sheet
  - Ring current
- **New gradient methods**
  - Rotation and curvature from $\nabla \boldsymbol{B}$ and $\nabla |\boldsymbol{B}|$
  - Dimensional derivative and spatial/temporal derivative
- **Other multi-point techniques**
  - Gradient of n
  - Partial currents (Harris sheet)
- **Summary**

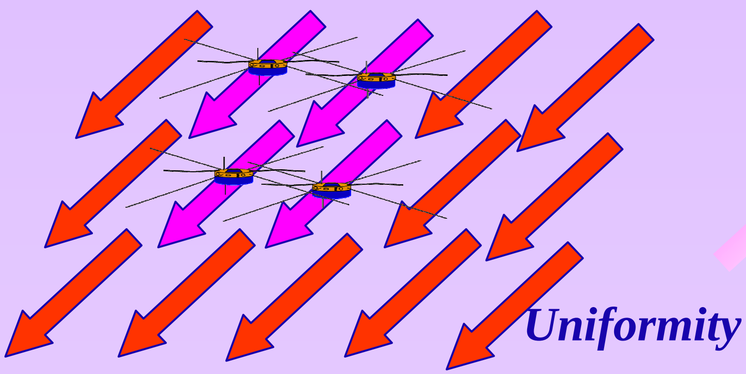
Cluster spatial/temporal problem:
issue is temporal evolution

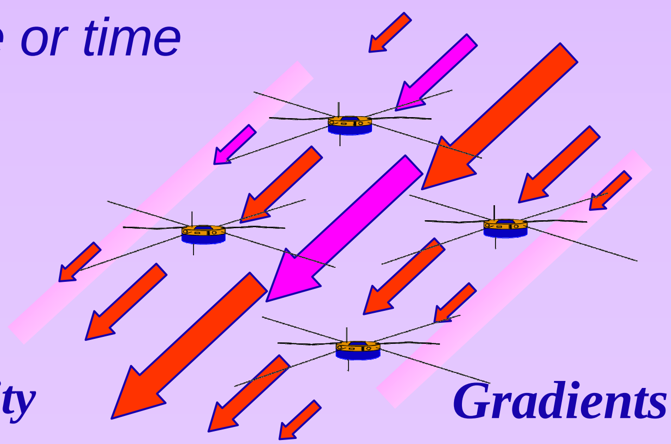
ISSI book on 4-spacecraft analysis methods

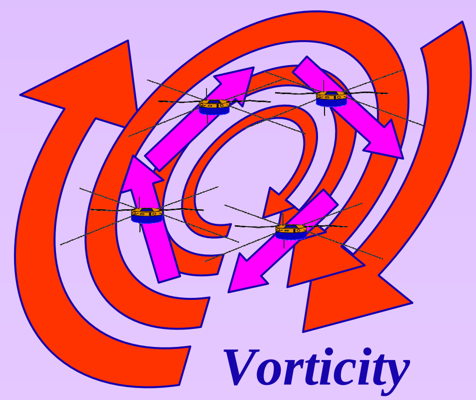
Spatial sampling: orbital constraints



Structure is not fixed in space or time



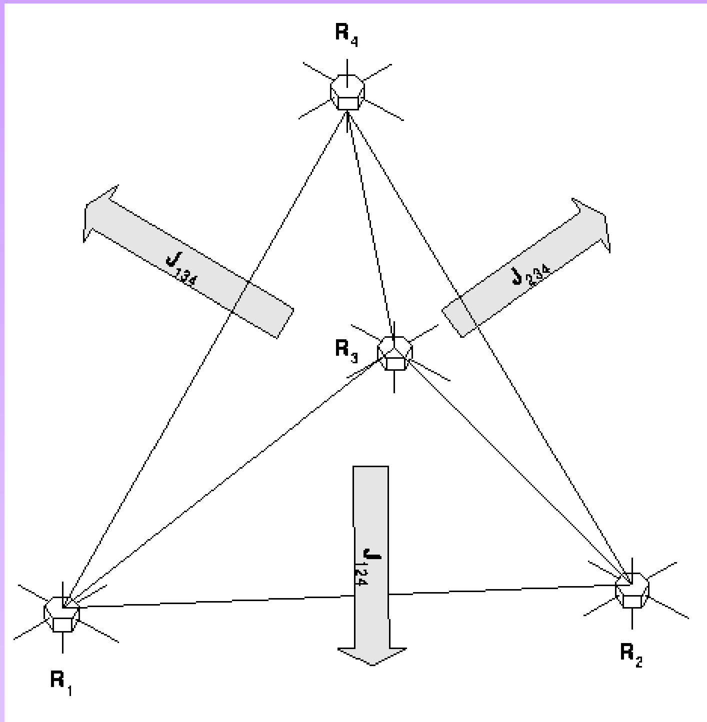




- **Curlometer**
  - MP currents and boundary thickness
  - Cusp boundaries
  - Currents in FTEs
  - Tail current sheet
  - Ring current

The Curlometer

Dunlop et al. 1988, Robert et al. 1998.



♦ Uses Ampère's law to estimate the average current density through the tetrahedron:

$$\mu_0 \mathbf{J} \cdot (\Delta \mathbf{R}_i \times \Delta \mathbf{R}_j) = \Delta \mathbf{B}_i \cdot \Delta \mathbf{R}_j - \Delta \mathbf{B}_j \cdot \Delta \mathbf{R}_i$$

we also have:

$$div(\mathbf{B})|\Delta \mathbf{R}_i \cdot \Delta \mathbf{R}_j \times \Delta \mathbf{R}_k| = |\sum_{cyclic} \Delta \mathbf{B}_i \cdot \Delta \mathbf{R}_j \times \Delta \mathbf{R}_k|$$

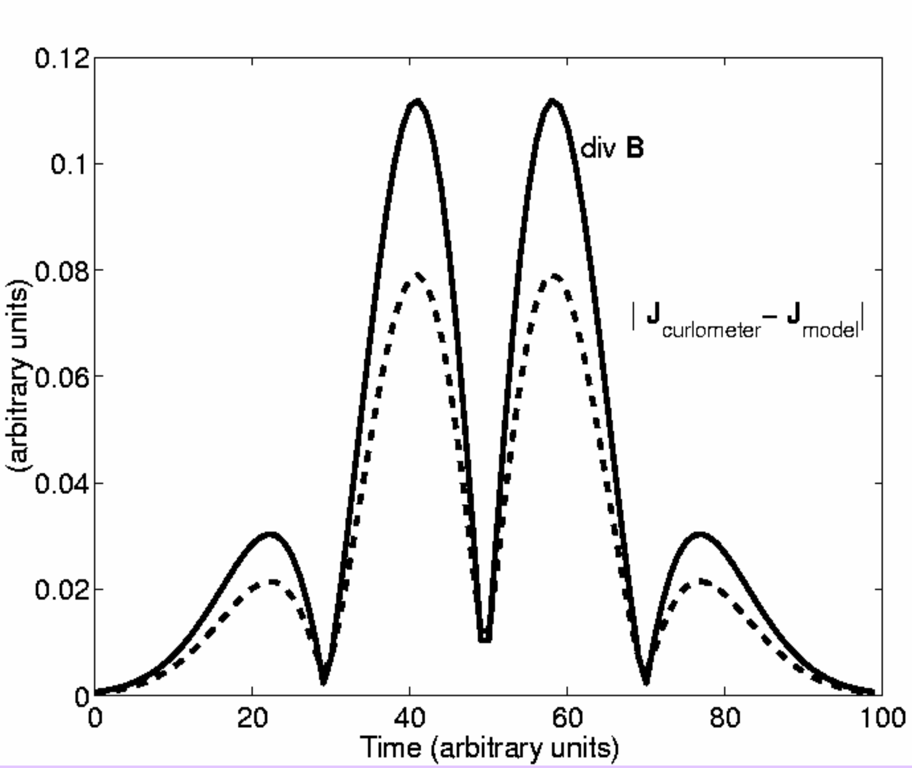
♦ This estimates ***J*** normal to the face 1ij of the tetrahedron.

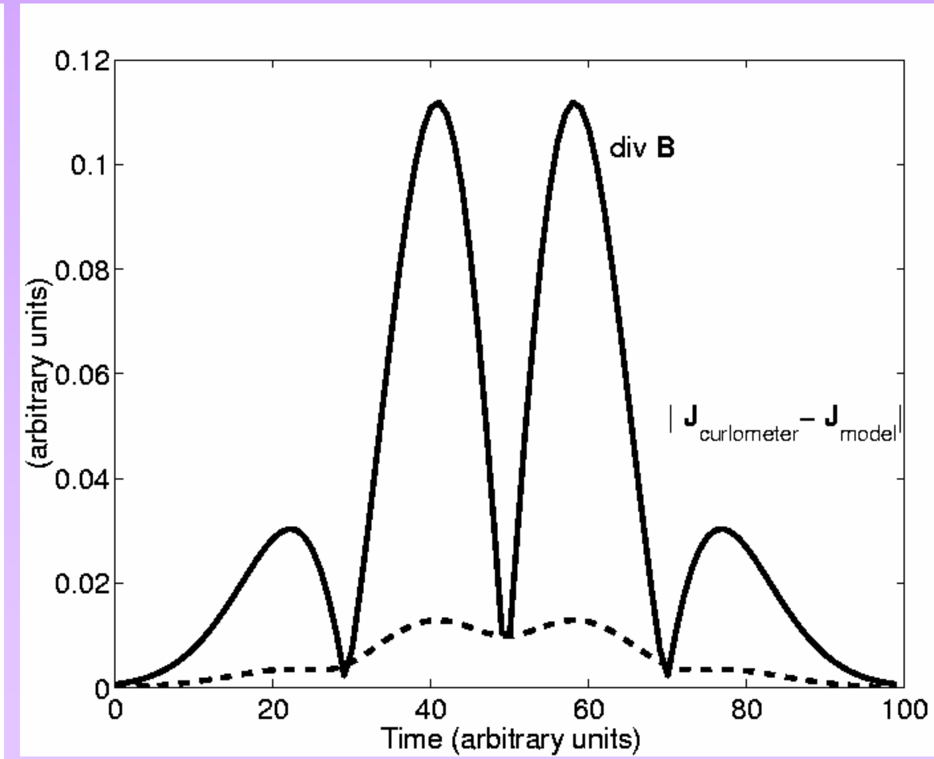
♦ Assumption: linear field variation between spacecraft.

- Calculation of *div(**B**)* provides a guide to the quality of the ***J*** estimate (spatial scale).
- Orientation the spacecraft configuration to the magnetic field structure is critical.
- Temporal scale (variations) complicate the effect of the estimates of ***J*** and *div(**B**)*.

Behaviour of div(B): model calculation

- *div(**B**)* and $|\boldsymbol{J}_{curlometer} - \boldsymbol{J}_{model}|$ calculated for a current sheet model
- For simple structures this effect turns out to be not critical (e.g. 1-D boundary)

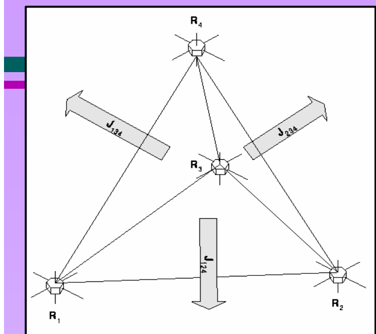




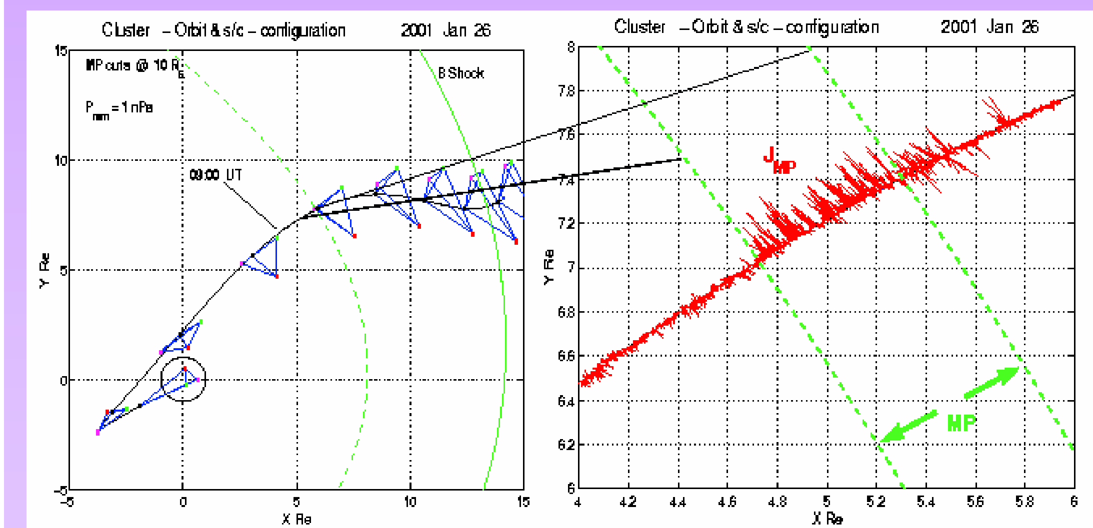
Regular configuration (relative scale: 0.2 to the model) Irregular configuration

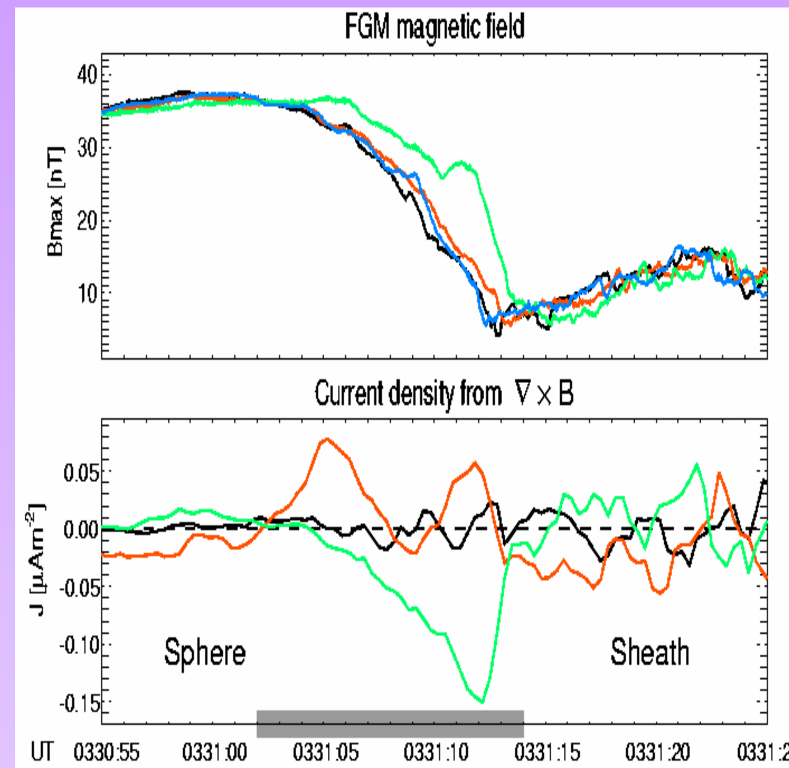
Currents in the magnetopause:

Dunlop and Balogh. 2005, Haarland et al. 2004, Xiao et al. 2004.



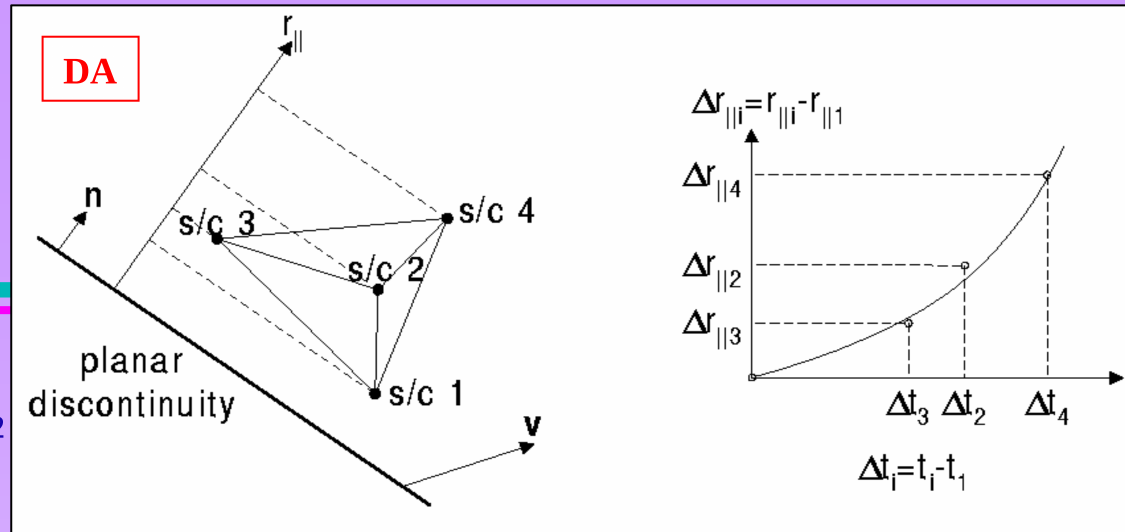
Curlometer



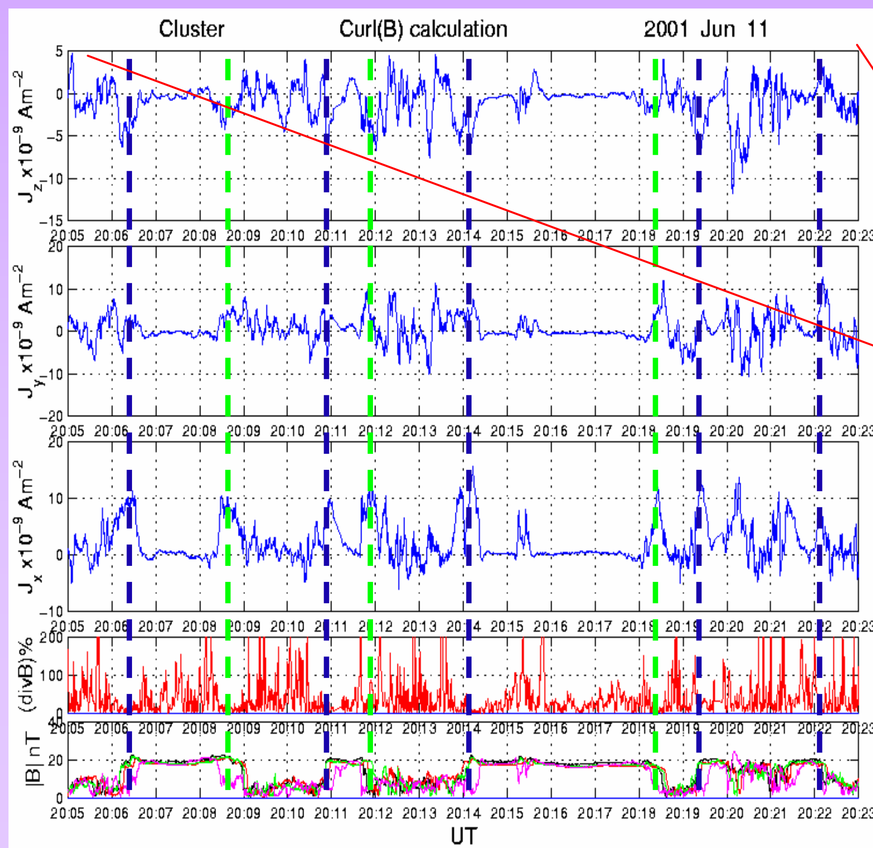


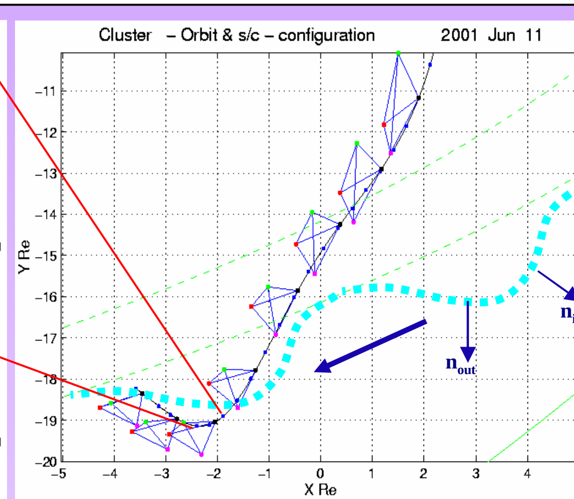
- Curlometer provides an unambiguous measure of the MP current: direction and magnitude
- Scaling of the current density generally confirmed by $^{\Delta B}/_{D}$ profiles across the MP (Chapman-Ferraro)
- Boundary motion can be reconstructed via minimisation of **J** obtained using curlometer (*div(**curlB**)=0*)
- Thin current layer, J~100 nAm^{-2}, can be accurately measured (smallest separations: 100 km),

Magnetopause II: Wavy current sheet

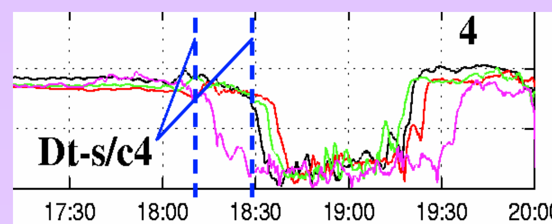


- Tilting of current sheet follows MP.
- Compare DA and curlB: J_{MP} ~15 nA/m^2
- Agrees with ΔB_{MP} across a thickness ΔD~1200 km.
- Scaling for variations in ΔD (previous event).
- Ranges from: 700-1400 km.



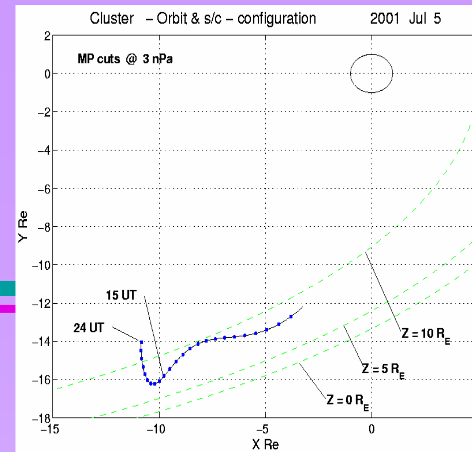


•The Δt's give const thickness for each s/c (within ±60 *km*).

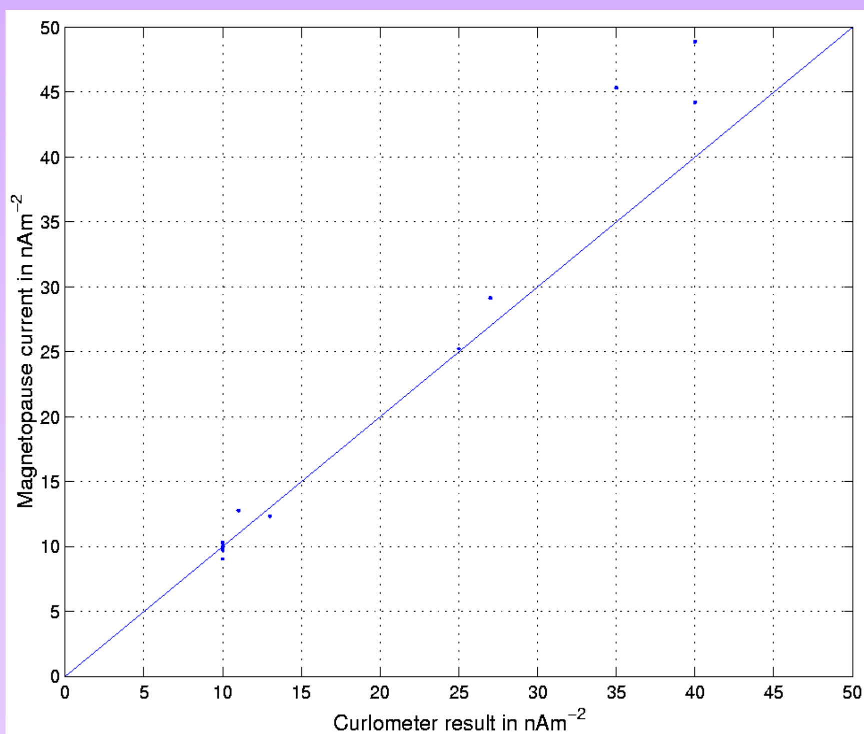


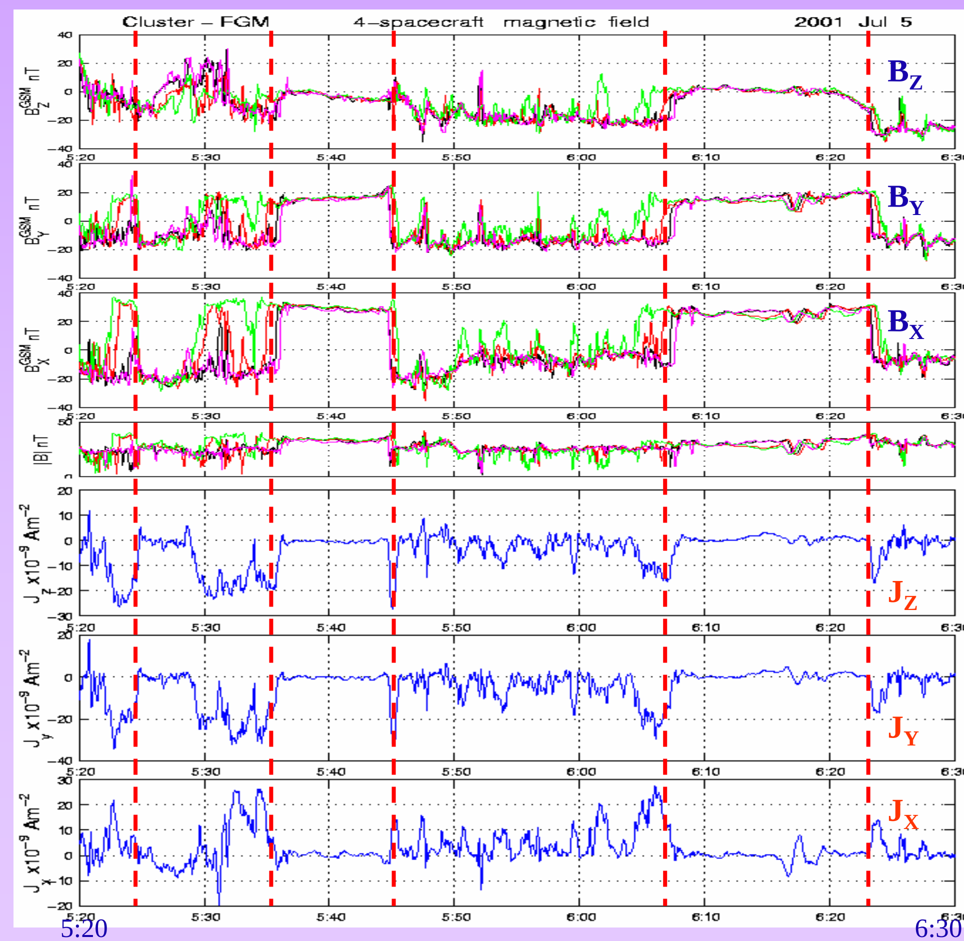
Magnetopause III:

Flank MP gave many crossings (LLBL)



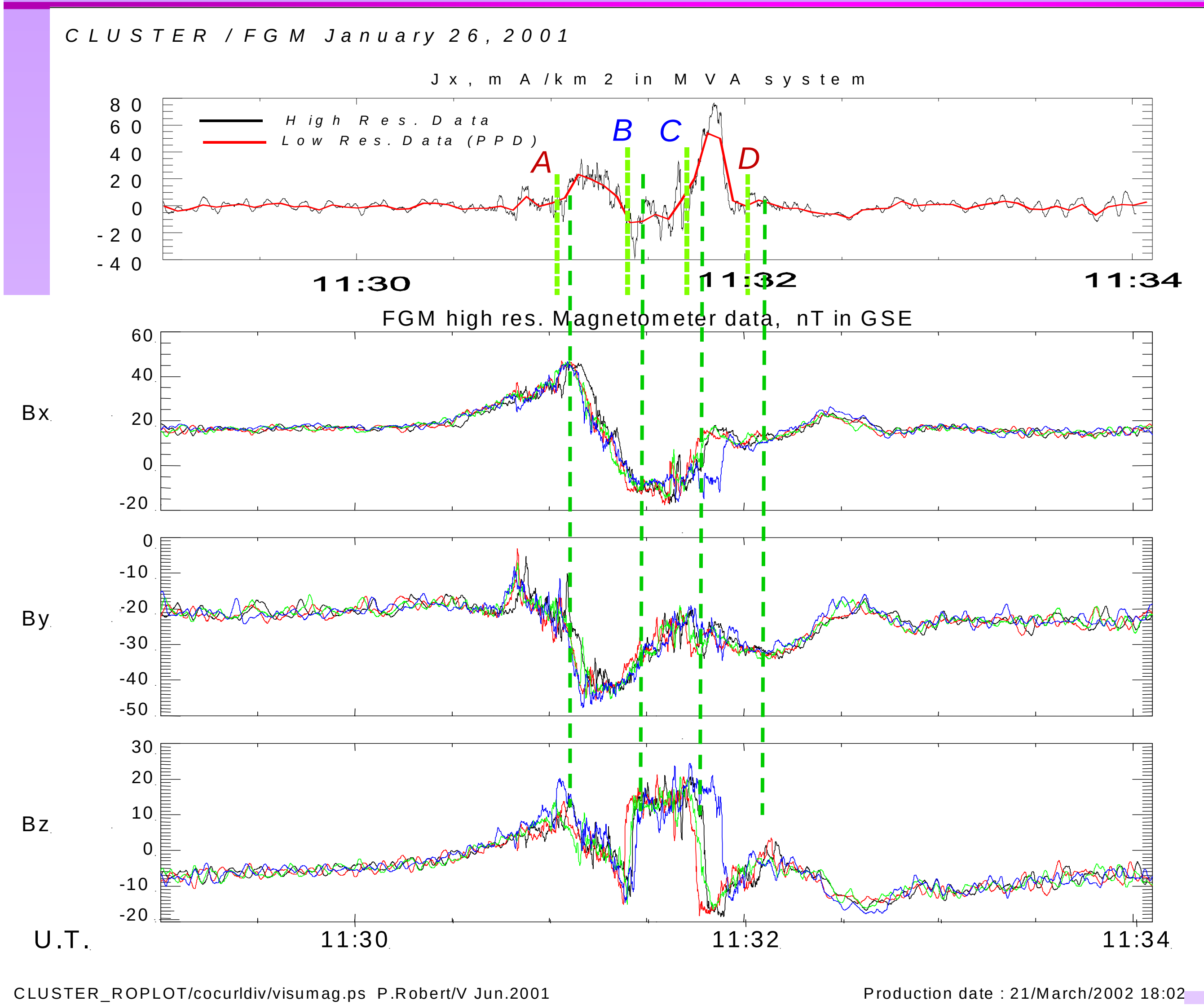
- Clear J direction and magnitude (in MP) at each crossing.
- Curl B estimate gives (in MP plane): <J_{MP}> =30 nA/m^2 (c.f. ΔB / ΔD).
- BL thickness varies throughout pass: scales for each crossing.
- Overestimate for large separation (thin layer)





Flux Transfer Events

Robert et al. 2003



- Isolated magnetosheath FTE
- Curlometer determination of currents within FTE – predominantly $J_{||}$
- Force-free double-current tubular flux rope fits data quite well

r_{AD} ~ 5200 km, I = 2300 kA

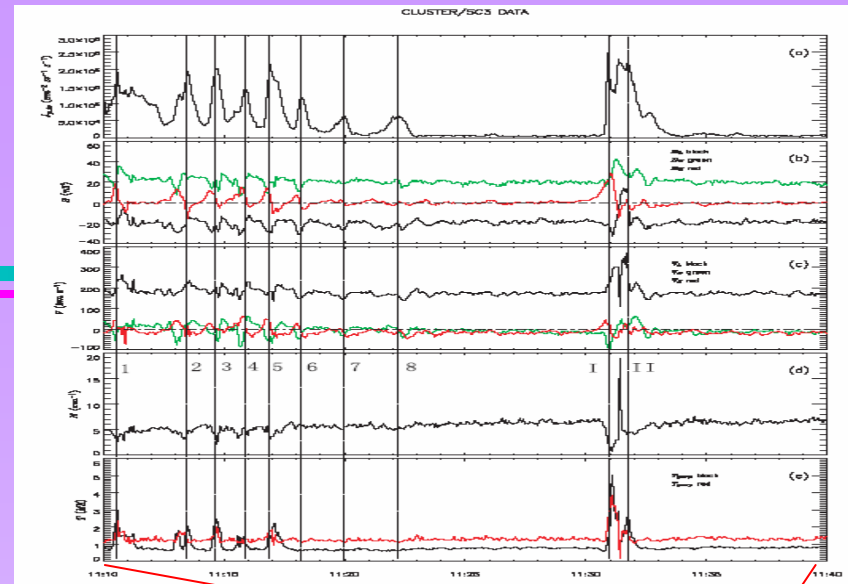
r_{BC} ~ 1400 km, I = 190 kA

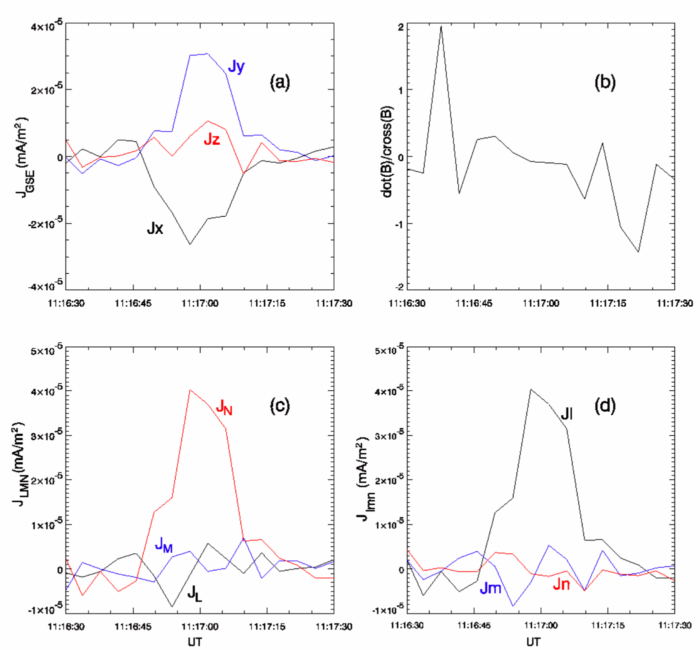
see: Robert presentation

FTE signatures II:

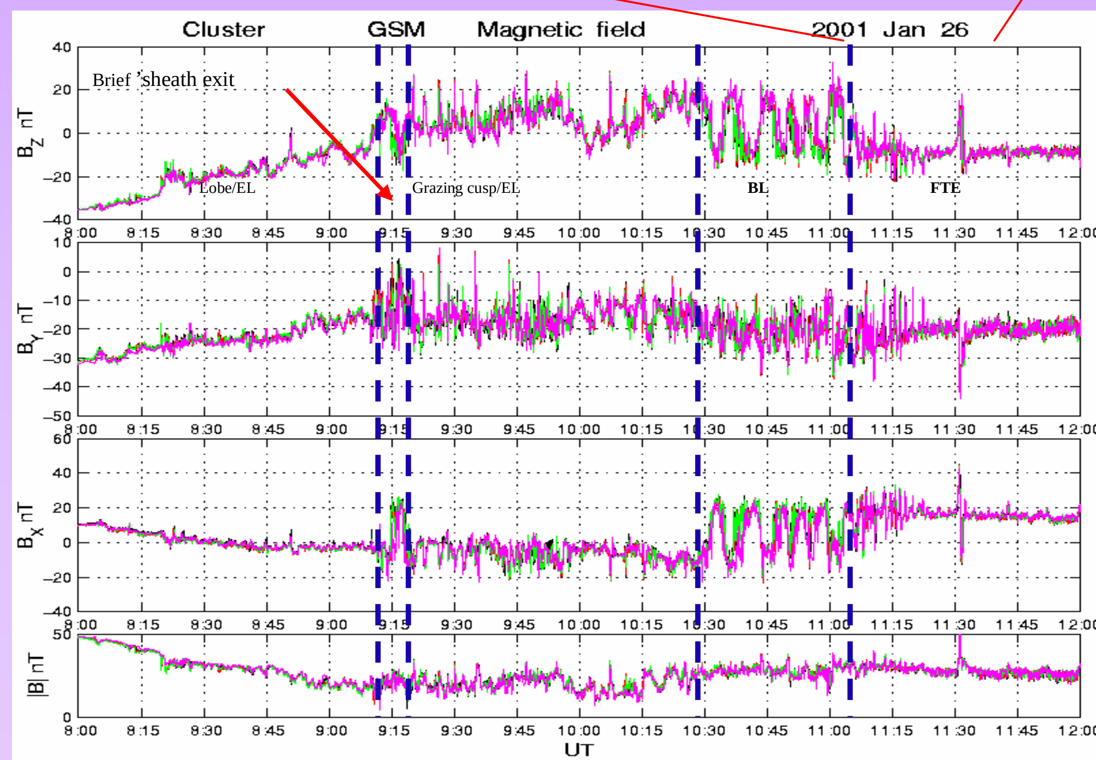
Pu et al, 2004; Zong et al. 2003, 2004.

- Train of FTEs contain high energy ions/electrons
- Coincident with bipolar MVA signatures
- Both MVA and 'CMVA' applied to FTE signatures
- Good alignment of J to the flux-tube axis





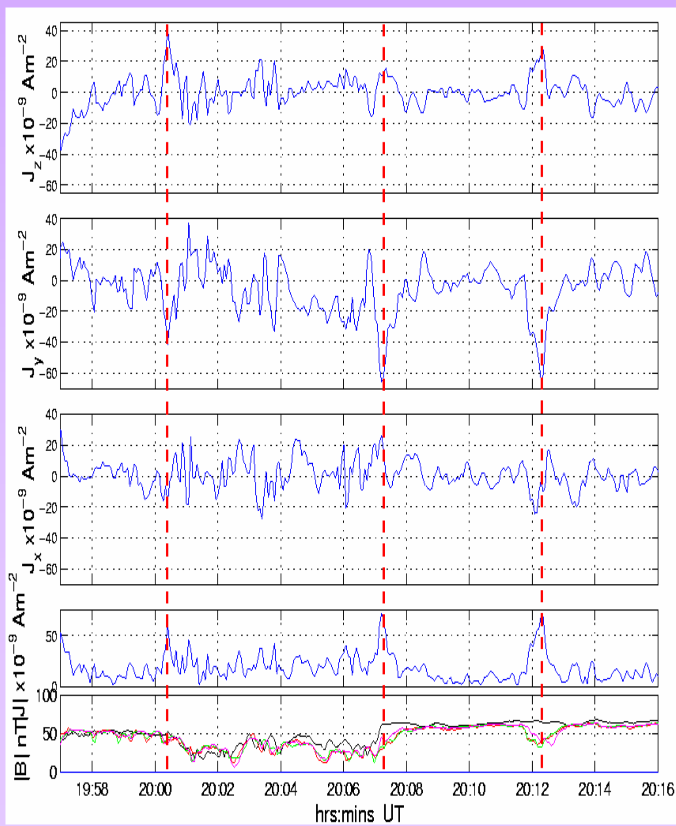
J_{GSE}, J_{LMN} and J_{GSE} projecting in B_{LMN} between 11:16:30 and 11:17:30,on Jan. 26, 2001

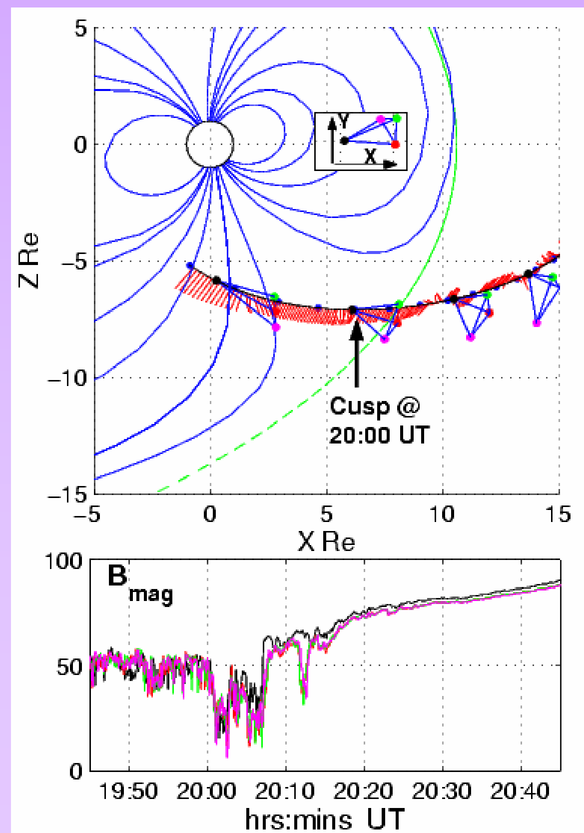


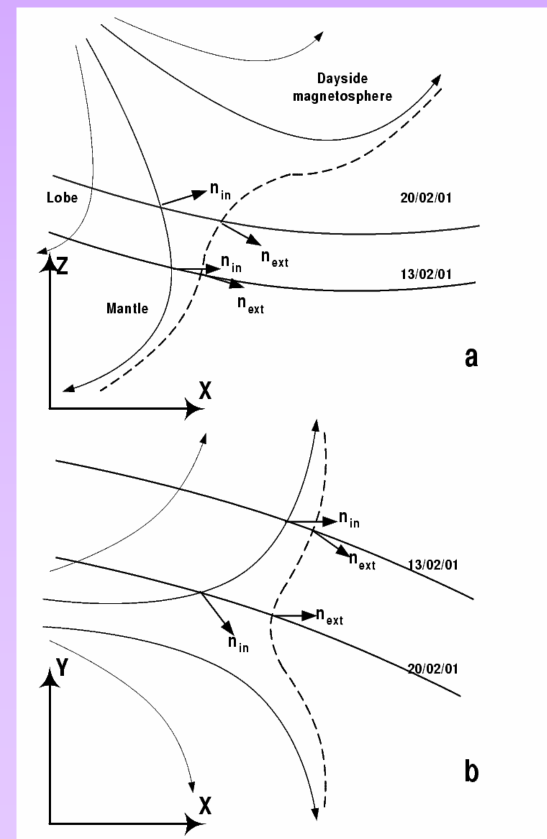
Currents in high-altitude cusp:

Lavraud et al. 2004, Cargill et al. 2004, Zhang et al. 2005, Dunlop et al. 2005, Taylor et al. 2005.

- Clear cusp boundaries (survey): outer (magnetosheath) and inner (magnetosphere,lobe)
- Boundary Currents: a significant current layer exists at both inner and outer boundaries
- Only the cusp/magnetosheath boundary, is associated with a corresponding ion jet



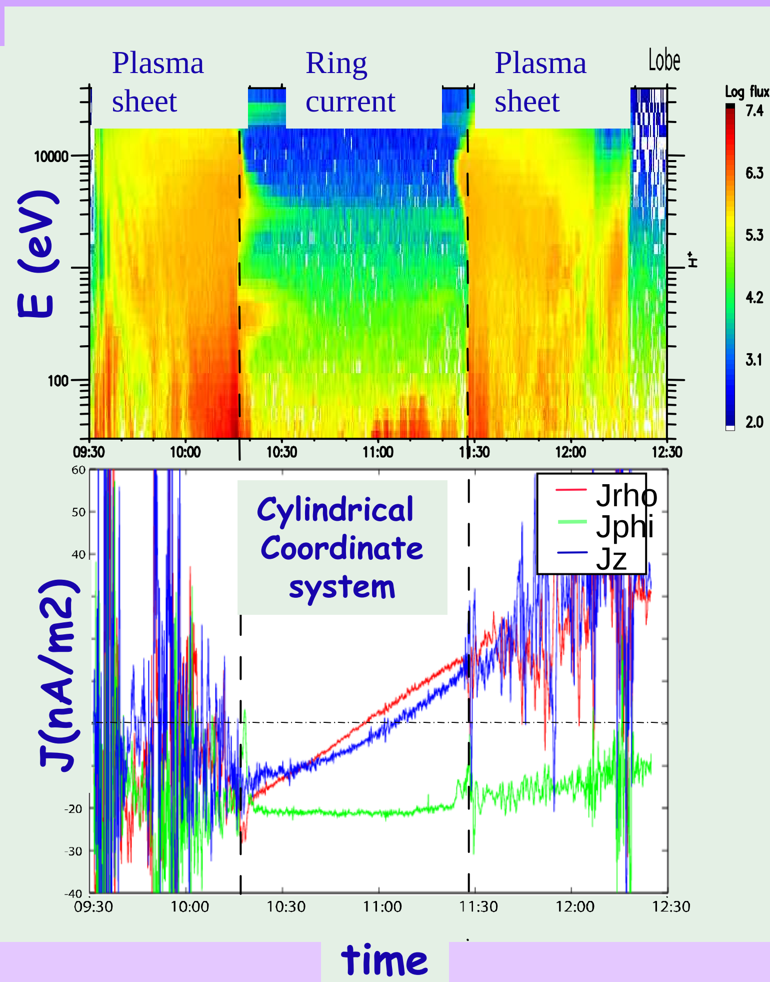




Ring current estimates:

Vallat et al. 2005

• During perigee (separation < 500 km): elongation along the Zsm axis => main error in component // to B).

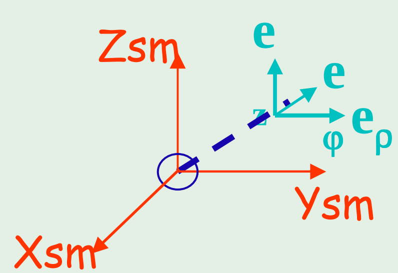


•Good estimate of the ring current (J components ⊥ B).

• Azimuthal (westward) at equator; FA at plasma sheet boundary

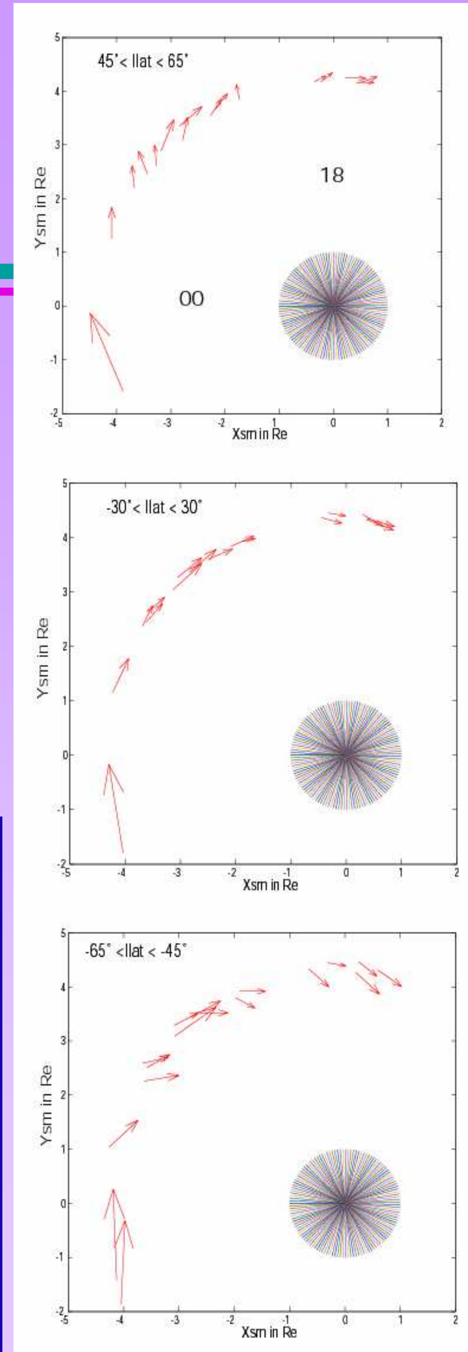
• Statistical confirmation

Transformation into cylindrical coordinate system



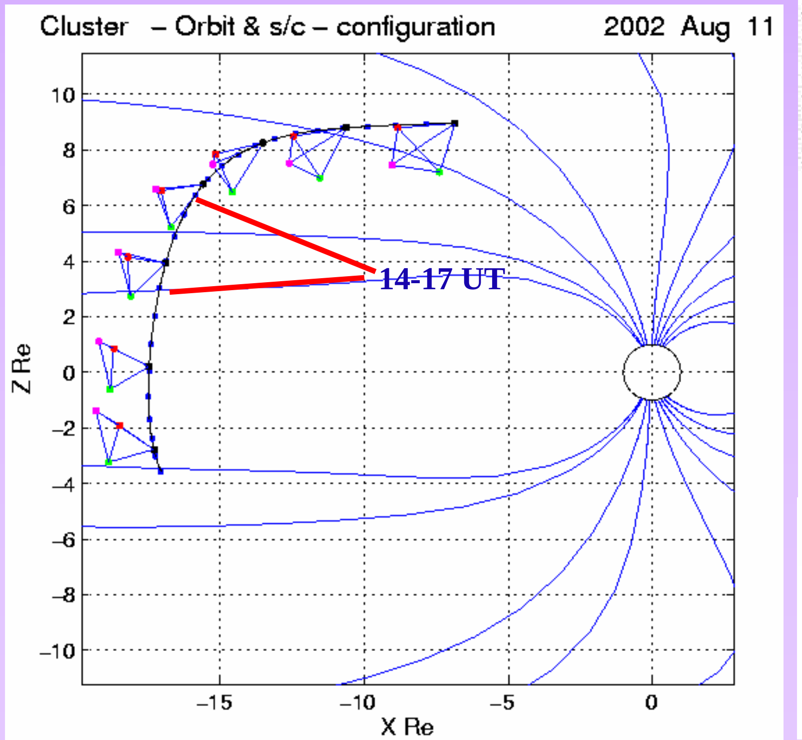
• Jρ : radial current.

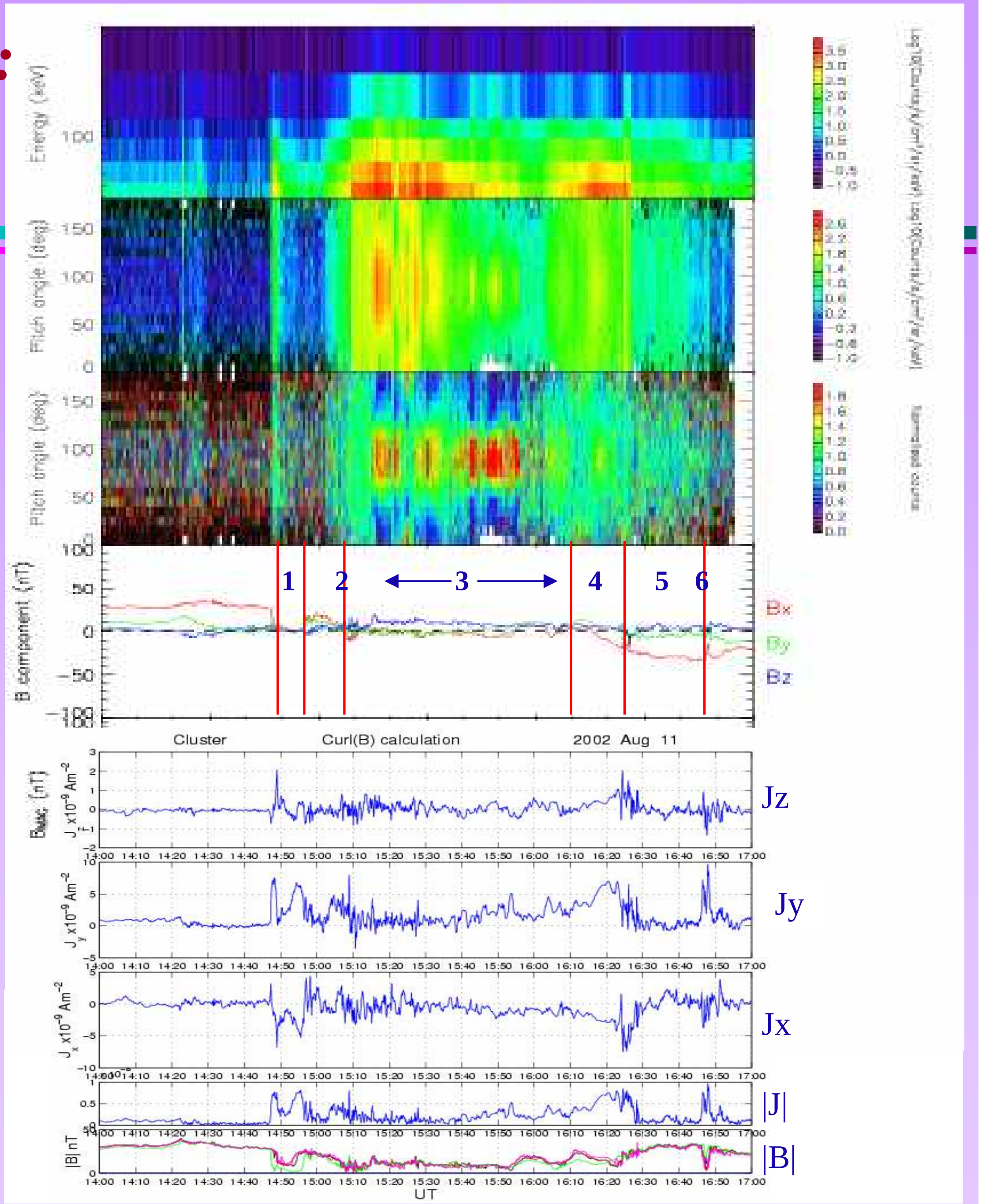
• Jφ : azimuthal current.



Magnetotail currents:
FA electrons and curl B

- Confirms change in current sheet location between beams and trapped signatures.
- Correlated with FGM curl B signature
- Hall current system?





- **New gradient methods**

  Rotation and curvature from $\nabla \mathbf{B}$ and $\nabla |\mathbf{B}|$

  Dimensional derivative and spatial/temporal derivative

MP orientation, motion and thickness:

Shi et al. 2005, Shen et al. 2003.

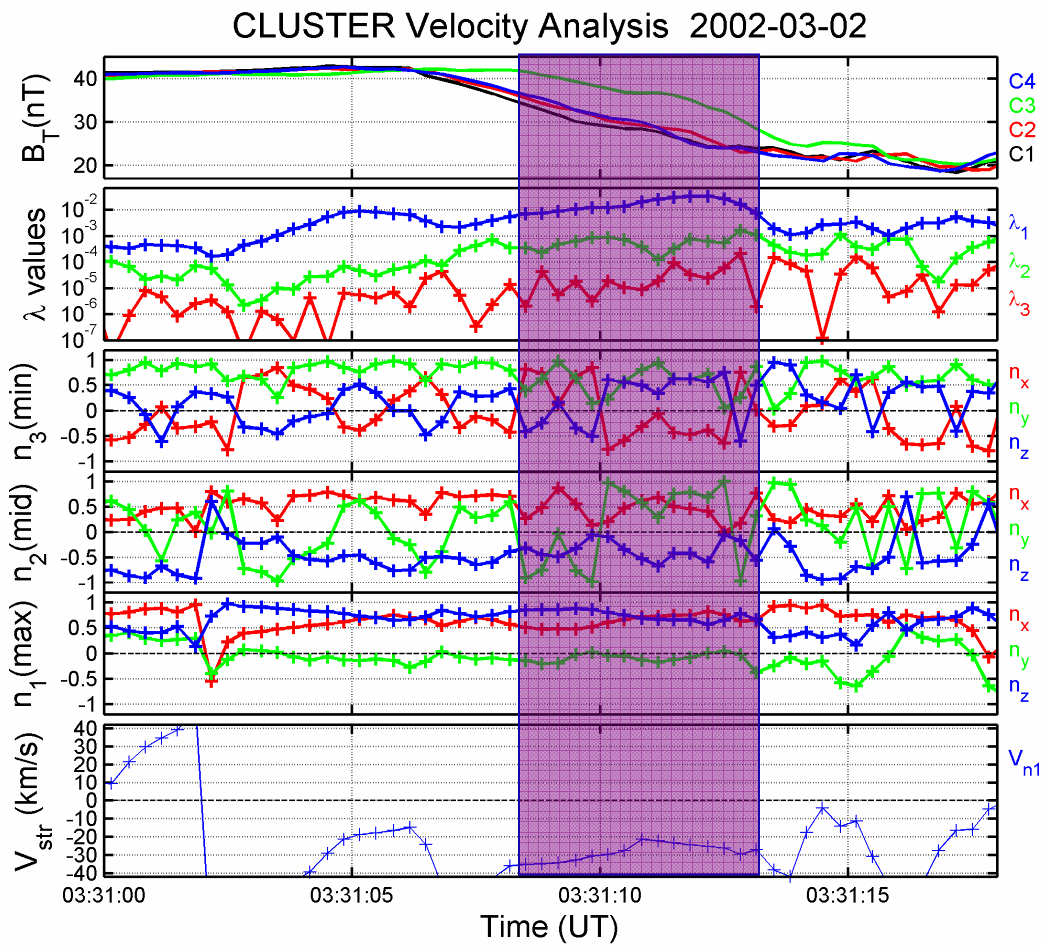
Minimum directional derivative (MDD) and spatio-temporal derivative (STD)

Determine the **dimensionality** and the directions by solving **eigenvalue** problem for **directional derivatives**

$$L = GG^T = (\nabla \vec{B})(\nabla \vec{B})^T$$

$$\lambda_1, n_1 \sim \lambda_2, n_2 \sim \lambda_3, n_3$$

1-D case



Directly solve the equation:

$$\frac{D\vec{B}}{Dt} + \vec{V}_{str} \cdot \nabla \vec{B} = 0$$

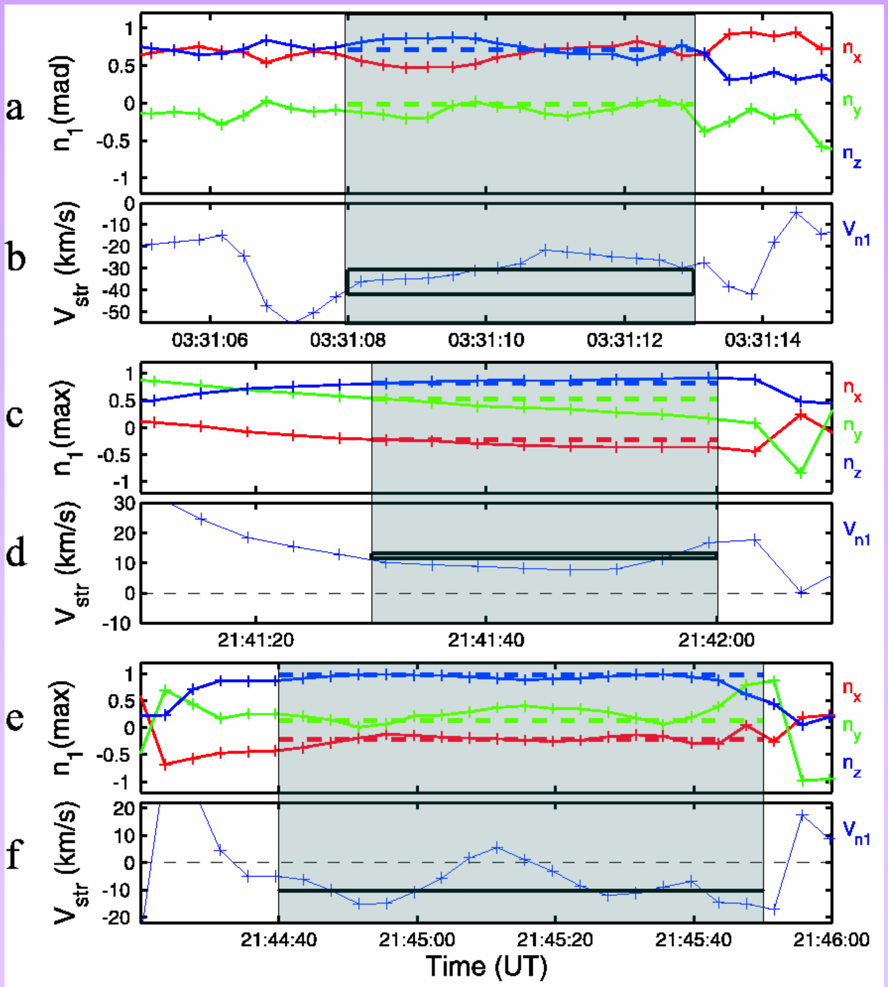
(quasi-stationary)

(*First term*: time variation of the field observed by the spacecraft)

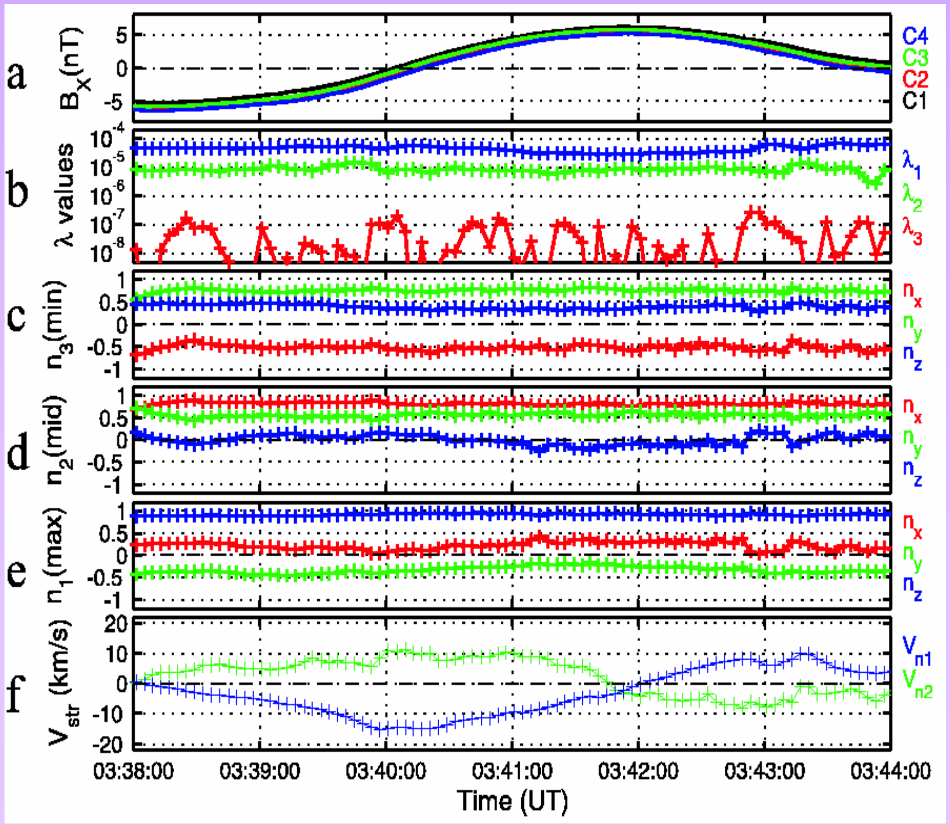
MP orientation, motion and thickness II:

Comparison to other boundary analysis and 2-D current sheet

•CTA and DA results:

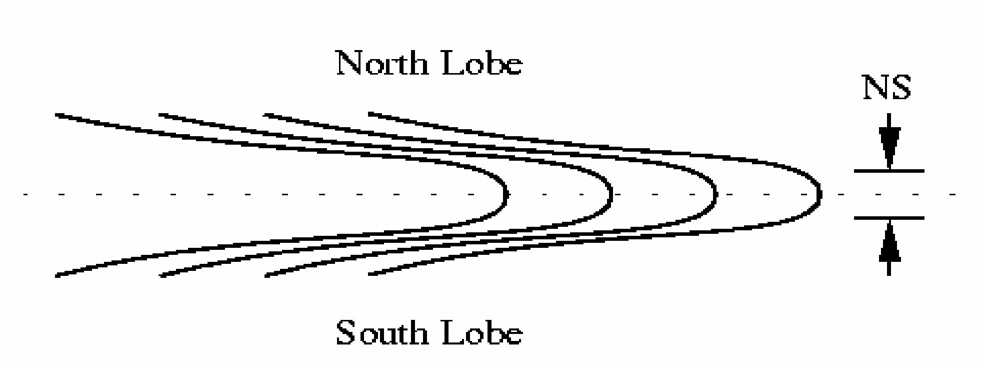


• $V_{1,2}$ shows reversal (flapping)

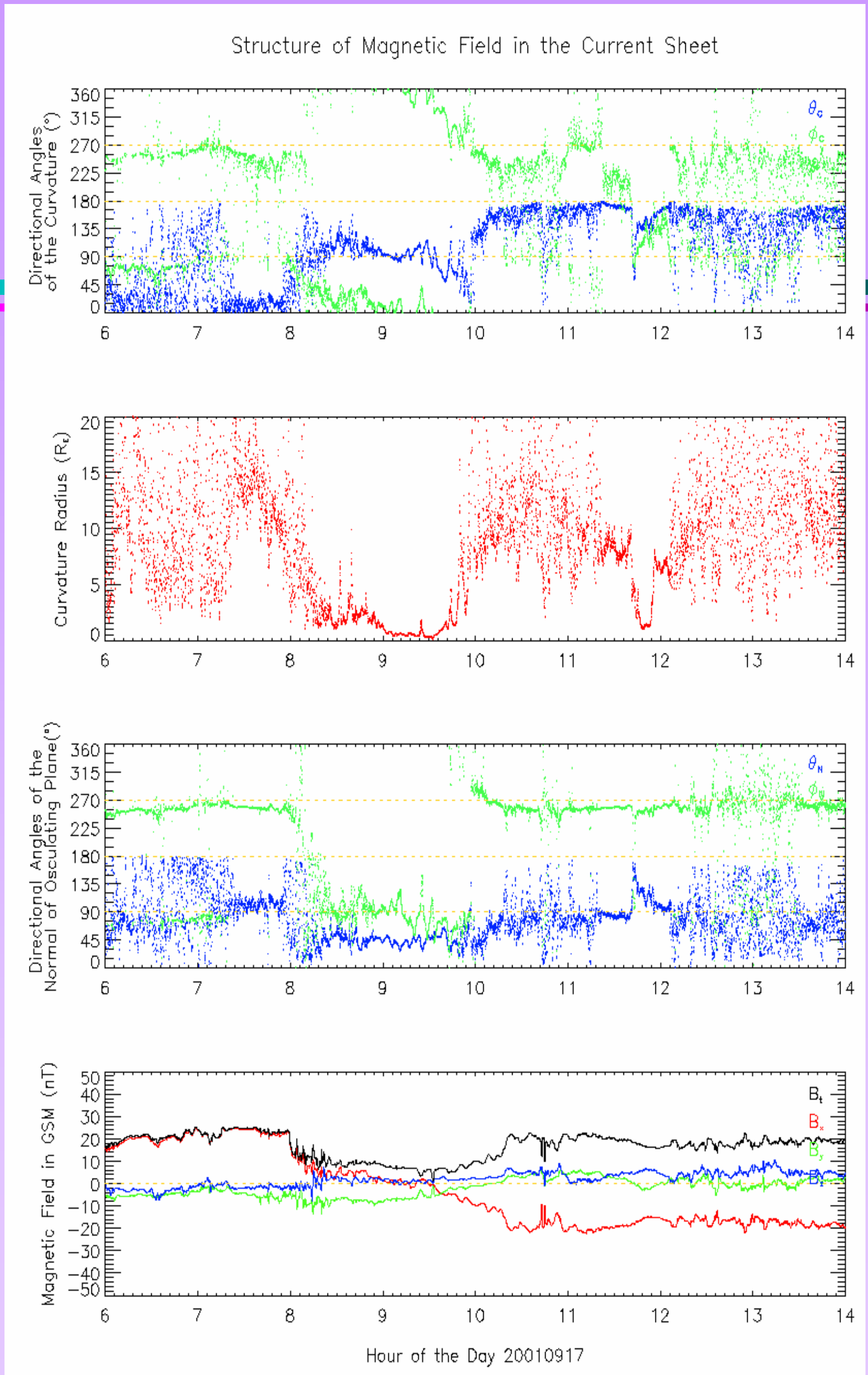


Rotation and curvature analysis:

Shen et al. 2003.



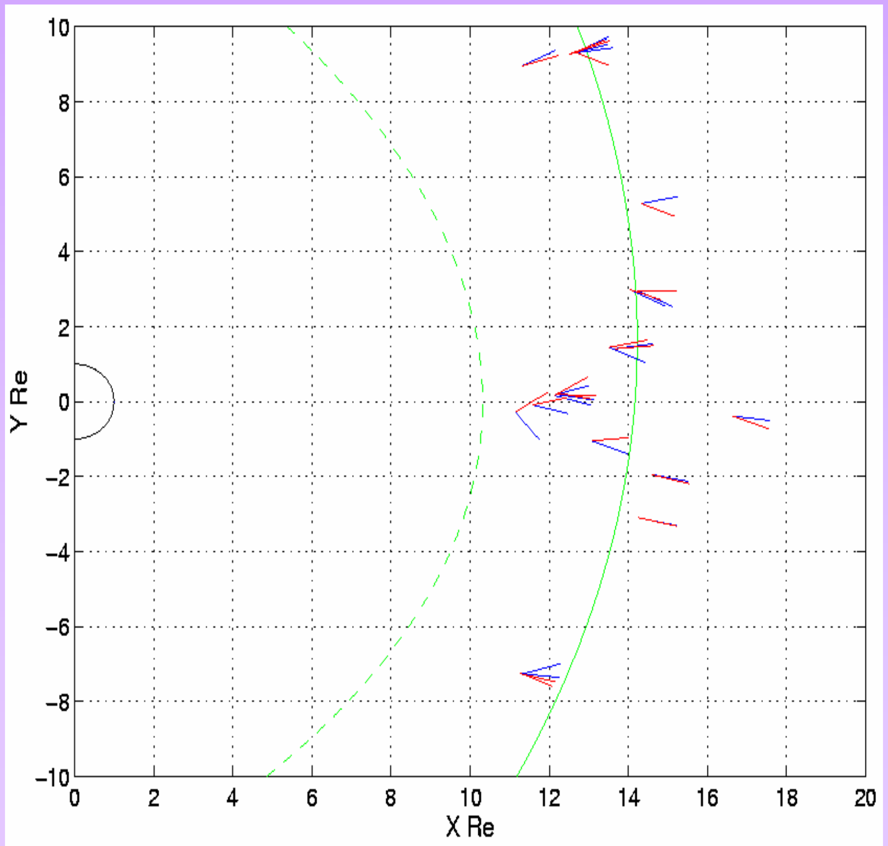
- Direct estimate of curvature
- Full rotation characteristics by method of maximum and minimum rotation rates (MRA)
- Find characteristic directions and curvature



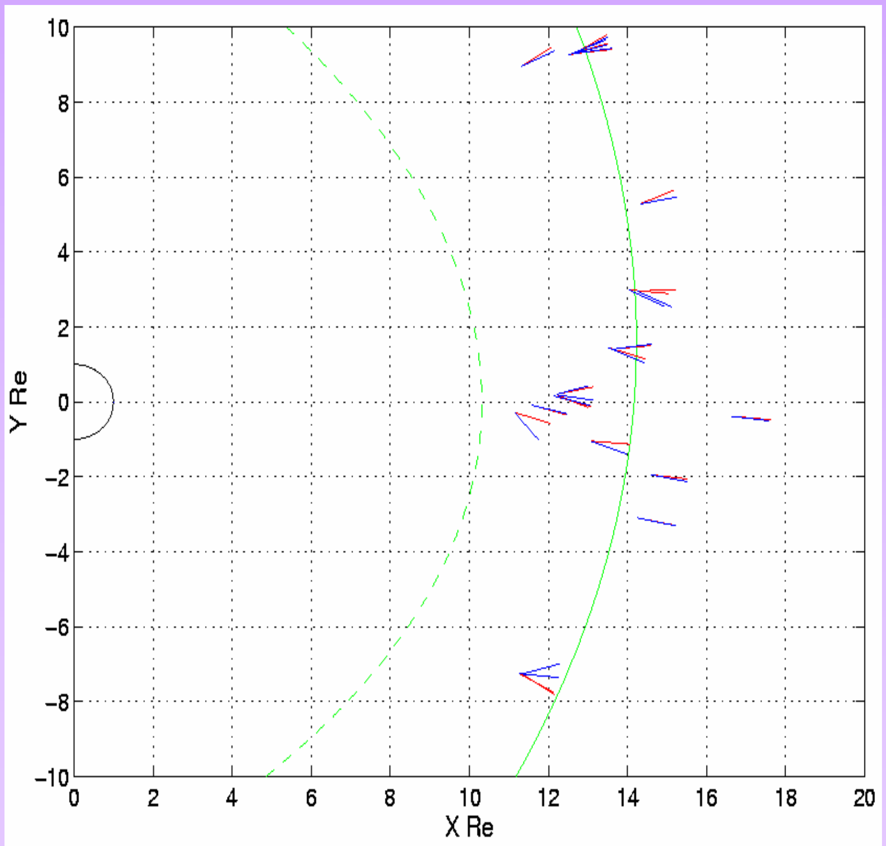
Gradient of magnetic pressure:

Shen et al. 2005.

- New approach for bow shock normals based on gradient of the magnetic pressure (to infer n)
- Selected Cluster crossing events during February and March of 2004
- Find at least as good representation as CVA and usually better than MVA or coplanarity



- comparison of *grad**B*** with coplanarity/TD



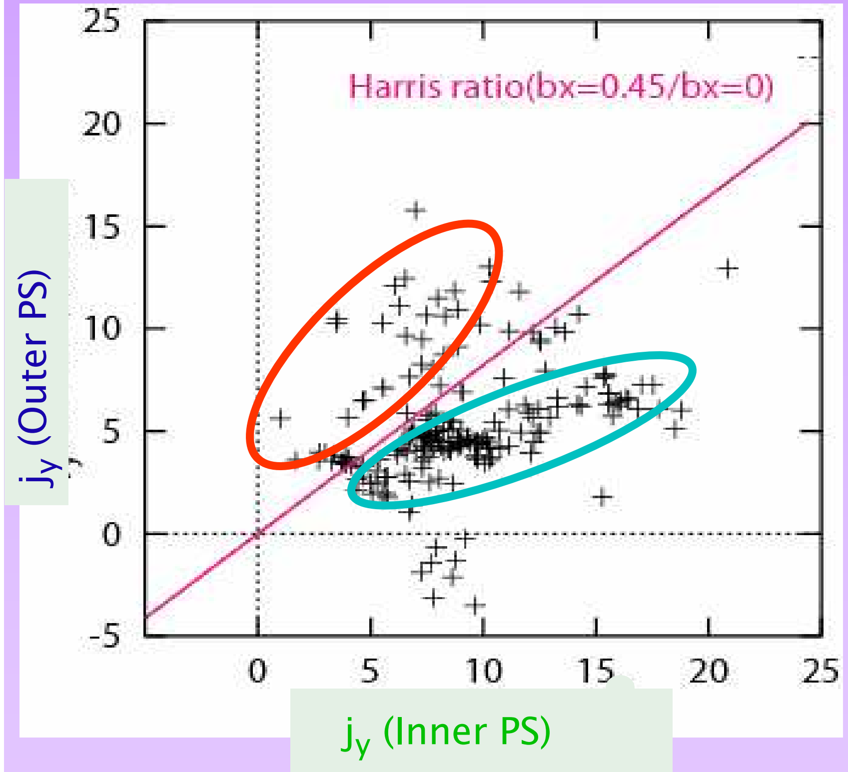
- comparison of *grad**B*** with timing (CVA)

- **Other multi-point techniques**
  - Gradient of n
  - Partial currents (Harris sheet)

Magnetotail currents II:
Asano et al. 2004, Nakamura et al. 2005.

2-s/c: ΔB/ΔD for planar current sheet; 3-s/c: Harris fit for profile in planar CS; 4-s/c: Curlometer

• Occurrence of Bifurcation

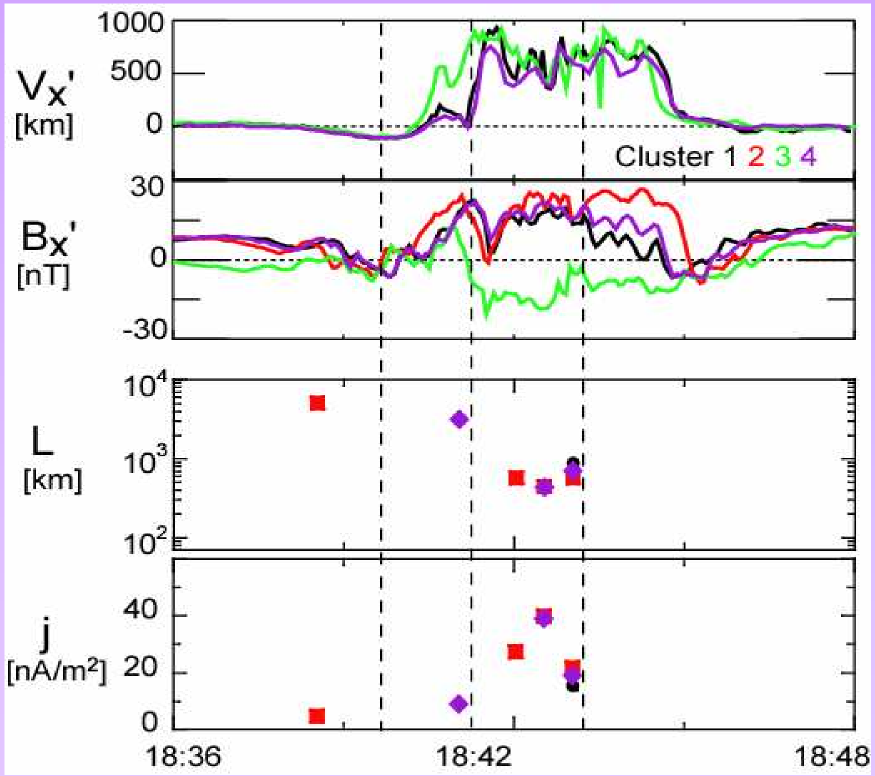


Compare NS to ‘off equator’ ratio

Some traversals show stronger JY in neutral sheet ⇒ embedded electron CS?

Larger J_Y in off-equatorial region observed ⇒Bifurcation

• Thin current sheet



• Harris, 3 S/C fit (BL, Z0, L): thin CS

• 4th satellite used for quality check

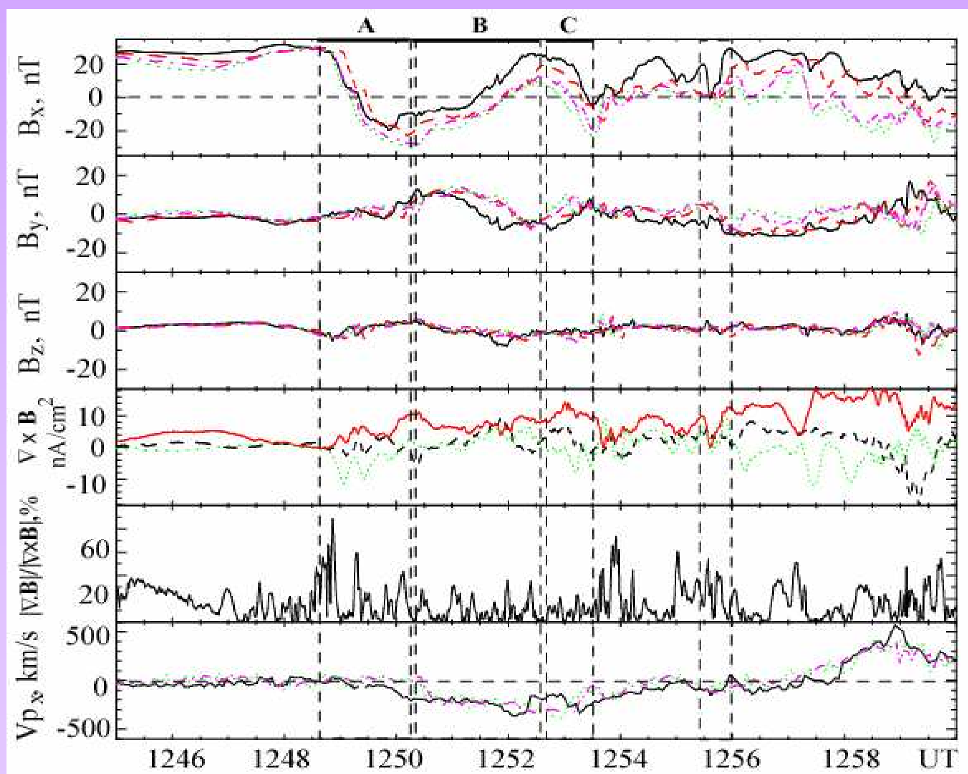
• Current increase: 10 ⇨ 40 nA/m², L:5000 ⇨500 km

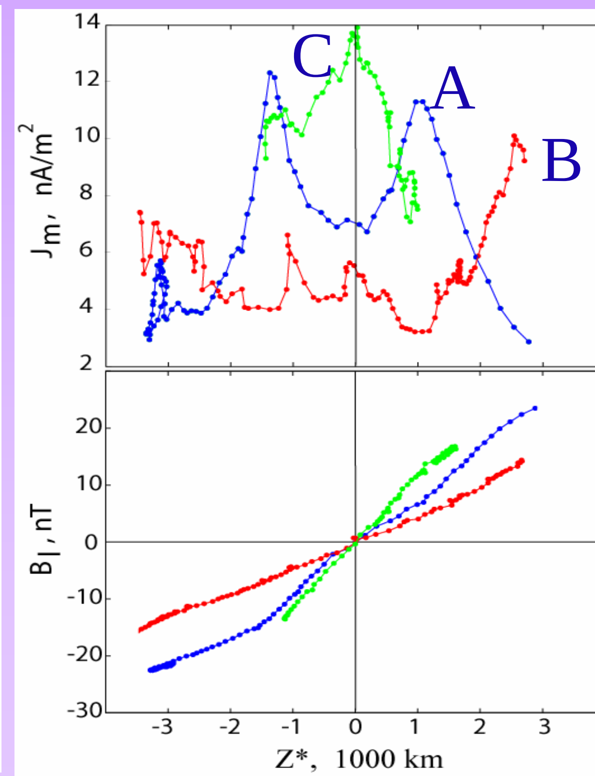
Magnetotail currents III:

Runov et al. 2004.

- Reconstruction of current sheet structure from crossing data
- Assume: spatial structure remains same during crossing:

→ $Bx(t+\Delta\tau) - Bx(t) = (dBx/dz)\ \Delta z/\Delta\tau$



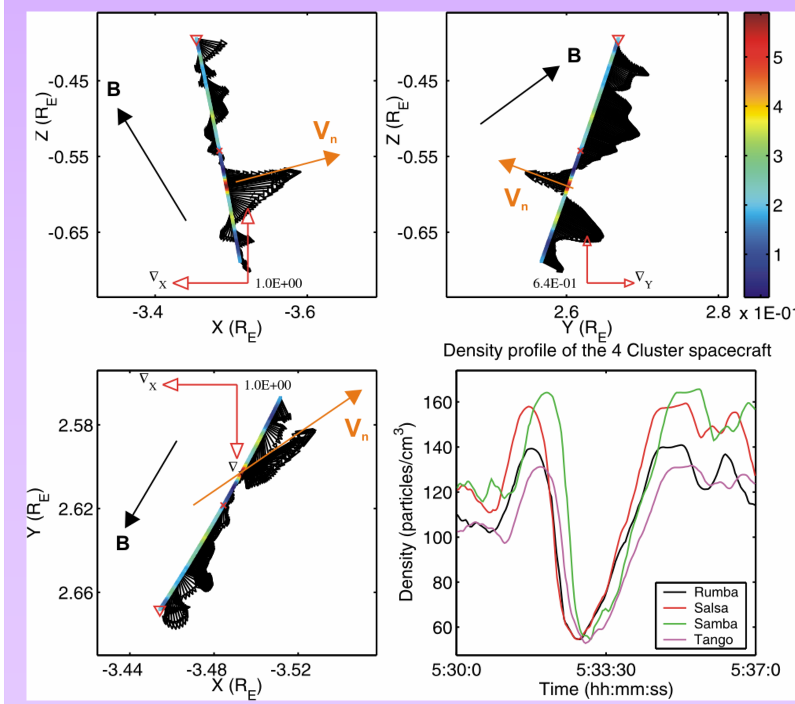


- Harris and bifurcated current sheet observed within 5 min.

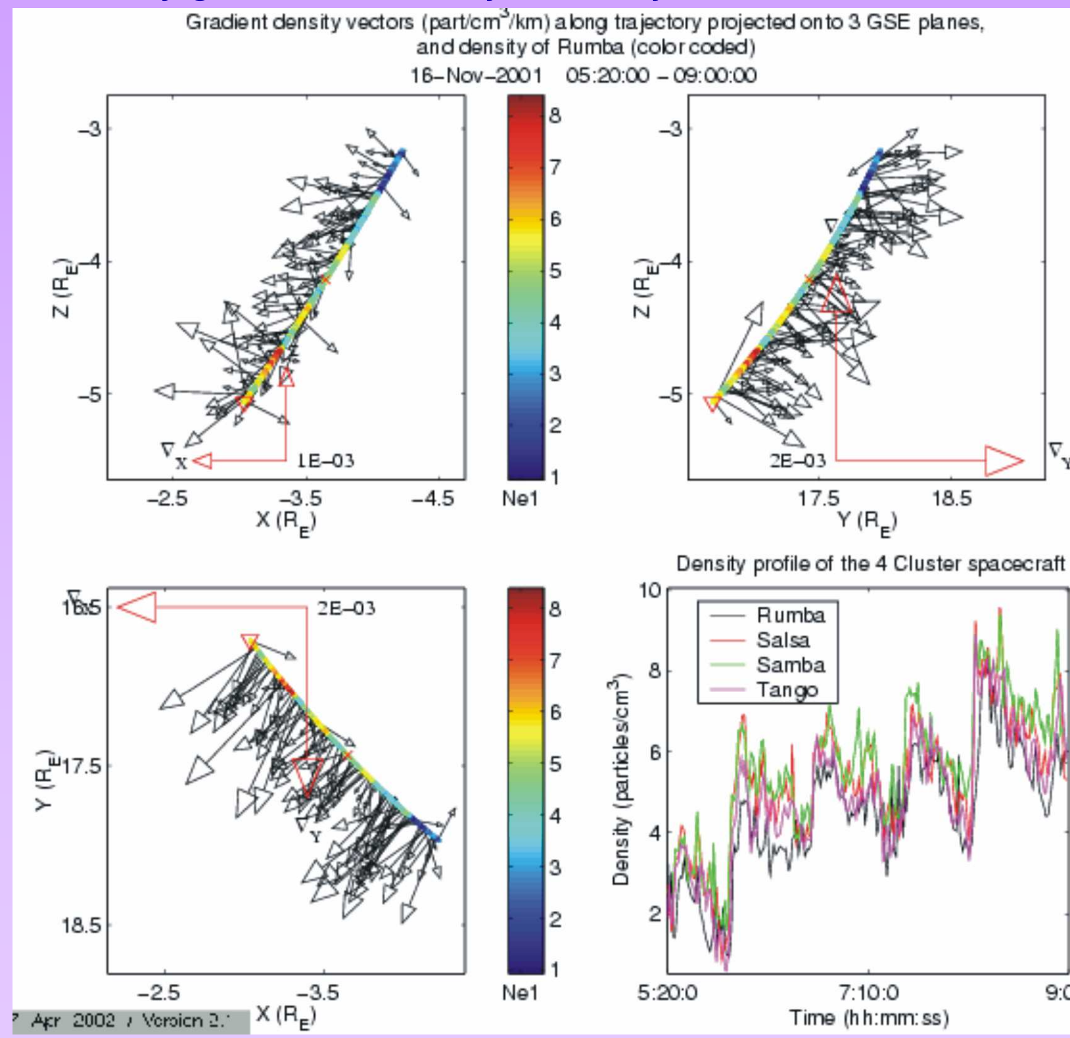
Gradient applied to n:

Darrouzet et al. 2002, Darrouzet et al. 2005.

- Density irregularity in the plasmasphere



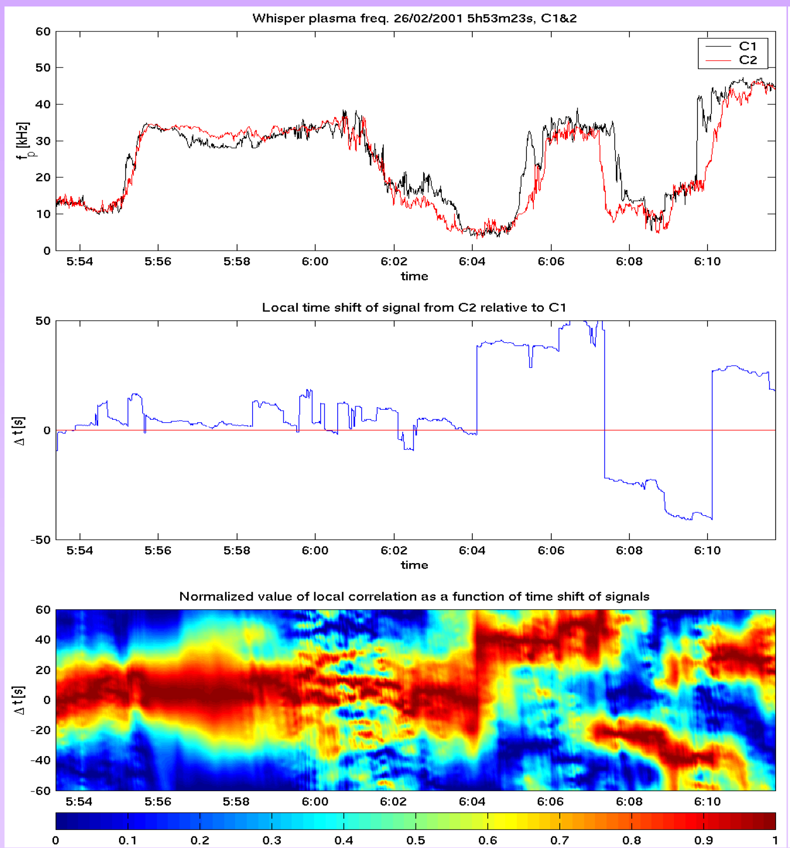
- Density gradient for a wavy boundary

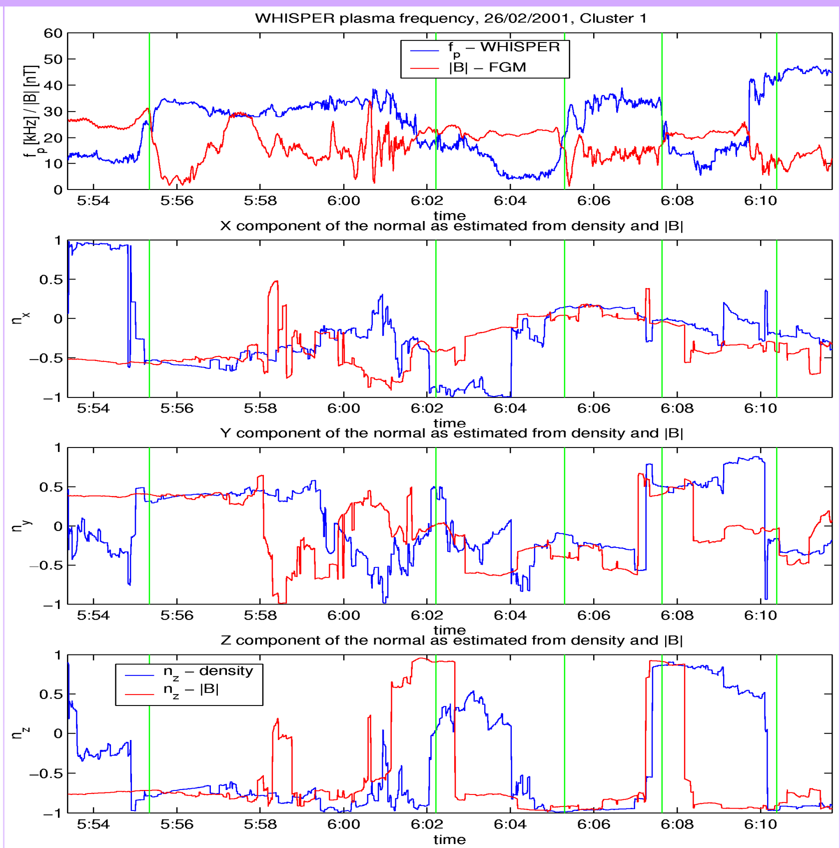


Timing issues:

Soucek et al. 2004

- Example of timing analysis, using a wavelet decomposition method.
- Crossings chosen for: constant velocity, planar boundary.
- Timing meaningfull at ‘stationary‘ MP boundary.
- Both instruments ‘see‘ similar boundary character.





Summary

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  - Currents in FTEs
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  - Ring current
- **New gradient methods**
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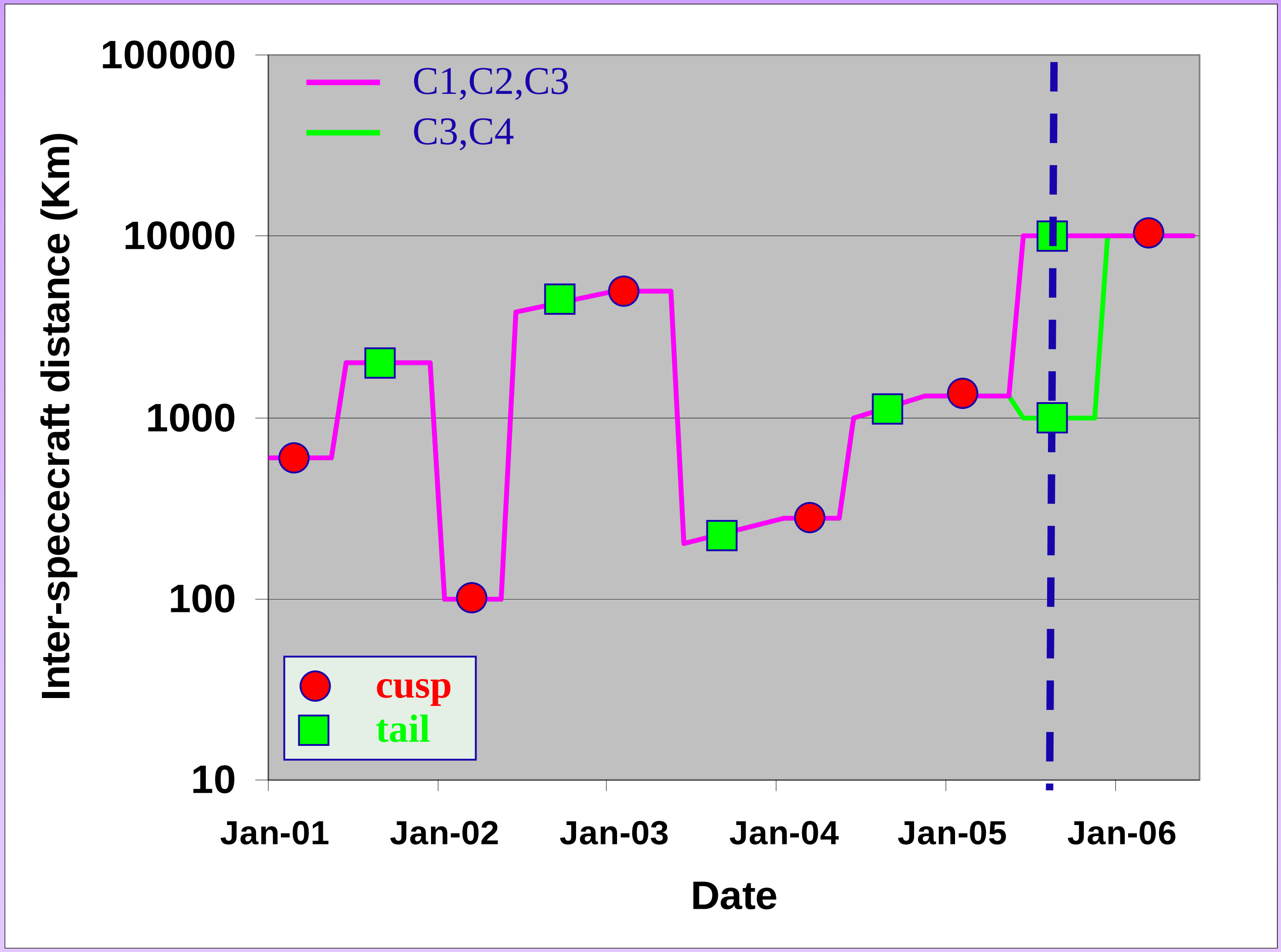
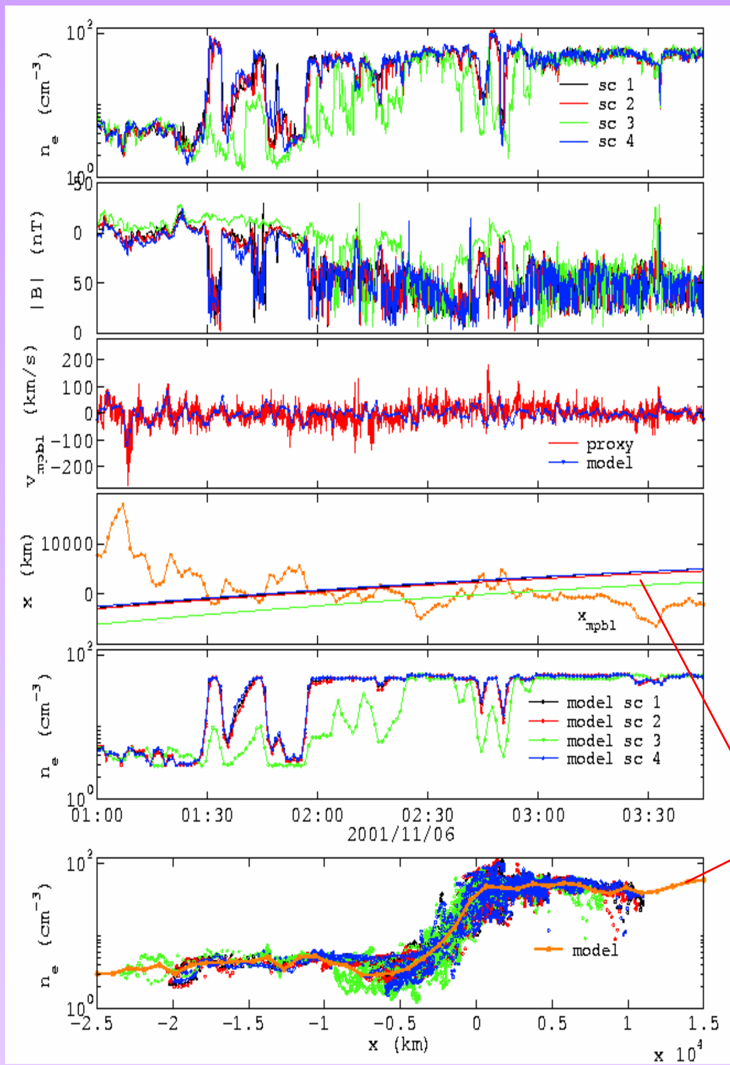
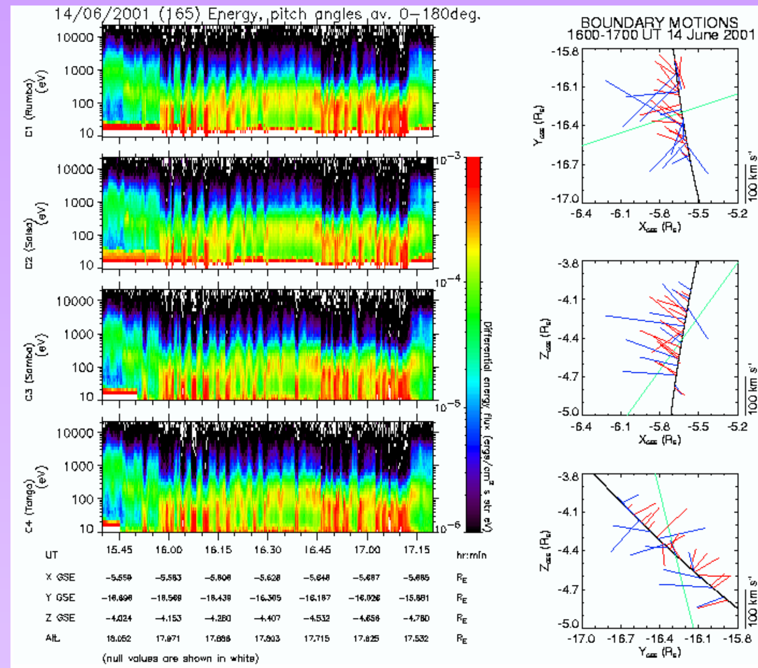
Cluster separation strategy

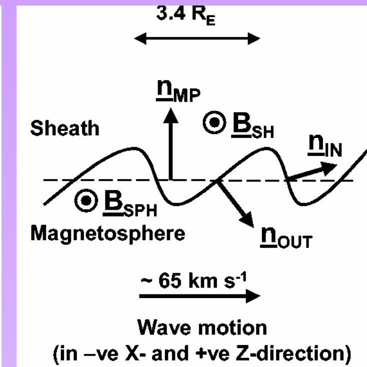


Surface waves II:

deKeyser et al. 2004, 2005, Owen et al. 2004.





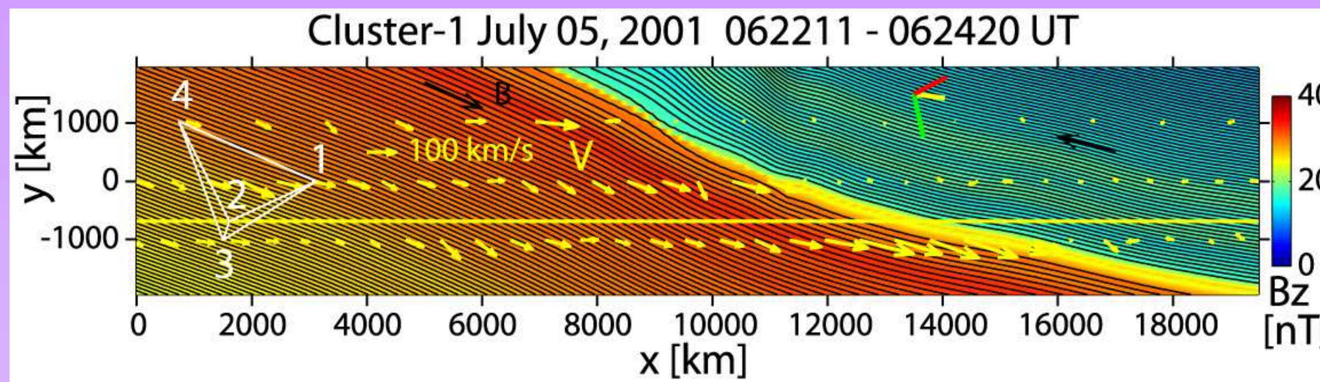


- Tracking an undulating boundary: reconstruction using plasma velocity (left) and confirmation via 4-s/c comparison and model
- Predicted boundary position is shown against orbit track and data can be resorted to 'x'.
- Q-periodic surface waves produce mixed plasma signatures
- 4-s/c boundary analysis shows clear boundary tilting consistent with K-H waves

Structures in the magnetopause:

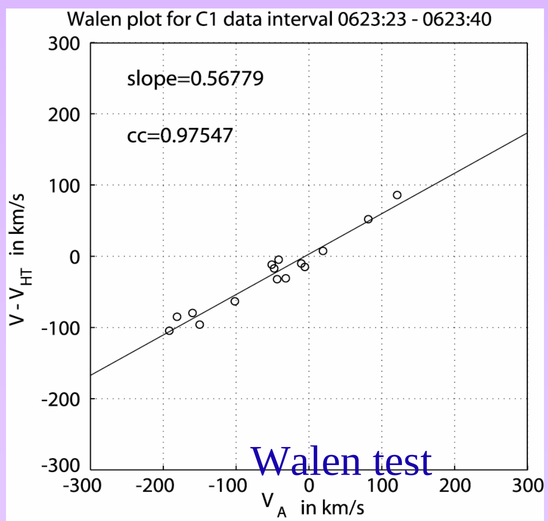
Sonnerup, Hasegawa et al. 2004, 2005.

- Grad-Shafranov, 2-D reconstruction (magnetic field maps): inter-comparison between 4 s/c
- High correlation with measured field and invariant direction

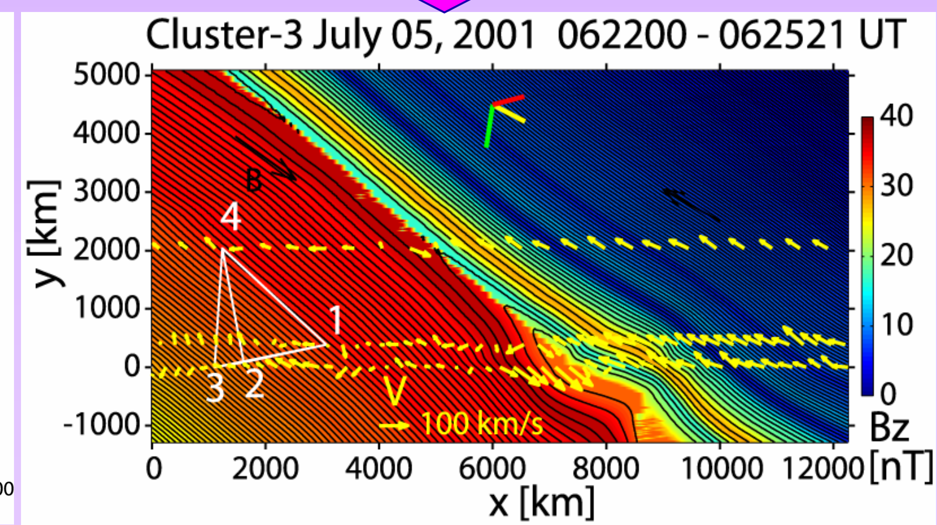


~30 sec

- Magnetopause structure was TD-type at the moment of the C1 traversal.
- By C3 crossing, current sheet thickens due to an increased number of reconnected field lines (RD-like)



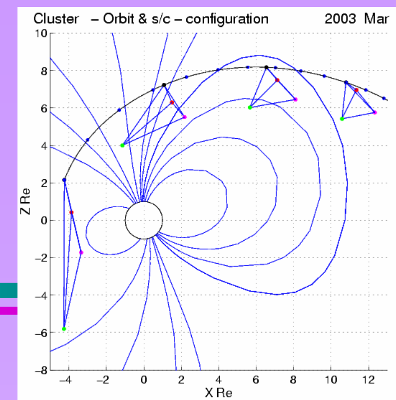
Walen test

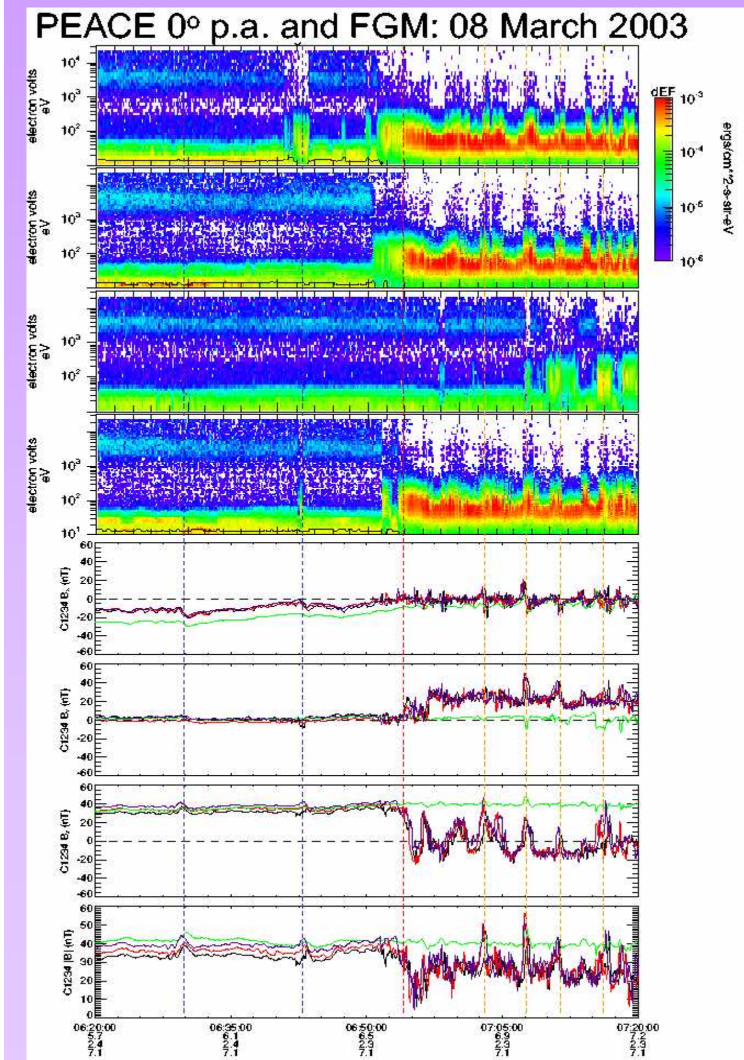


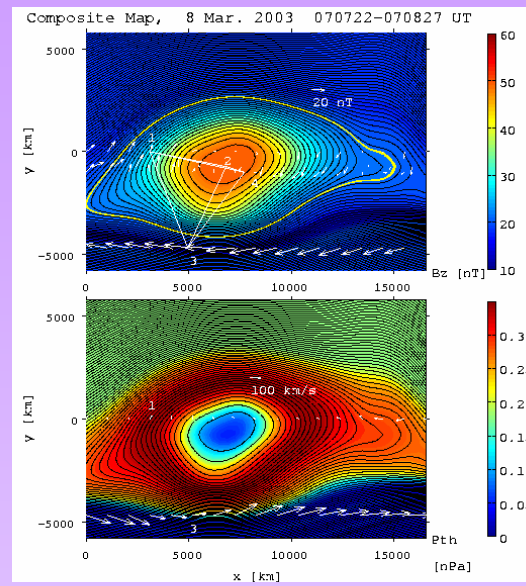
- Results reveal: embedded structure, islands, rapid changes from crossing to crossing

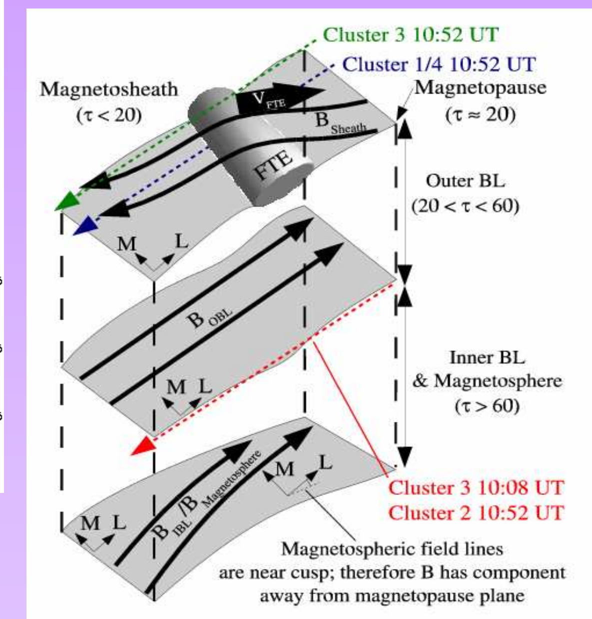
Structure of FTEs:

Sonnerup et al. 2004, Owen et al. 2005, Fear et al. 2005.









- 4 s/c sampling of FTE structure (either side of the boundary)
- spacecraft 3 remains in magnetosphere as spacecraft 1,2,4 exit into magnetosheath: all observe FTE like signatures
- 2-D reconstruction of this event confirms interpretation of a bulge (of high magnetic, low plasma pressure) on the MP
- Similar events show different sampling conditions