The k-filtering Applied to Wave Electric and Magnetic Field Measurements from Cluster

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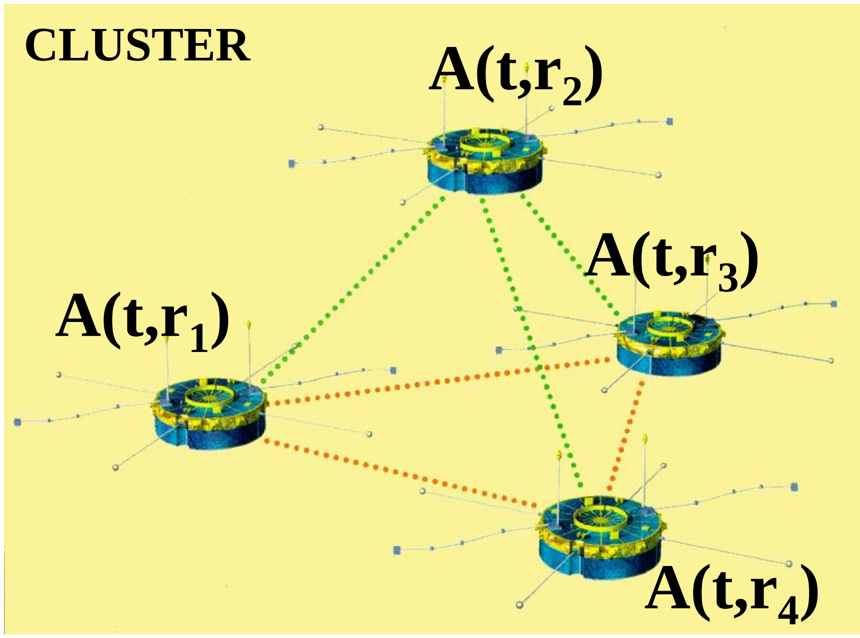
OUTLINES

- The k-filtering technique
  - The basics
  - Examples
- 3D characterization of the magnetosheath ULF turbulence
  - Using magnetic field measurements only
  - Combining electric and magnetic field measurements
- Conclusion

5th anniversary of Cluster in Space – 19-23 September 2005, ESA/ESTEC, The Netherlands

The k-Filtering technique

A multi-spacecraft data analysis technique allowing to estimate the wave-field energy distribution as a function of ω and **k**.



$$\mathbf{A}(t,\mathbf{r}) = \begin{pmatrix} A_1(t,\mathbf{r}) \\ A_2(t,\mathbf{r}) \\ \vdots \\ A_L(t,\mathbf{r}) \end{pmatrix}$$

Examples :

1) $\mathbf{A}(t,\mathbf{r}) = \mathbf{B}(t,\mathbf{r})$

2) $\mathbf{A}(t,\mathbf{r}) = \mathbf{E}(t,\mathbf{r})$

3) $\mathbf{A}(t,\mathbf{r}) = \begin{bmatrix} \mathbf{E}(t,\mathbf{r}) \\ c\mathbf{B}(t,\mathbf{r}) \end{bmatrix}$

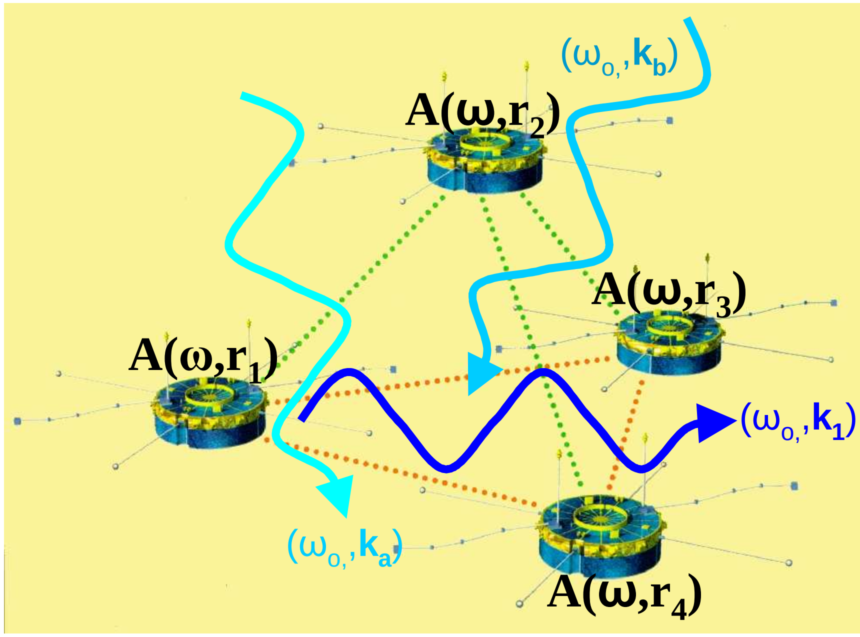
$$\mathbf{A}(t,\mathbf{r}) = \int_{\omega}\int_{\mathbf{k}} \mathbf{A}(\omega,\mathbf{k}) e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} d\omega\, d\mathbf{k}$$

$$P_{\mathbf{A}}(\omega,\mathbf{k}) = trace\left\langle \mathbf{A}(\omega,\mathbf{k})\mathbf{A}^{\mathrm{T}}(\omega,\mathbf{k}) \right\rangle$$

Notations:

$$\mathbf{A}(t) = \begin{pmatrix} \mathbf{A}(t,\mathbf{r}_1) \\ \mathbf{A}(t,\mathbf{r}_2) \\ \mathbf{A}(t,\mathbf{r}_3) \\ \mathbf{A}(t,\mathbf{r}_4) \end{pmatrix} \Rightarrow \mathbf{A}(\omega) = \begin{pmatrix} \mathbf{A}(\omega,\mathbf{r}_1) \\ \mathbf{A}(\omega,\mathbf{r}_2) \\ \mathbf{A}(\omega,\mathbf{r}_3) \\ \mathbf{A}(\omega,\mathbf{r}_4) \end{pmatrix} \Rightarrow \mathbf{M}_{\mathbf{A}}(\omega) = \left\langle \mathbf{A}(\omega)\mathbf{A}^{\mathrm{T}}(\omega) \right\rangle$$

Filter bank approach



$$\mathbf{A}(\omega) = \begin{pmatrix} \mathbf{A}(\omega,\mathbf{r}_1) \\ \mathbf{A}(\omega,\mathbf{r}_2) \\ \mathbf{A}(\omega,\mathbf{r}_3) \\ \mathbf{A}(\omega,\mathbf{r}_4) \end{pmatrix}$$

$$\mathbf{A}(\omega,\mathbf{k}_1) = \mathbf{F}^{\mathrm{T}}(\omega,\mathbf{k}_1)\mathbf{A}(\omega)$$

A(ω_o) → Filter $(\omega_o,\mathbf{k_0})$ → $P_A(\omega_o,\mathbf{k_o})$ = 0

A(ω_o) → Filter $(\omega_o,\mathbf{k_1})$ → $P_A(\omega_o,\mathbf{k_1})$ ≠ 0

A(ω_o) → Filter $(\omega_o,\mathbf{k_2})$ → $P_A(\omega_o,\mathbf{k_2})$ = 0

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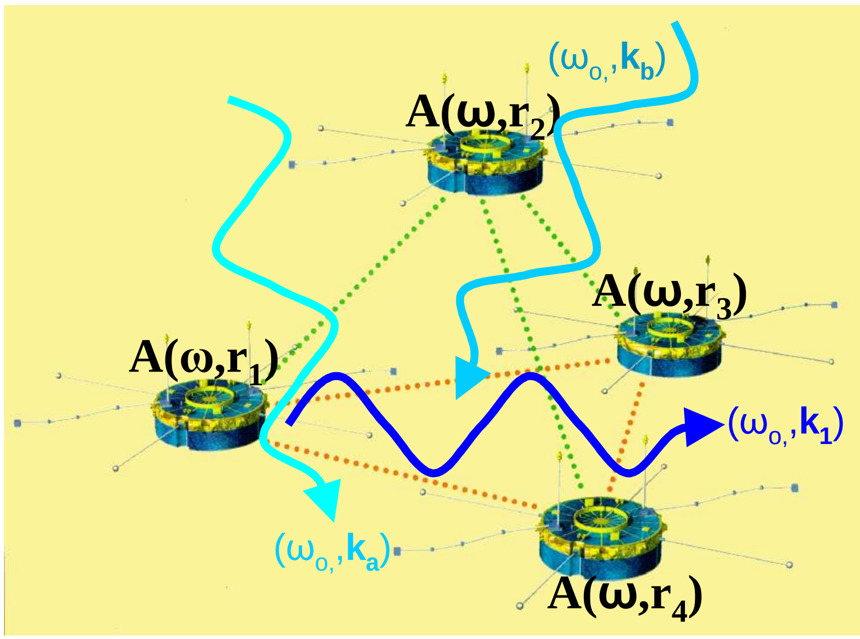
P(ω,k) Estimation



$$\mathbf{A}(\omega,\mathbf{r}) = \int_{\mathbf{k}} \mathbf{A}(\omega,\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}$$

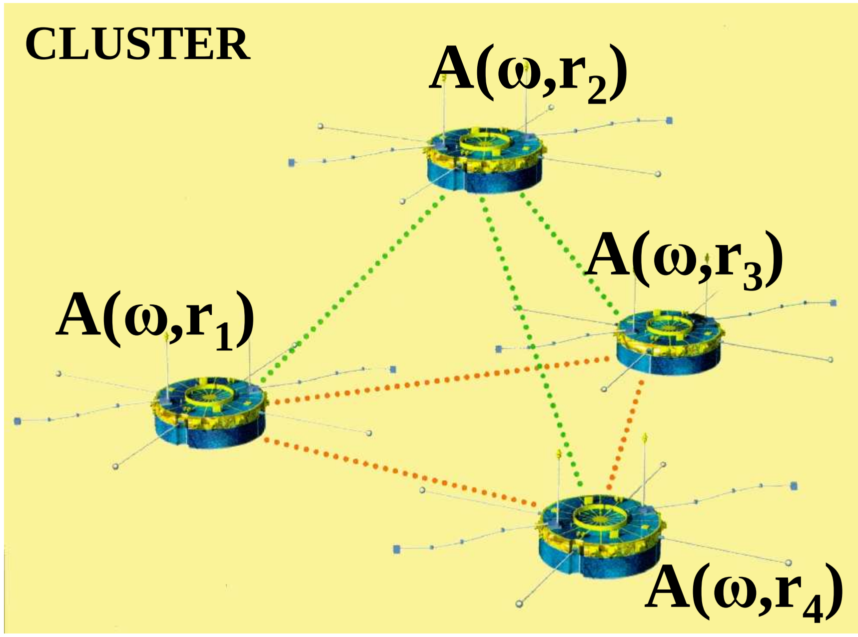
$$\mathbf{H}(\mathbf{k}) = \begin{pmatrix} \mathbf{I}_L \, e^{-i\mathbf{k}\cdot\mathbf{r}_1} \\ \mathbf{I}_L \, e^{-i\mathbf{k}\cdot\mathbf{r}_2} \\ \mathbf{I}_L \, e^{-i\mathbf{k}\cdot\mathbf{r}_3} \\ \mathbf{I}_L \, e^{-i\mathbf{k}\cdot\mathbf{r}_4} \end{pmatrix}$$

$$\mathbf{A}(\omega) = \int_{\mathbf{k}} \mathbf{H}(\mathbf{k}) \mathbf{A}(\omega,\mathbf{k}) \, d\mathbf{k}$$

$$P(\omega,\mathbf{k}) = trace\left\{\mathbf{F}^{\mathrm{T}}(\omega,\mathbf{k})\mathbf{M}_{\mathrm{A}}(\omega)\mathbf{F}(\omega,\mathbf{k})\right\} = \text{minimum}$$

$$\text{with} \quad \mathbf{F}^{\mathrm{T}}(\omega,\mathbf{k})\mathbf{H}(\mathbf{k})\mathbf{A}(\omega,\mathbf{k}) = \mathbf{A}(\omega,\mathbf{k})$$

P(ω,k) Estimation



$$\mathbf{A}(\omega) = \begin{pmatrix} \mathbf{A}(\omega,\mathbf{r}_1) \\ \mathbf{A}(\omega,\mathbf{r}_2) \\ \mathbf{A}(\omega,\mathbf{r}_3) \\ \mathbf{A}(\omega,\mathbf{r}_4) \end{pmatrix}$$

$$\mathbf{M}_{\mathbf{A}}(\omega) = \left\langle \mathbf{A}(\omega)\mathbf{A}^{\mathrm{T}}(\omega) \right\rangle$$

Cluster data

Cluster configuration

$$\mathrm{P}_{\mathrm{A}}(\omega,\mathbf{k}) = \mathrm{trace}[(\mathbf{H}^{\mathrm{T}}(\mathbf{k})\,(\mathbf{M}_{\mathbf{A}(\omega)})^{-1}\mathbf{H}(\mathbf{k}))^{-1}]$$

P(ω,k) Estimation

The constraining matrix $C_A(\omega,k)$

The extra information we have from physical laws can be included to further enhanced the k-filtering

Example: $\mathbf{A}(t,\mathbf{r}) = \begin{bmatrix} \mathbf{E}(t,\mathbf{r}) \\ c\mathbf{B}(t,\mathbf{r}) \end{bmatrix}$ and Faraday's law

$$\nabla \times \mathbf{E}(t,\mathbf{r}) = -\partial_t \mathbf{B}(t,\mathbf{r}) \quad \Longrightarrow \quad \mathbf{k} \times \mathbf{E}(\omega,\mathbf{k}) = \omega \mathbf{B}(\omega,\mathbf{k})$$

$$\mathbf{A}(\omega,\mathbf{k}) = \begin{pmatrix} E_x(\omega,\mathbf{k}) \\ E_y(\omega,\mathbf{k}) \\ E_z(\omega,\mathbf{k}) \\ cB_x(\omega,\mathbf{k}) \\ cB_y(\omega,\mathbf{k}) \\ cB_z(\omega,\mathbf{k}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -ck_z/\omega & ck_y/\omega \\ ck_z/\omega & 0 & -ck_x/\omega \\ -ck_y/\omega & ck_x/\omega & 0 \end{pmatrix} \begin{pmatrix} E_x(\omega,\mathbf{k}) \\ E_y(\omega,\mathbf{k}) \\ E_z(\omega,\mathbf{k}) \end{pmatrix} = \mathbf{C_A}(\omega,\mathbf{k}) \begin{pmatrix} E_x(\omega,\mathbf{k}) \\ E_y(\omega,\mathbf{k}) \\ E_z(\omega,\mathbf{k}) \end{pmatrix}$$

$$P_A(\omega,\mathbf{k}) = \mathrm{trace}\left[\mathbf{C}_{\mathbf{A}(\omega,\mathbf{k})} \left(\mathbf{C}_{\mathbf{A}}{}^{\mathbf{T}}{}_{(\omega,\mathbf{k})}\, \mathbf{H}^{\mathbf{T}}{}_{(\mathbf{k})} \left(\mathbf{M}_{\mathbf{A}(\omega)}\right)^{-1} \mathbf{H}_{(\mathbf{k})}\, \mathbf{C}_{\mathbf{A}(\omega,\mathbf{k})}\right)^{-1} \mathbf{C}_{\mathbf{A}}{}^{\mathbf{T}}{}_{(\omega,\mathbf{k})}\right]$$

(Pinçon and Lefeuvre, JGR,1991 ; Pinçon et al., ISSI SR-001,1998)

Validity of P(w,k)

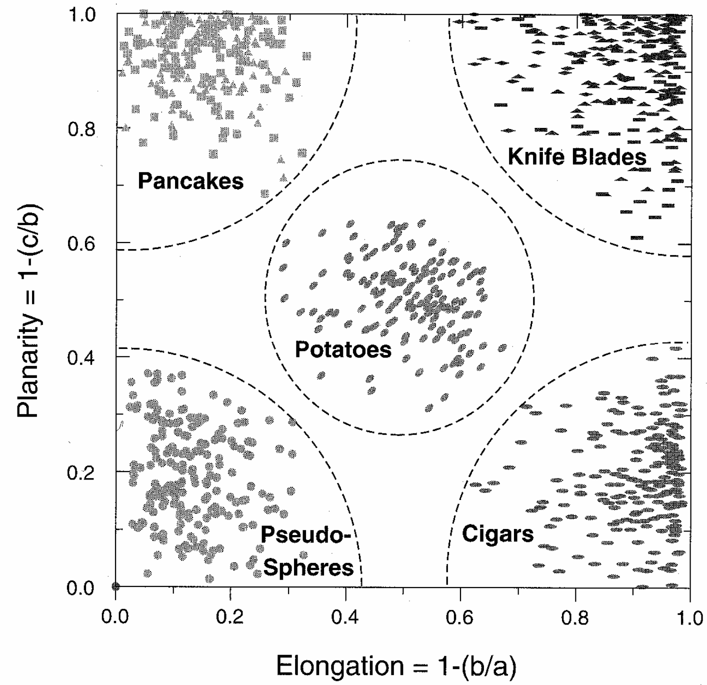
Wave-field requirements

- The wave-field is stationary
- The wave-field is homogeneous
- No aliasing

To avoid spatial aliasing the wave-field has to be free of wavelengths smaller than the minimum inter-spacecraft distance

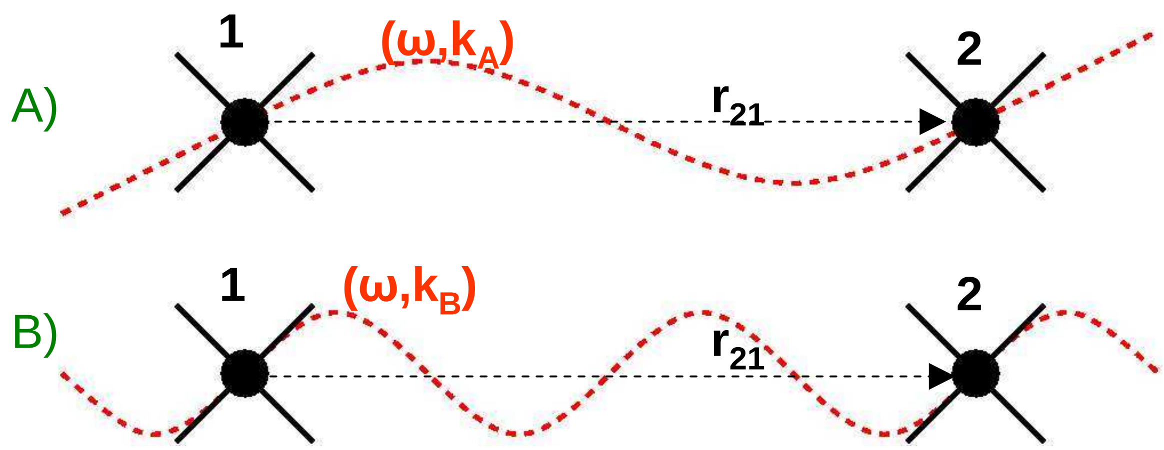
Geometric requirements

- 3D analysis ⇔ 3D geometry



(P. Robert et al., ISSI Scientific Report SR-001, 1998)

Spatial Aliasing



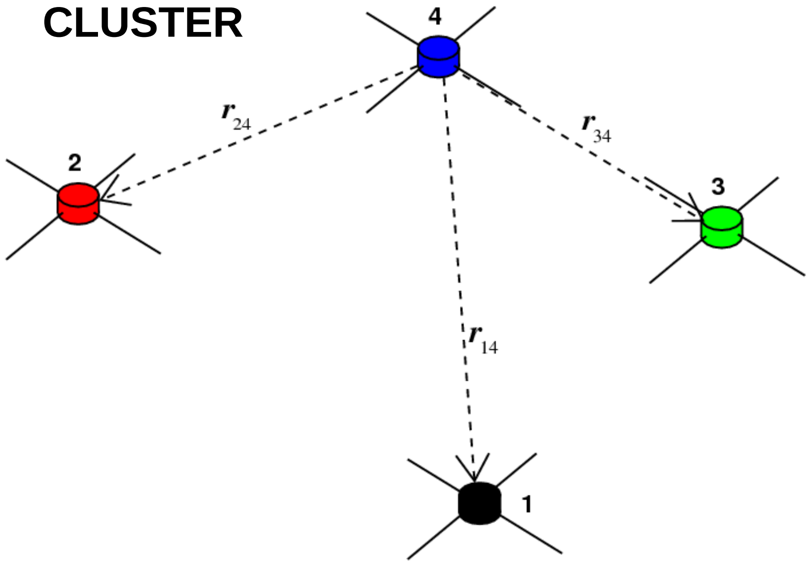
Two satellites cannot distinguish between $\mathbf{k}_A$ and $\mathbf{k}_B$ if:

$$\Delta\mathbf{k}\cdot\mathbf{r}_{21}=2\pi n$$

($\Delta\mathbf{k}=\mathbf{k}_B-\mathbf{k}_A$ and n is an integer)

Spatial Aliasing

CLUSTER

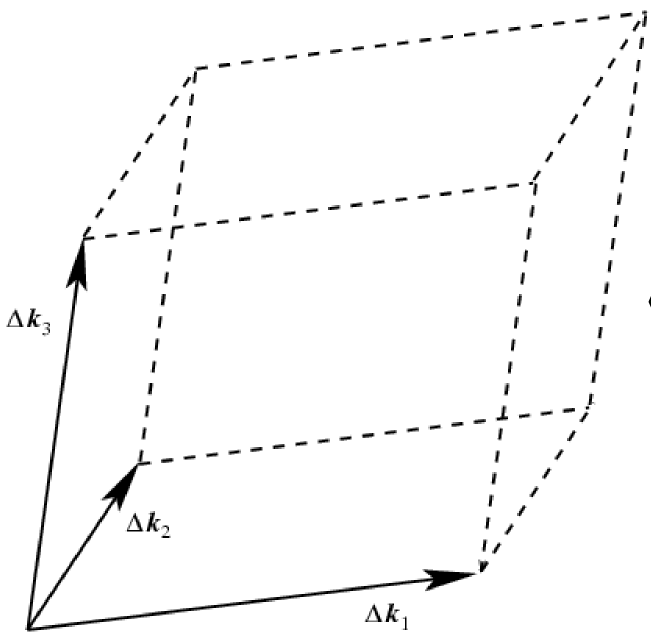


Spatial aliasing if:

$\Delta \mathbf{k} \cdot \mathbf{r}_{\alpha\beta} = 2\pi n$

(n is an integer ; α and β ∈{1,2,3,4})

$$\Delta \mathbf{k} = n_1 \Delta \mathbf{k}_1 + n_2 \Delta \mathbf{k}_2 + n_3 \Delta \mathbf{k}_3$$



$$\Delta \mathbf{k}_1 = \frac{2\pi}{V}(\mathbf{r}_{24} \times \mathbf{r}_{34})$$

$$\Delta \mathbf{k}_2 = \frac{2\pi}{V}(\mathbf{r}_{34} \times \mathbf{r}_{14})$$

$$\Delta \mathbf{k}_3 = \frac{2\pi}{V}(\mathbf{r}_{14} \times \mathbf{r}_{24})$$

$$V = \mathbf{r}_{14} \cdot (\mathbf{r}_{24} \times \mathbf{r}_{34})$$

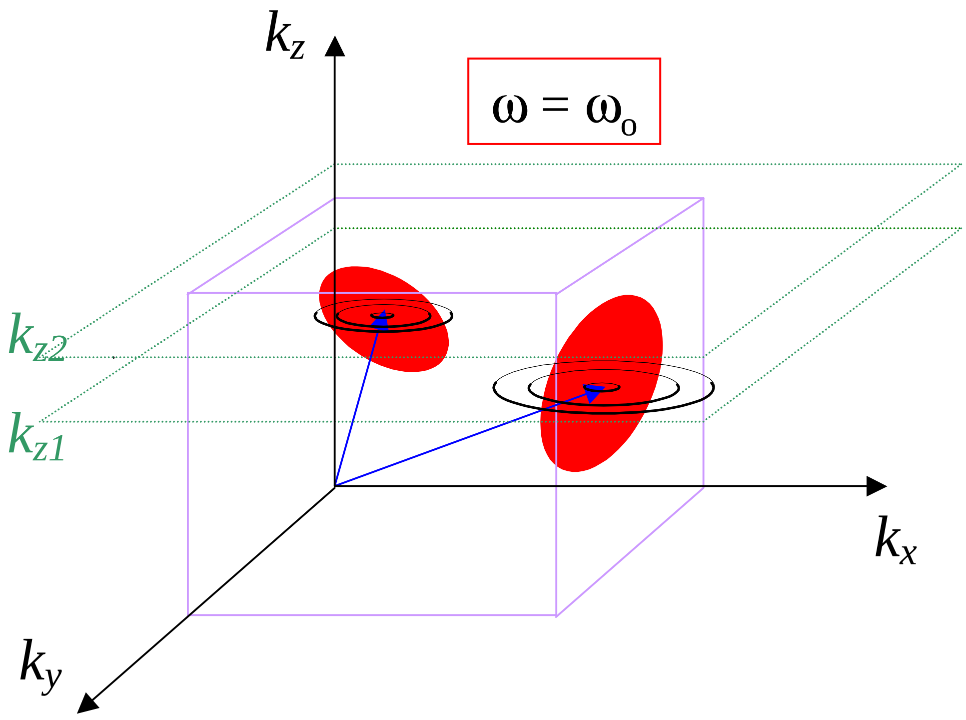
Neubauer and Glassmeier, JGR,1990

Representation of the wave-field energy distribution

$P(\omega, k_x, k_y, k_z)$: 4 variables !

- For a given ω we split the 3D-k domain in slices corresponding to different values of k_z.

- For each slice we adopt a contour line representation.

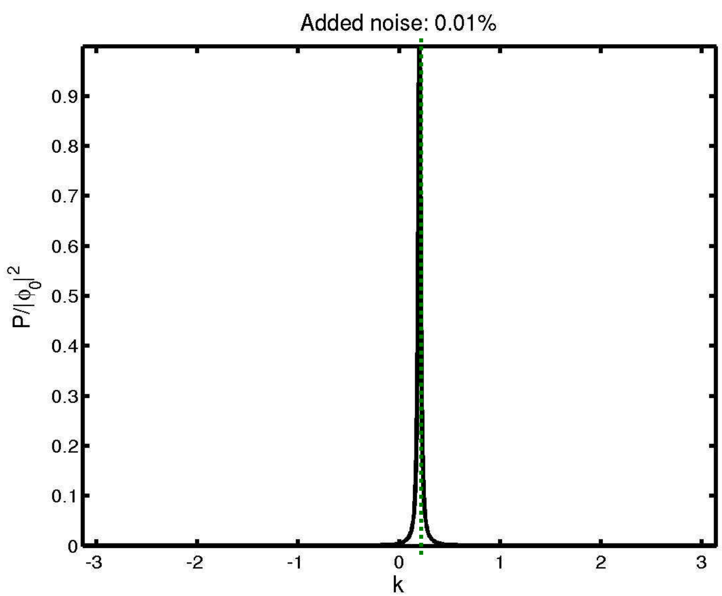


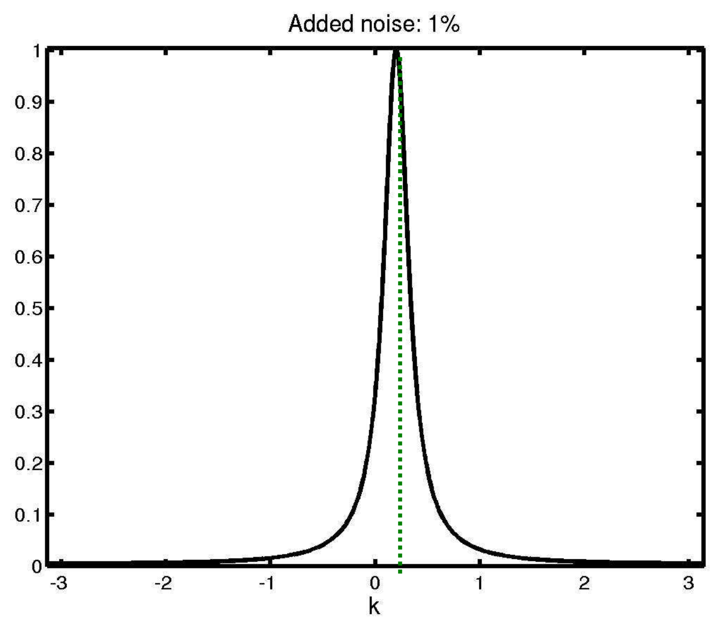
Simple 1D examples

Two satellites (at x_1 and x_2) measuring one field quantity $\Phi(t,x)$

A) The wave field is given by: $\Phi(t,x) = \Phi_o \exp[i(\omega_o t - k_o x)]$ + noise

$$P(\omega_o, k) = |\phi_o|^2 \frac{\varepsilon(2+\varepsilon)}{2(1+\varepsilon-\cos[(k-k_o)(x_1-x_2)])}$$





$k_o = 0.2$

$x_1 - x_2 = 1$

$\Phi_o = 1$

$P(\omega,k)$ is periodic in k

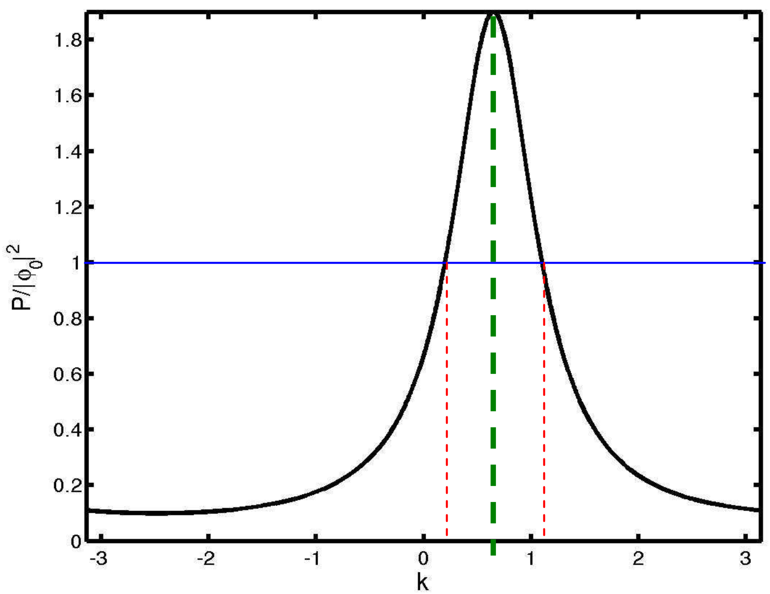
Simple 1D examples

B) The wave field is given by: $\Phi(t,x) = \Phi_1 \exp[i(\omega_o t - k_1 x)] + \Phi_2 \exp[i(\omega_o t - k_2 x)]$

The two waves are assumed to not be phase coherent

$$P(\omega_o, k) = \frac{|\phi_1(\omega_o)|^2 |\phi_2(\omega_o)|^2 \left(1 - \cos[(k_1 - k_2)\Delta x]\right)}{|\phi_1(\omega_o)|^2 \left(1 - \cos[(k_1 - k)\Delta x]\right) + |\phi_2(\omega_o)|^2 \left(1 - \cos[(k_2 - k)\Delta x]\right)}$$

$P(\omega_o,k)$: solution with only one peak.



$k_1 = 0.2$

$k_2 = 1.1$

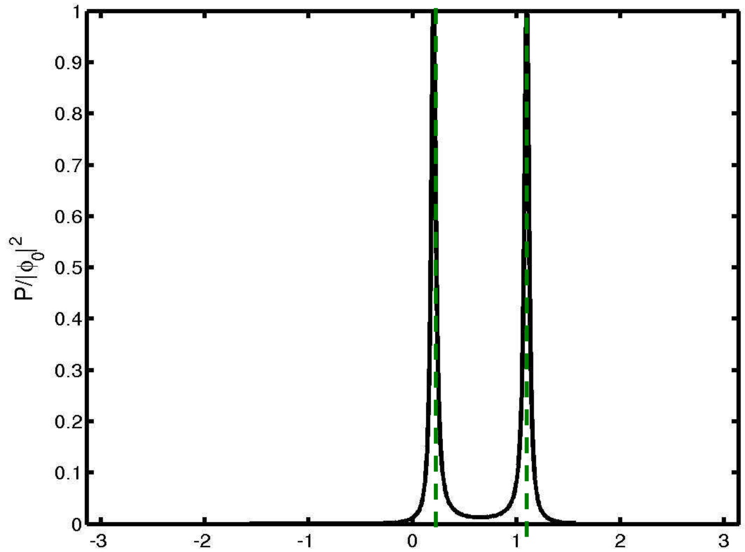
$\Delta x = x_1 - x_2 = 1$

$\Phi_1 = \Phi_2 = 1$

Simple 1D examples

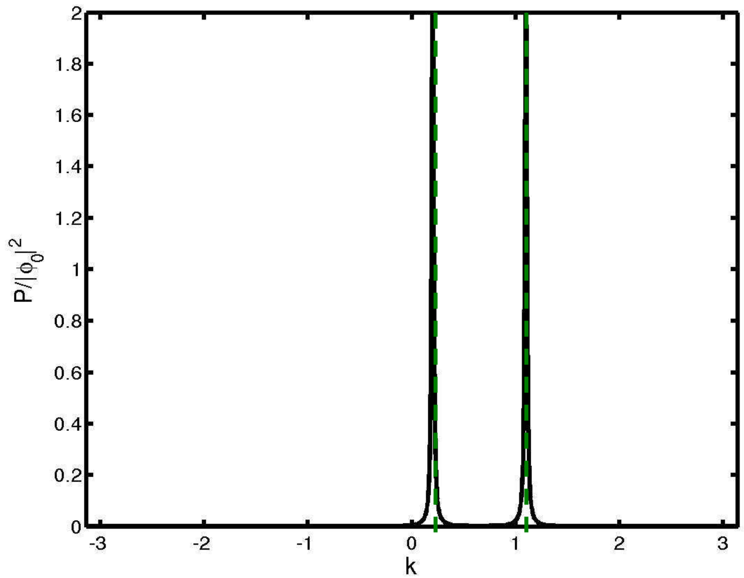
C) Same as B), one field quantity but using three satellites

The more satellites we use, the better resolution we get



D) Same as B), two satellites but using two field quantities

The more field quantities we use, the better resolution we get



Application:
Study of ULF wave fluctuations in the magnetosheath

Theoretical argument

• Intense ULF wave fluctuations are observed in the magnetosheath near the magnetopause.

• These fluctuations are expected to play an important role in the transfers between the solar wind and the magnetosphere.

Experimental issues

Interpretation of the fluctuations:

• MHD waves, mirror mode, low hybrid waves.
• Weak turbulence.
• Strong turbulence.

Properties of the fluctuations:

• No monochromatic waves.
• Continuous spectra.
• No clear polarization.

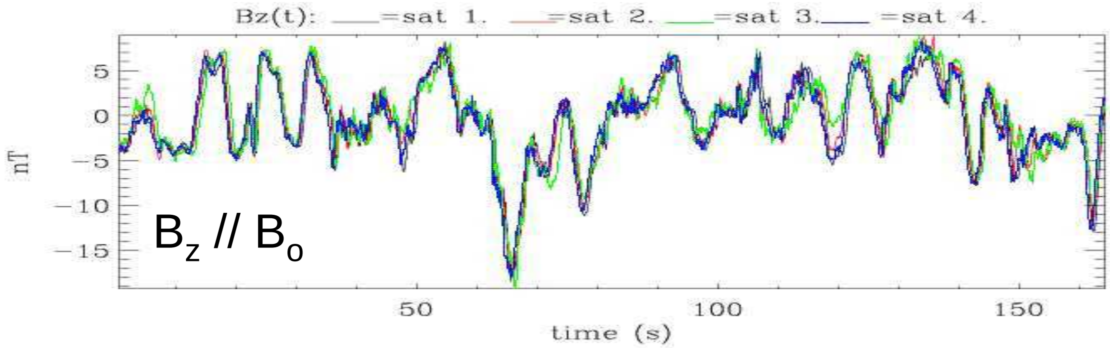
Cluster data

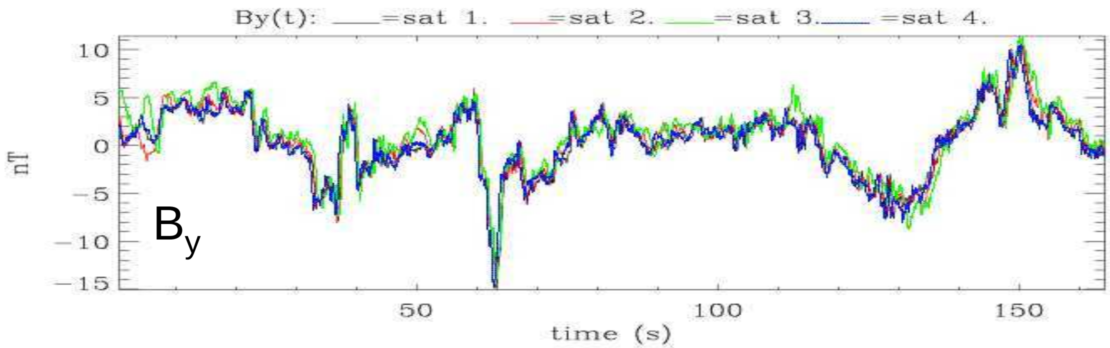
E and B Cluster measurements
+
K-filtering technique

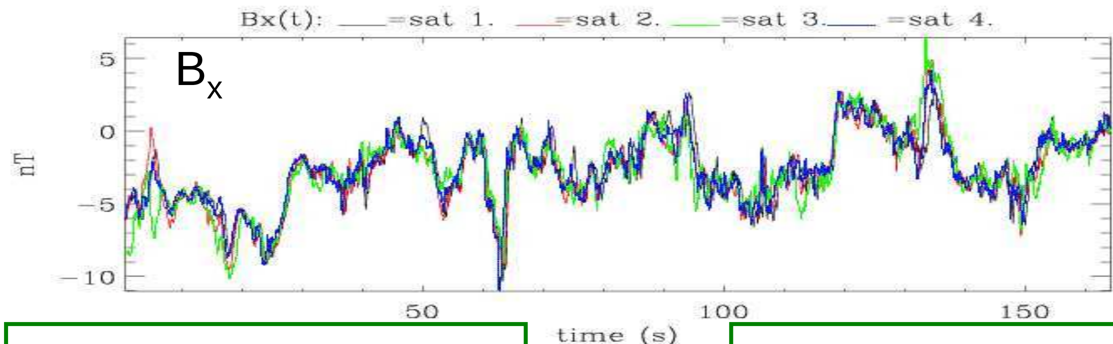
→ **3D-characterization of ULF wave-field fluctuations**

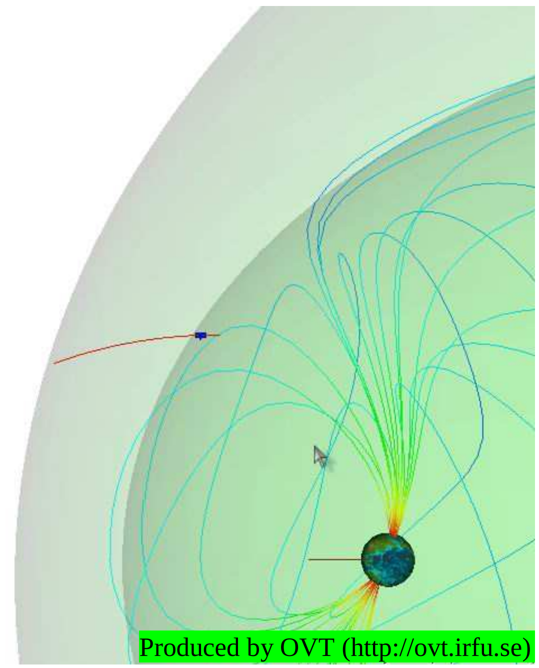
FGM data related to the selected event

2002-02-18 [05:34:01 – 05:36:45] UT









X_{GSE}=5.6 R_E

Y_{GSE}=4.6 R_E

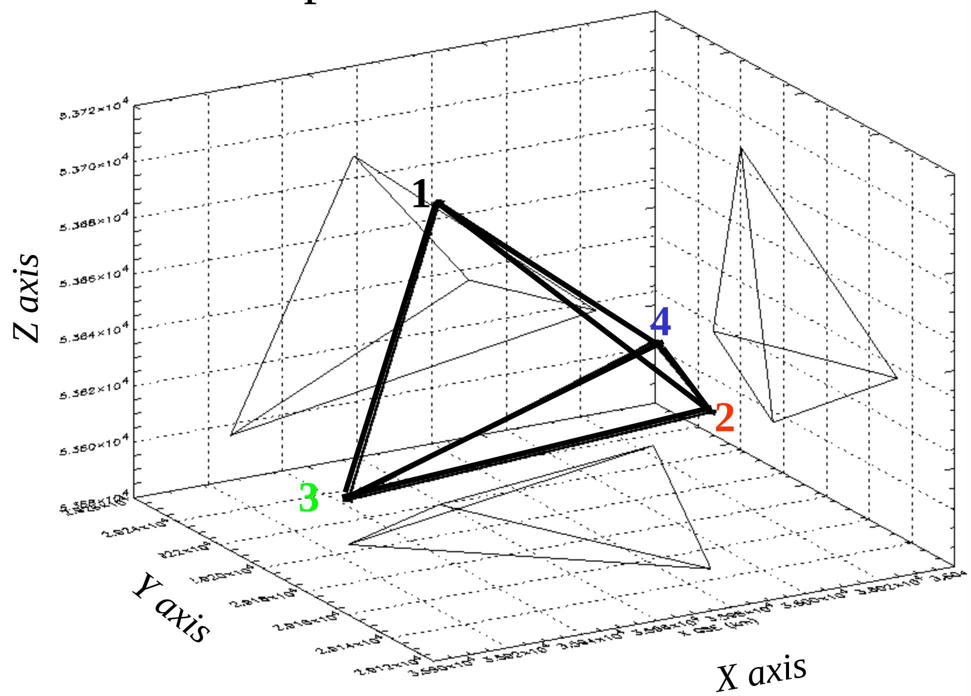
Z_{GSE}=8.4 R_E

- The wave-field is stationary

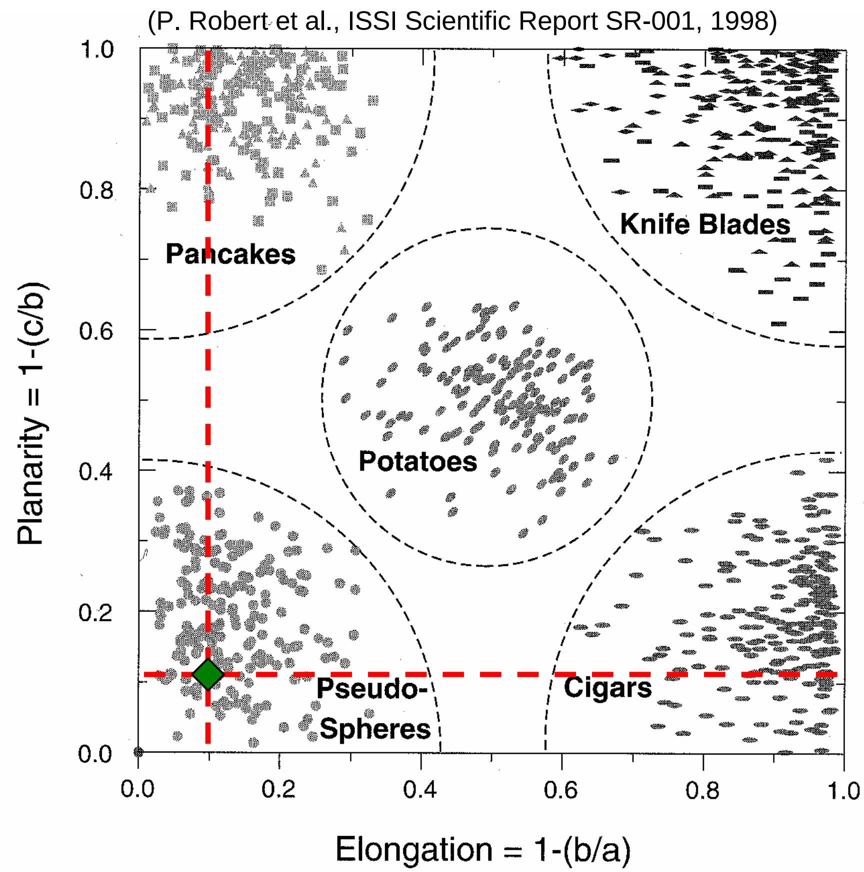
- The wave-field is homogeneous

Geometrical shape of the tetrahedron during the event

Interspacecraft distance ~ 100 km



GSE coordinate system (km)



$$\mathbf{r}_1 = \begin{matrix}35959\\29220\\53688\end{matrix} \quad \mathbf{r}_2 = \begin{matrix}35990\\29147\\53636\end{matrix} \quad \mathbf{r}_3 = \begin{matrix}35926\\29205\\53596\end{matrix} \quad \mathbf{r}_4 = \begin{matrix}36024\\29233\\53617\end{matrix}$$

Elongation ≤ 0.1

Planarity ≤ 0.1

Magnetosheath plasma parameters during the selected event

From CIS, FGM, and WHISPER experiments →

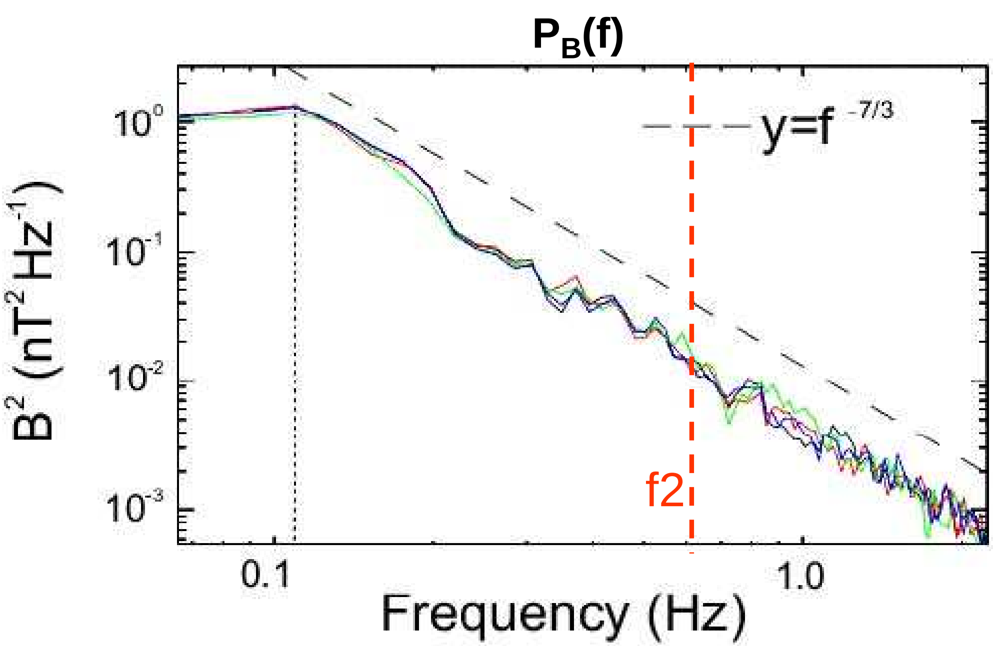
$n = 36\ cm^{-3}$

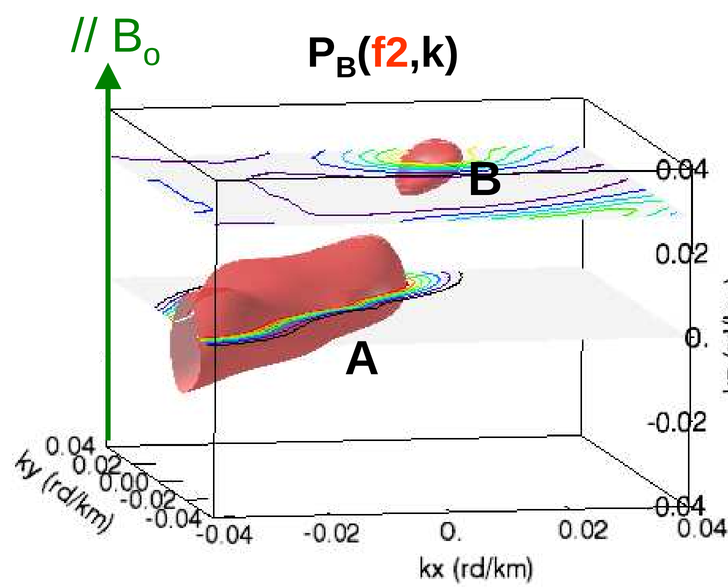
$T_{i//} = 140$ eV, $T_{i\perp} = 170$ eV

$V_A = 78$ km/s, $f_{ci} = 0.33$ Hz, $\rho = 79$ km

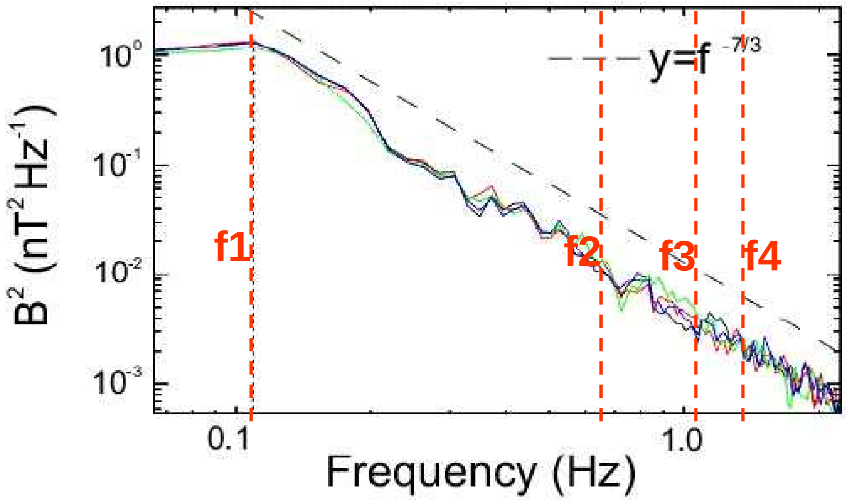
$\beta_{i//} = 4.5$, $\beta_{i\perp} = 5.4$

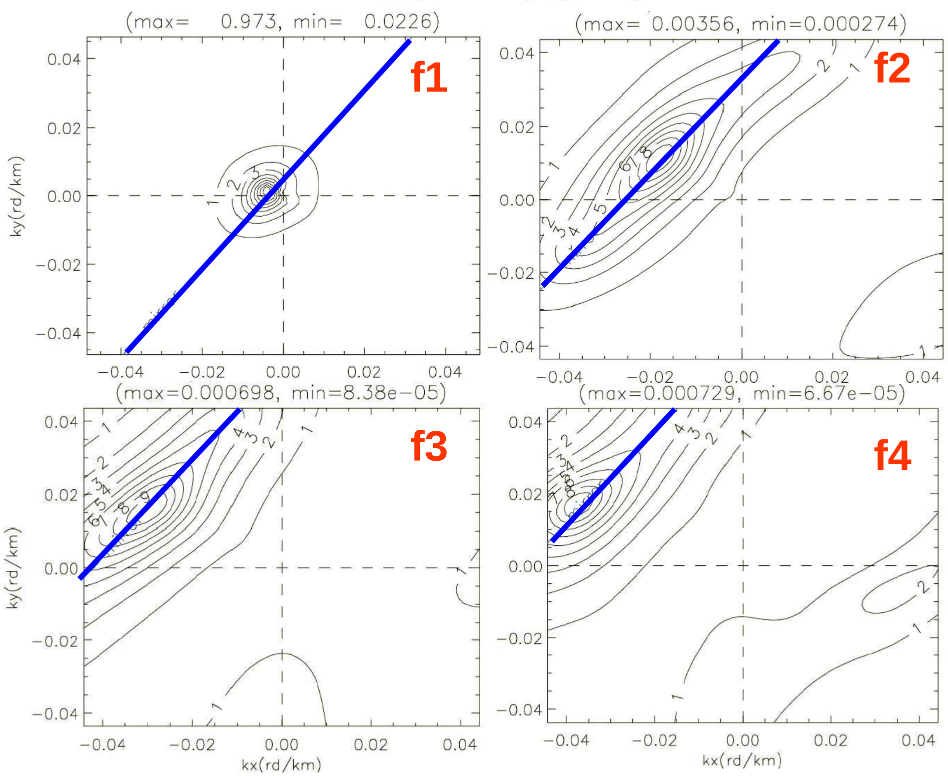
Power spectra of the ULF magnetic fluctuations

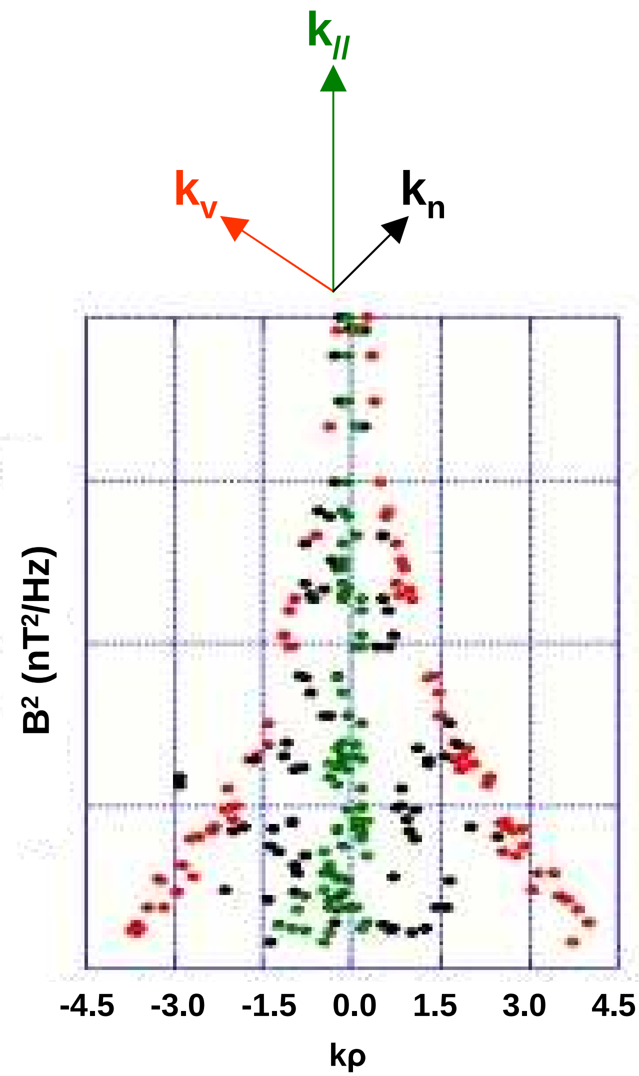




3D characterization of ULF magnetic fluctuations







(F. Sahraoui et al., submitted PRL, 2005)

3D characterization of ULF magnetic + electric fluctuations

Problems:

- Normalisation of **A**(ω)

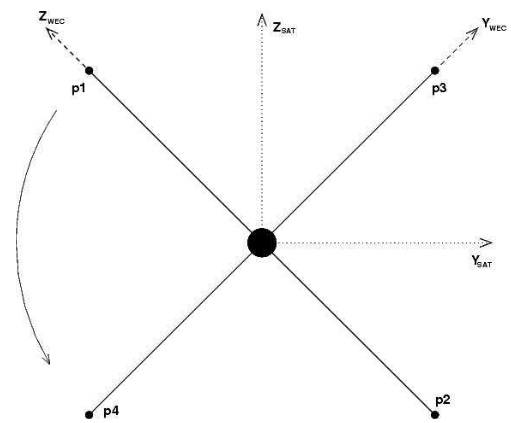
$$\mathbf{A}(\omega,\mathbf{r}) = \begin{bmatrix} E_x(\omega,\mathbf{r}) \\ E_y(\omega,\mathbf{r}) \\ R(\omega)\mathbf{B}(\omega,\mathbf{r}) \end{bmatrix} \quad \text{with} \quad R(\omega) = \sqrt{\frac{\left\langle |E_x(\omega,\mathbf{r})|^2 + |E_y(\omega,\mathbf{r})|^2 \right\rangle_{\mathbf{r}}}{\left\langle |B_x(\omega,\mathbf{r})|^2 + |B_y(\omega,\mathbf{r})|^2 + |B_z(\omega,\mathbf{r})|^2 \right\rangle_{\mathbf{r}}}}$$

- EFW data: only two electric components

Constraining matrix $\mathbf{C}_A$ derived from:

$$\mathbf{k} \cdot \mathbf{B}(\omega,\mathbf{k}) = 0$$

$$\omega B_z = k_x E_y - k_y E_x$$



- Technical problems on the EFW instruments:

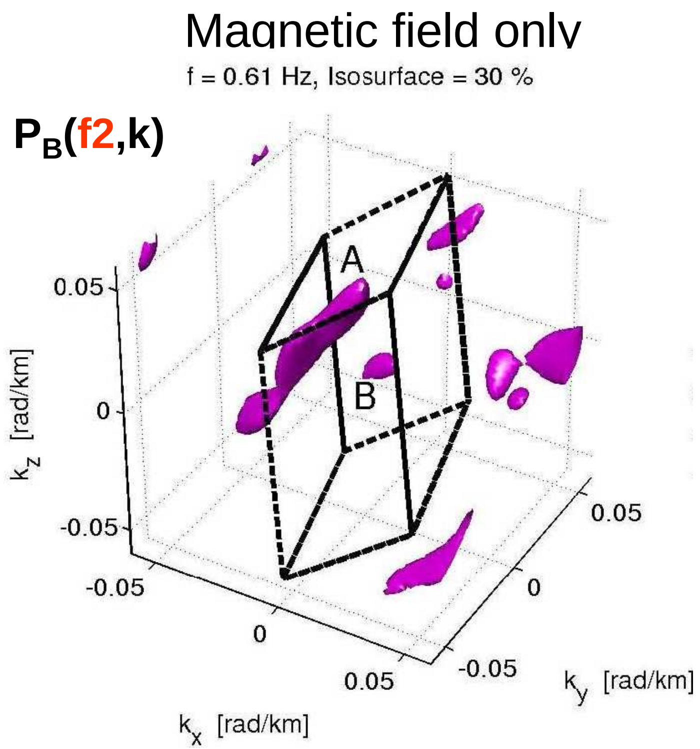
2001-07-25: 10 Hz filter problem on S/C2, probe 3
2001-12-28: probe failure on S/C1, probe 1
2002-07-29: probe failure on S/C3, probe 1

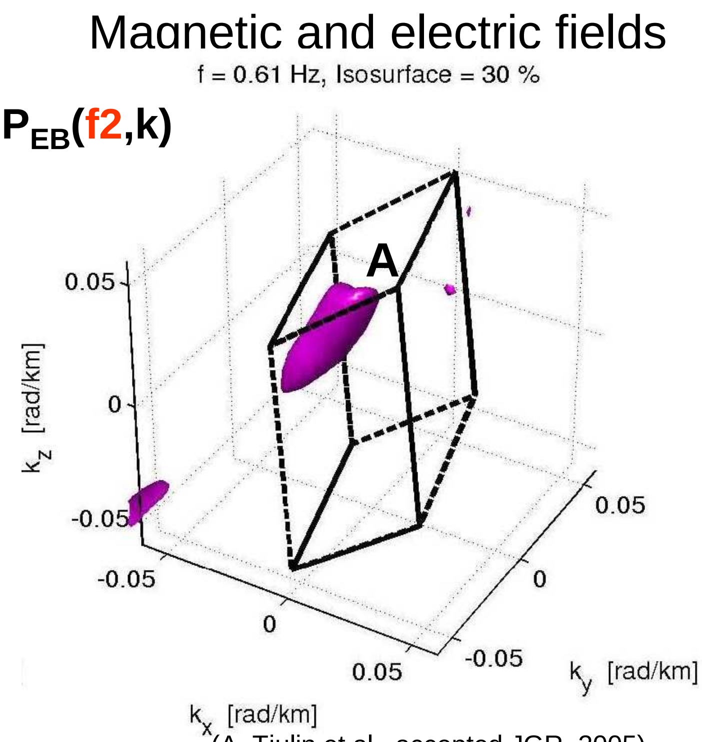
Non identical S/C:

Modification of the k-filtering equations

- Special spin effect cleaning for EFW data

3D characterization of ULF magnetic + electric fluctuations





(A. Tjulin et al., accepted JGR, 2005)

Advantages:

- **Drastic reduction of aliasing effects outside the validity domain**

- **No more aliased peak (B) inside the validity domain**

-**Higher resolution: two local maxima within (A)**

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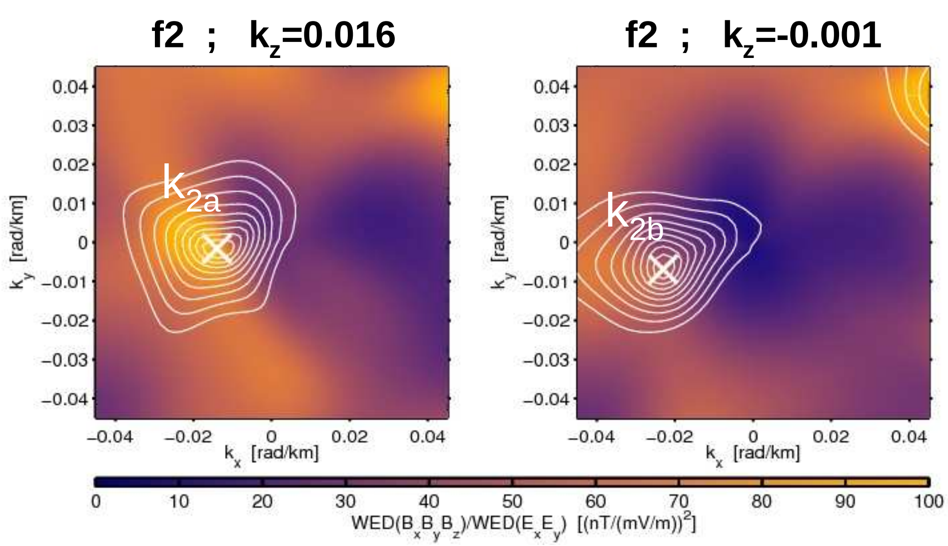
3D characterization of ULF magnetic + electric fluctuations

LPCE

	f1=0.37	f2=0.61	f3=1.12	(Hz)
B:	K_1= -0.0110, -0.0024, 0.0053	K_2= -0.0167, -0.0040, 0.0068	K_3= -0.0307, -0.0094, 0.0144	(rad/km) GSE
E+B:	K_1= -0.0099, -0.0023, 0.0064	K_{2a}= -0.0137, -0.0016, 0.0159 ; K_{2b}= -0.0230, -0.0069, -0.0014	K_3= -0.0325, -0.0077, 0.0086	

$$K_2 \approx \langle K_{2a} + K_{2b} \rangle$$

Additional information: ratio between magnetic and electric field energy



Ratio for k_{2a}=97 $(nT/(mV/m))^2$

Ratio for k_{2b}=52 $(nT/(mV/m))^2$

K_{2b} is more "electrostatic" than K_{2a}

Conclusion

The k-filtering technique is a method based on simultaneous multi-point measurements to characterize the wave-field fluctuations in space plasmas in terms of the wave-field energy distribution in the frequency and k vector space.

- Application to 3D characterization of the magnetosheath ULF turbulence
  - ⇨ Wave field energy is dominated by mirror modes at all frequencies
  - ⇨ Wave field energy is cascading from large scale to smaller scales along the plasma flow.
- Combining E and B measurements:
  - ⇨ Less aliasing effects
  - ⇨ Better resolution
  - ⇨ Additional interesting information

The K-filtering technique is a rather complex tool and cannot be used routinely