Discontinuity Analysis with Cluster

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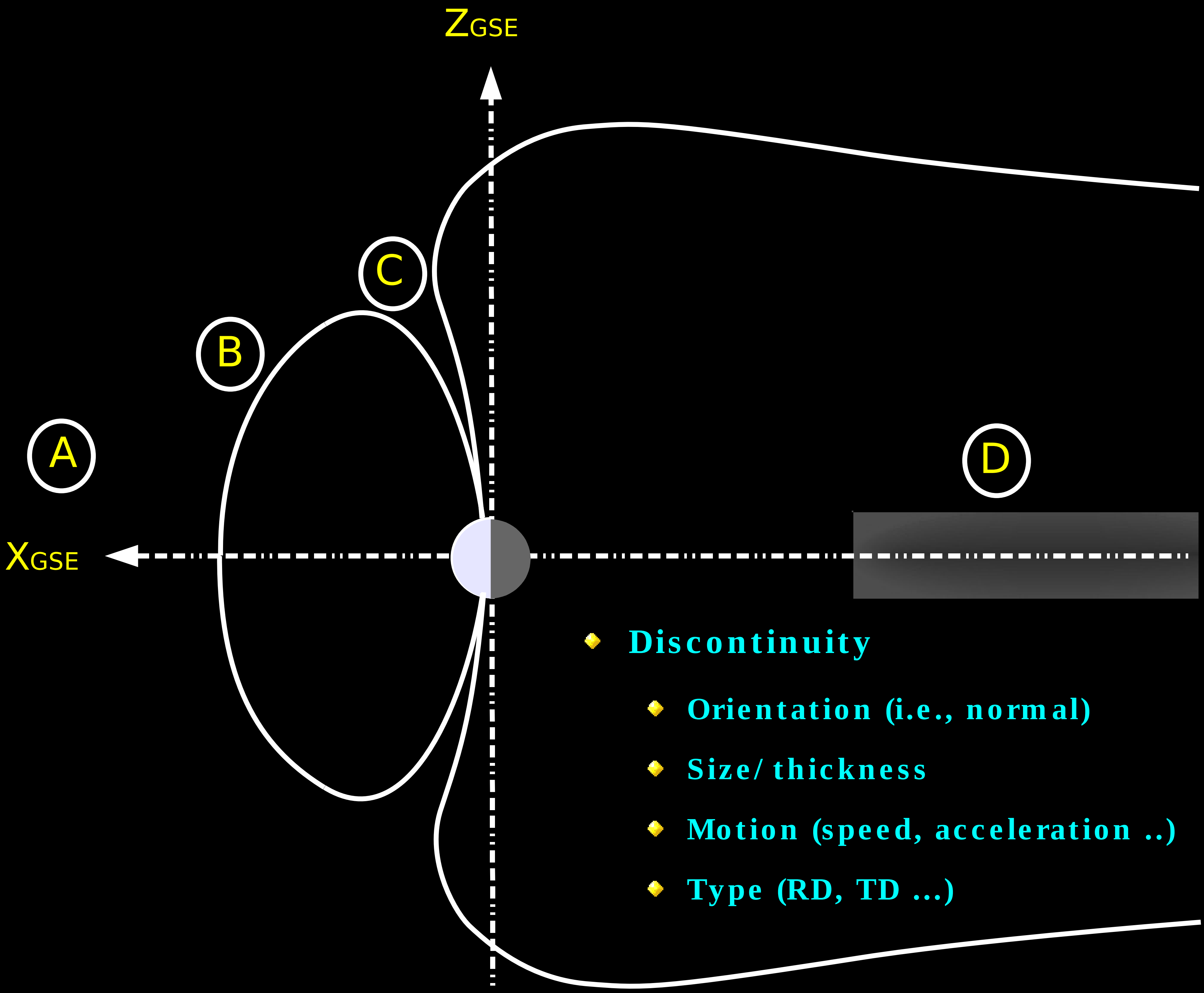
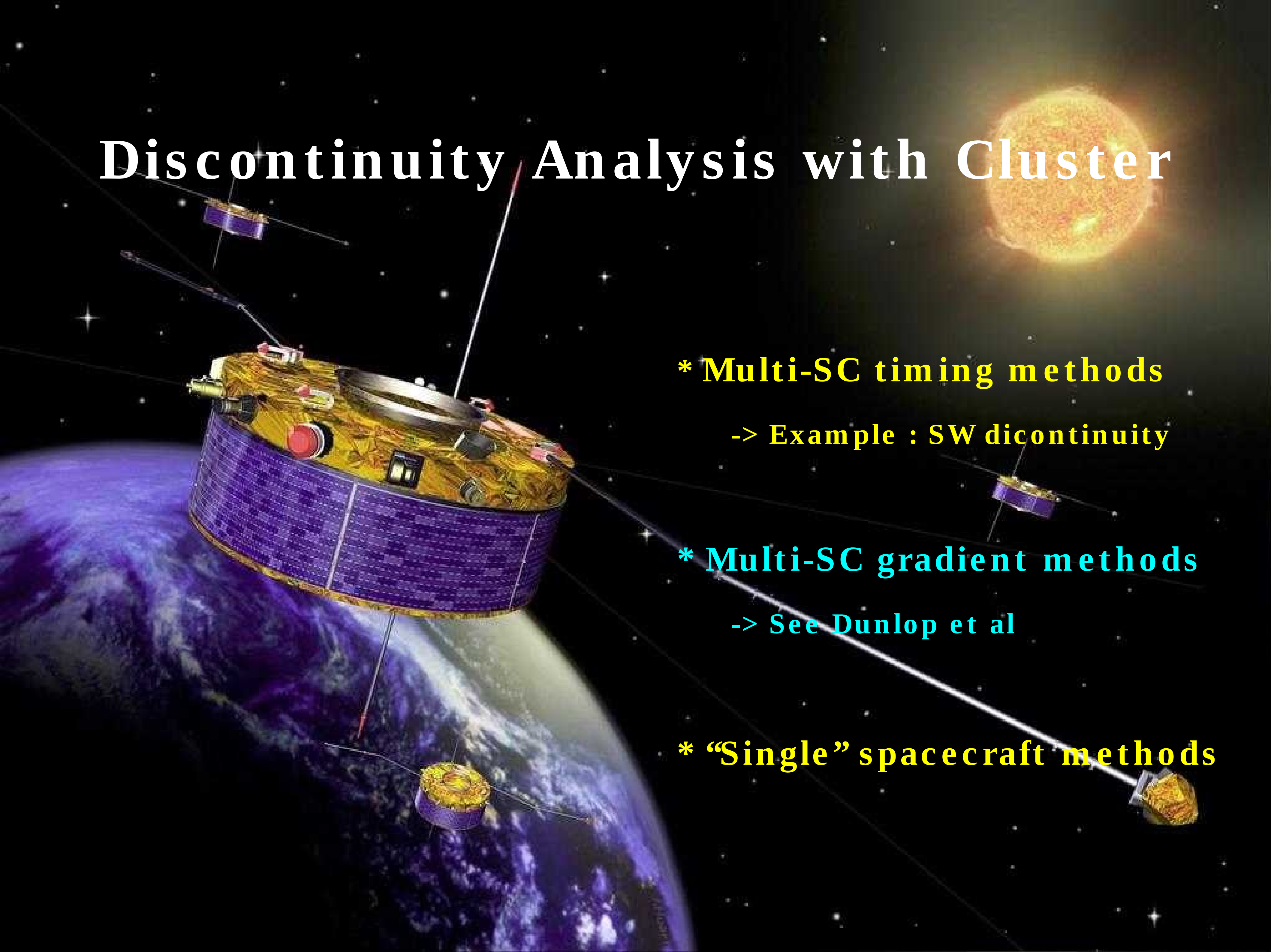


- Discontinuity
  - Orientation (i.e., normal)
  - Size/ thickness
  - Motion (speed, acceleration ..)
  - Type (RD, TD ...)

Timing Methods		Normal	Velocity	Acc.
CVA	(Constant Velocity Approach)	*YES*	*YES*	*NO*
CTA	(Constant Thickness Approach)	*YES*	*YES*	*YES*
MTV	(Minimum Thickness Variation)	*YES*	*YES*	*YES*
MVV	(Minimum Velocity Variation)	*YES*	*YES*	*YES*
DA	(Discontinuity Analyzer)	*YES*	*YES*	*YES*
Gradient Methods				
GRA	(Gradient of any Quantity)	*YES*	*NO*	*NO*
MVAJ	(Minimum Variance of **J**)	***YES***	***YES***	*NO*
MVAcE	(Minimum Variance of $\nabla \times \mathbf{E}$)	***YES***	***YES***	*NO*
MDD	(Minimum Directional Derivative)	*YES*	*NO*	*NO*
STD	(Spatio Temporal Derivative)	*NO*	*YES*	*YES*
Single-spacecraft Methods				
MVAB	(Minimum Variance of B)	***YES***	***NO***	***NO***
HT	(deHoffmann-Teller Analysis)	***NO***	***YES***	***YES***
MFR	(Minimum Faraday Residue)	***YES***	***YES***	***NO***
MMR	(Minimum Massflow Residue)	***YES***	***YES***	***NO***
MLMR	(Minimum Linear Momentum Residue)	***YES***	***YES***	***NO***
MTER	(Minimum Total Energy Residue)	***YES***	***YES***	***NO***
MER	(Minimum Entropy Residue)	***YES***	***YES***	***NO***
COM	(Combination of above)	***YES***	***YES***	***NO***

Discontinuity Analysis with Cluster

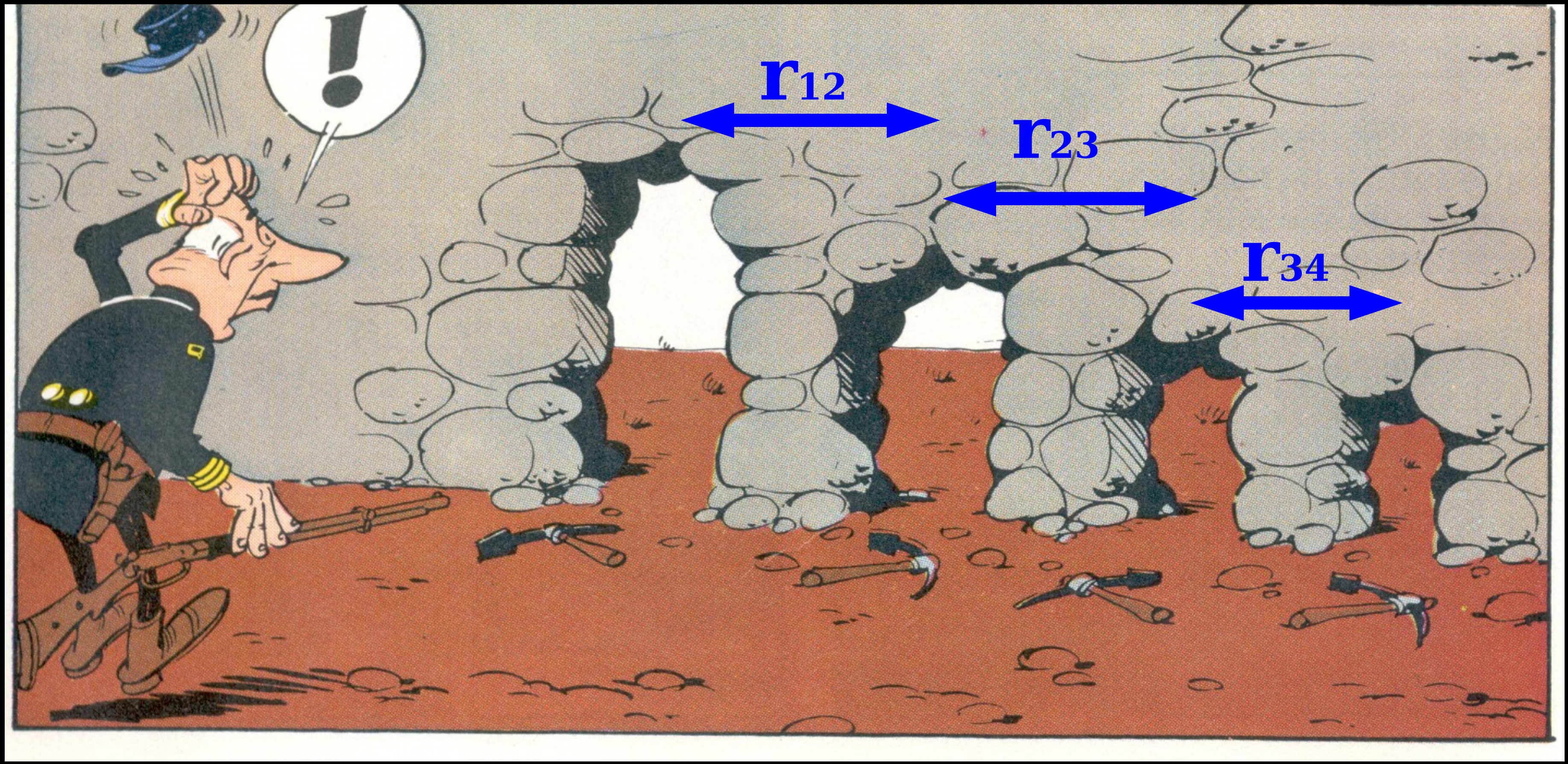


* Multi-SC timing methods
  -> Example : SW dicontinuity

* Multi-SC gradient methods
  -> See Dunlop et al

* “Single” spacecraft methods

Timing Methods : Principle



Orientation, Velocity :

- find out WHEN each SC crossed discontinuity

Thickness, Acceleration :

- find DURATION of crossing

Simple approach :

$$\begin{bmatrix} \mathbf{r}_{12} \\ \mathbf{r}_{13} \\ \mathbf{r}_{14} \end{bmatrix} \cdot \frac{1}{V} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} t_{12} \\ t_{13} \\ t_{14} \end{bmatrix}$$

Caveats :

- no acceleration

-> not good for wavy MP, cusp, magnetotail

-> use DA or CTA

Example – SW discontinuities

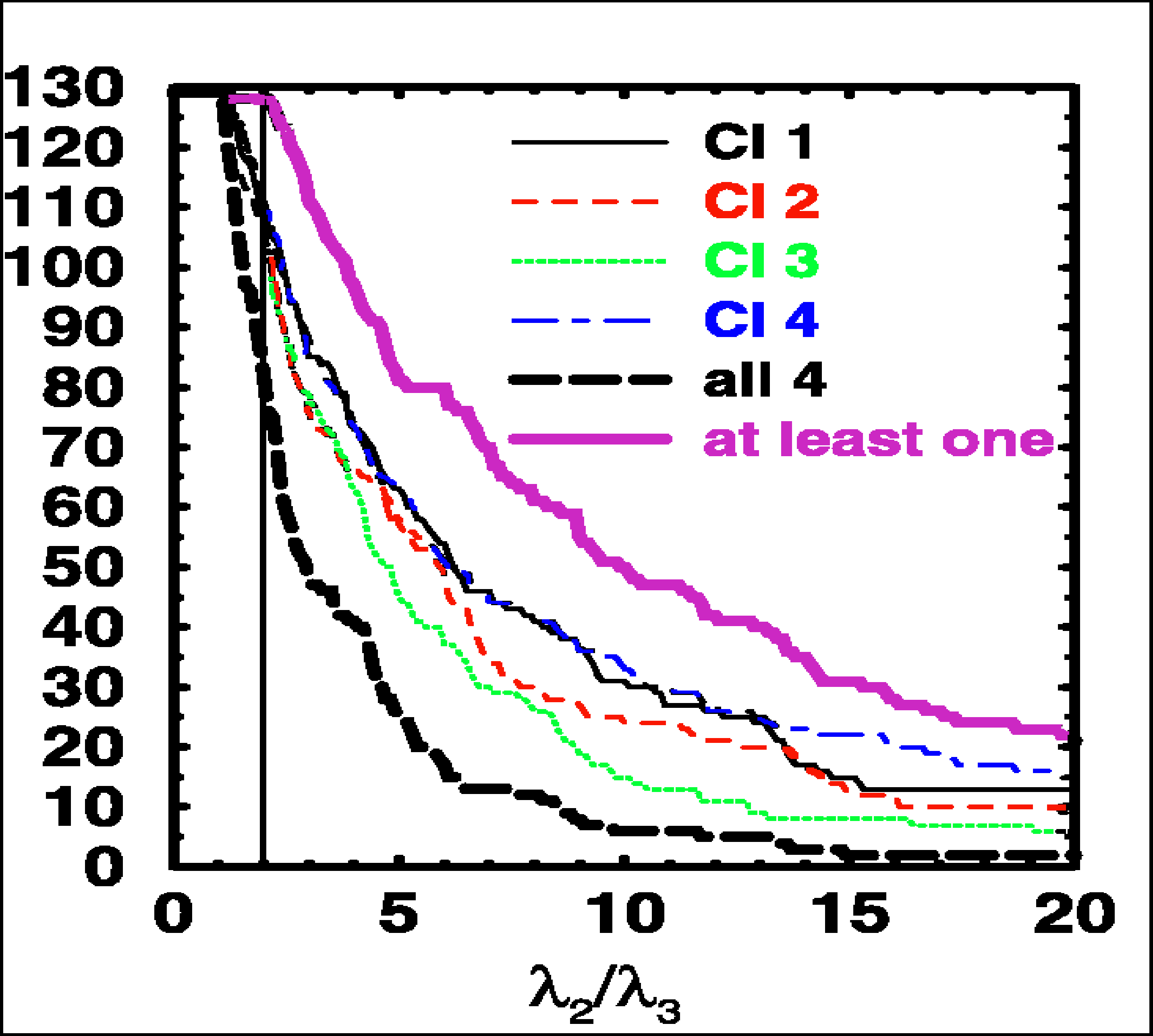
Knetter et al, (2003), 2004, studied 129 SW discontinuities :

- Tested 3 methods :
  - MVAB (Minimum Variance Analysis of B-field)
  - $\mathbf{n} = \mathbf{B}_1 \times \mathbf{B}_2$
  - Timing (Triangulation – i.e., Constant Velocity Approach)

- Thickness of discontinuities

- Classified as TD or RD (Tangential / Rotational Dicontinuity)
  - RD : $|B_n| / B_{max} \geq 0.4$; $|[B]| / B_{max} < 0.2$
  - TD : $|B_n| / B_{max} < 0.4$; $|[B]| / B_{max} \geq 0.2$

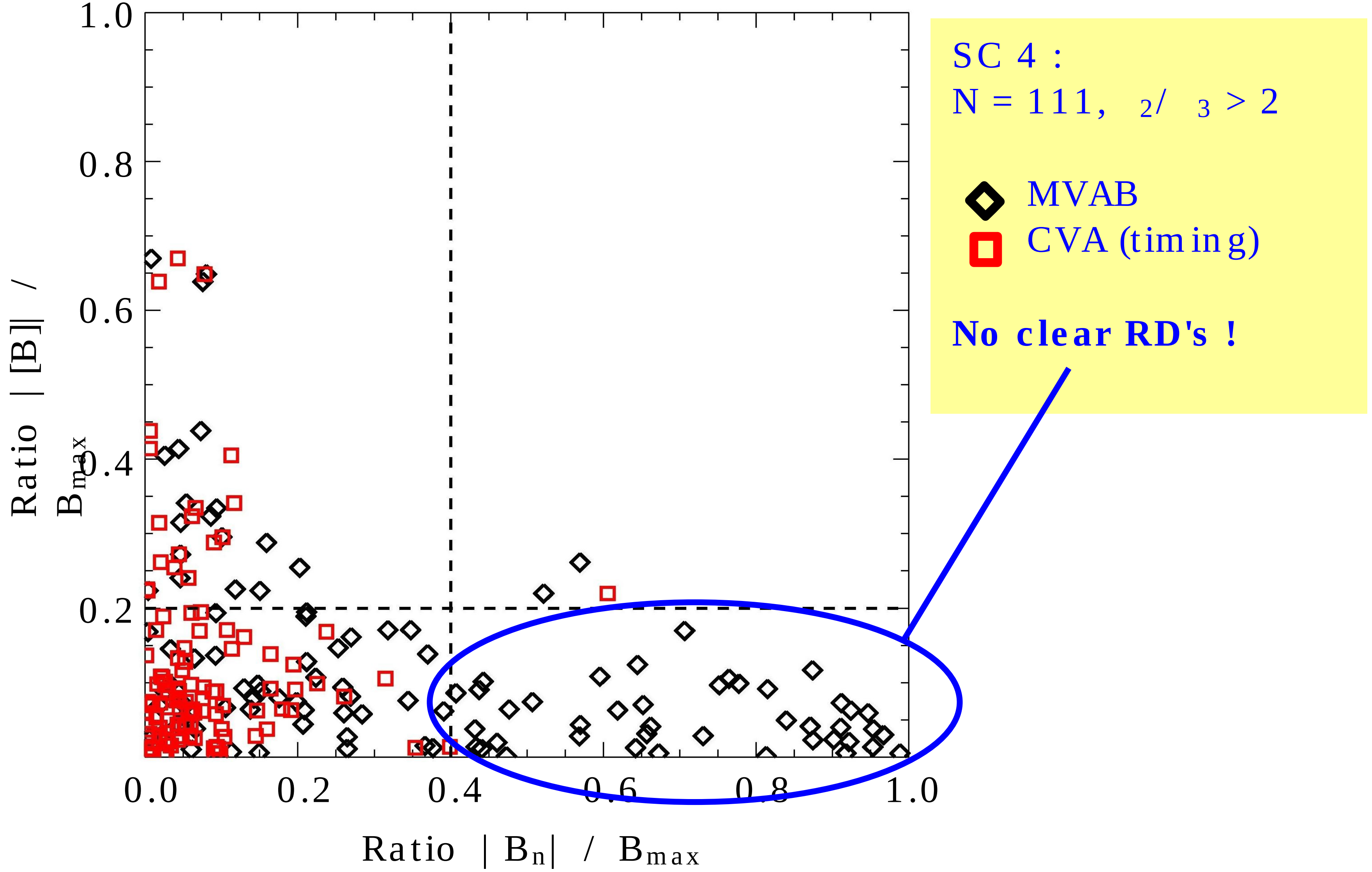
Problem : MVA not always reliable



After Knetter et al, 2004

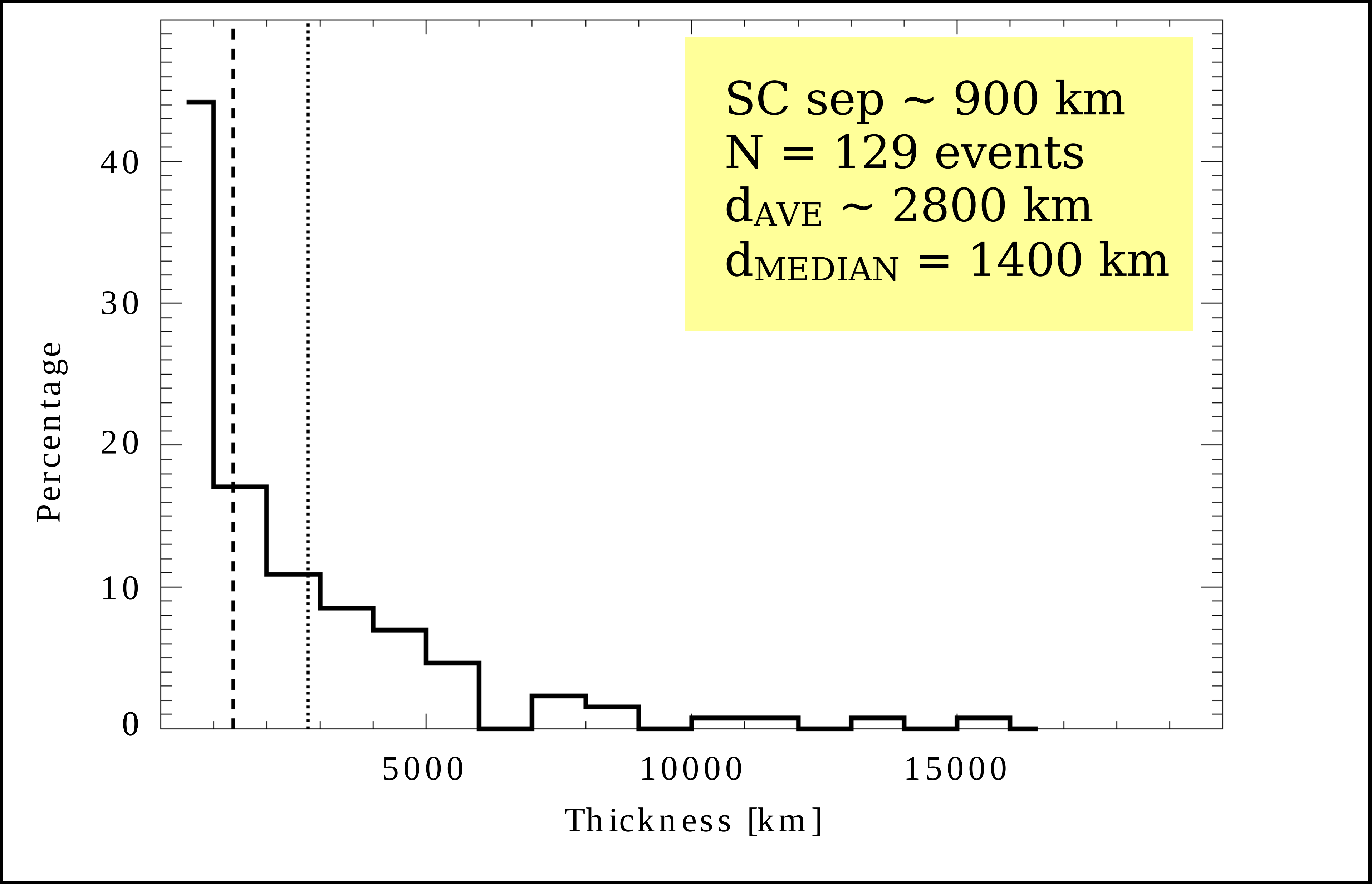
RD : | Bn| / Bmax ≥ 0.4; | [B]/ Bmax < 0.2

TD : | Bn| / Bmax < 0.4; | [B]/ Bmax ≥ 0.2



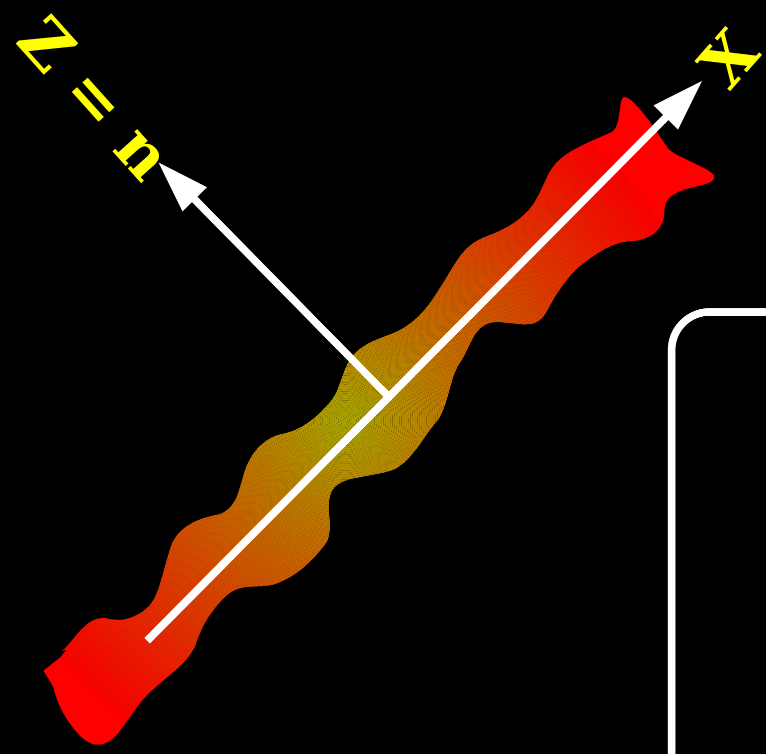
After Knetter et al, 2004

SW discontinuities - thickness



After Knetter et al, 2004

“Single” Spacecraft Methods : MVA

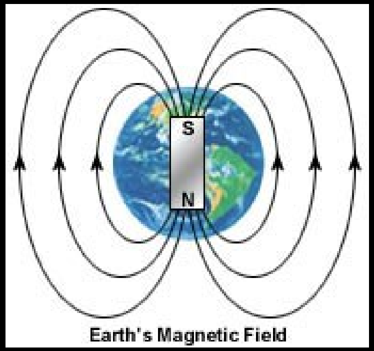


$$\sigma^2 = \frac{1}{M}\sum_{m=1}^{M} \left| \left(\boldsymbol{B} - \langle \boldsymbol{B} \rangle \right) \cdot \boldsymbol{n} \right|^2$$

$$Q_{ij}^{B} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle$$

- Find **eigenvalues** and **eigenvectors** of Q_{ij}^{B}

Minimum variance – other applications



$$\nabla \cdot \boldsymbol{B} = 0$$ (conservation of magnetic poles)

$$\nabla \cdot \boldsymbol{J} = 0$$ (conservation of charge)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0$$ (conservation of mass flux)

. . .

$$\frac{\partial \eta}{\partial t} + \nabla \cdot q = 0$$ (conservation of any quantity q)

Generic Residue Analysis

$$Q_{ij}^{GENERIC} = \left\langle (\Delta q_{ki} - U_i \Delta \eta_k)\ (\Delta q_{kj} - U_j \Delta \eta_k) \right\rangle$$

Q^{MVAB}

Q^{MVAJ}

Q^{MMR}

Q^{MFR}

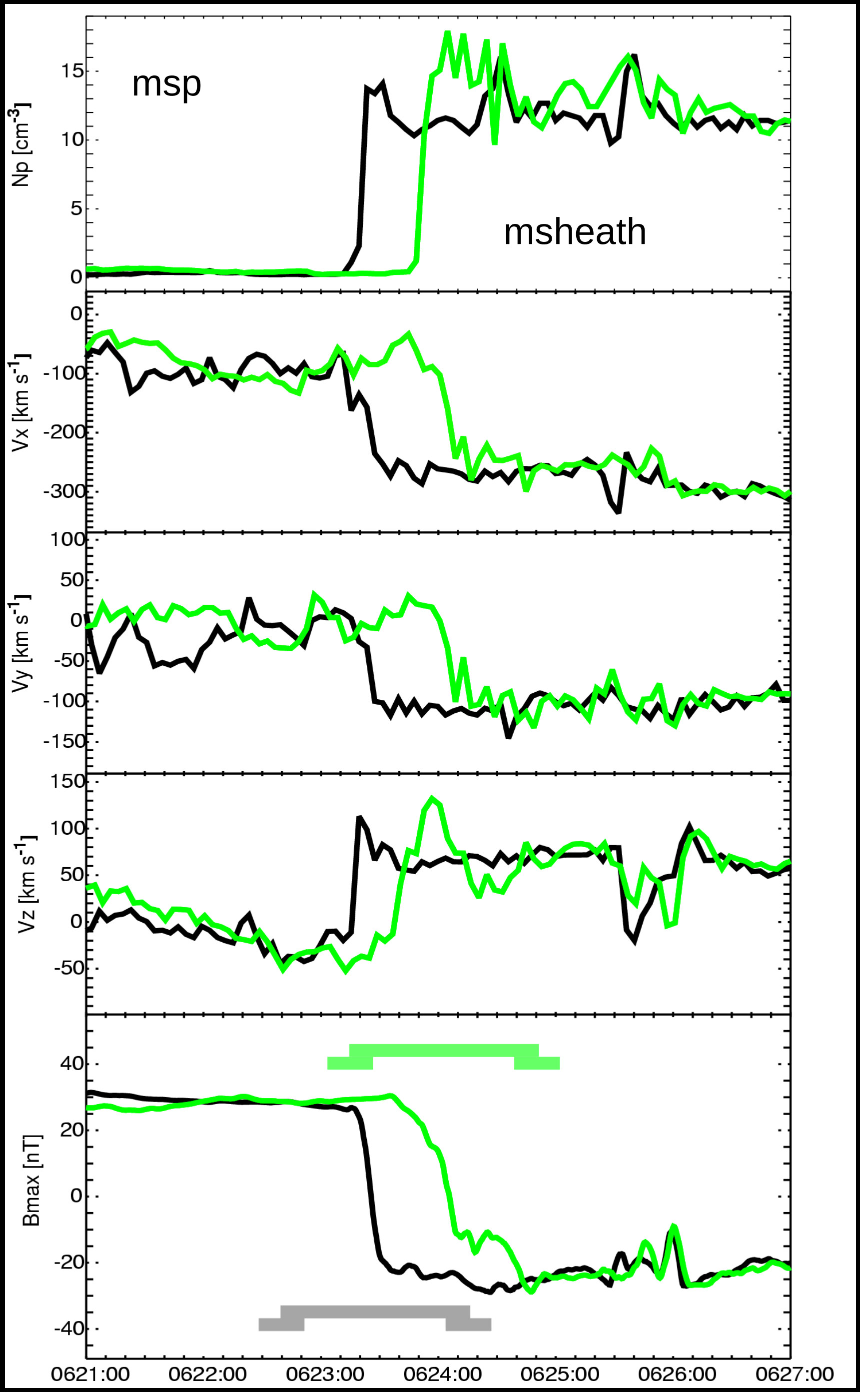
Q^{MTER}

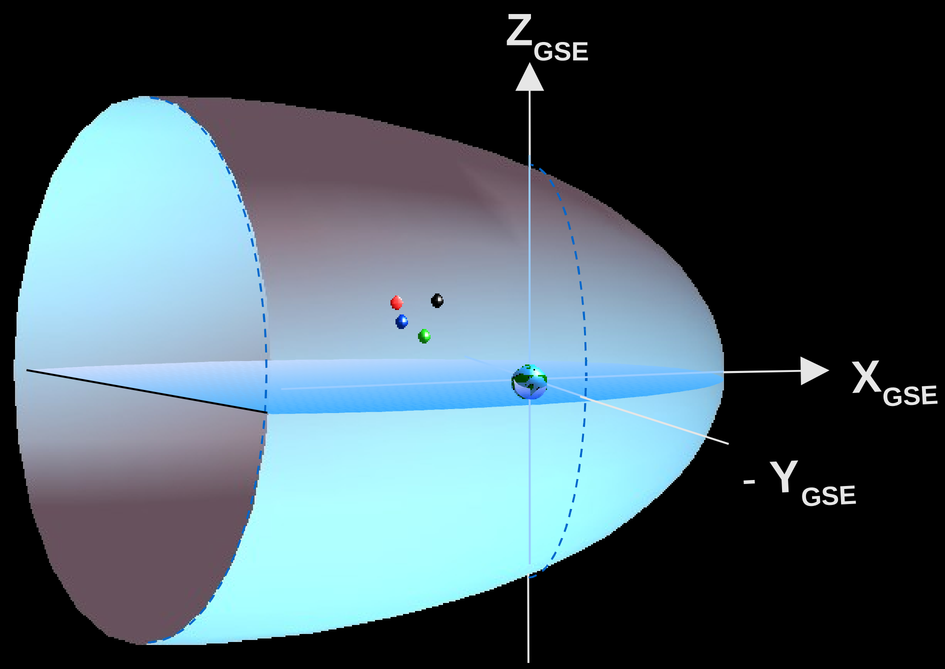
. . .

Orientation of discontinuity from any method :

Find eigenvalues and eigenvectors of Q

Example : Magnetopause crossing



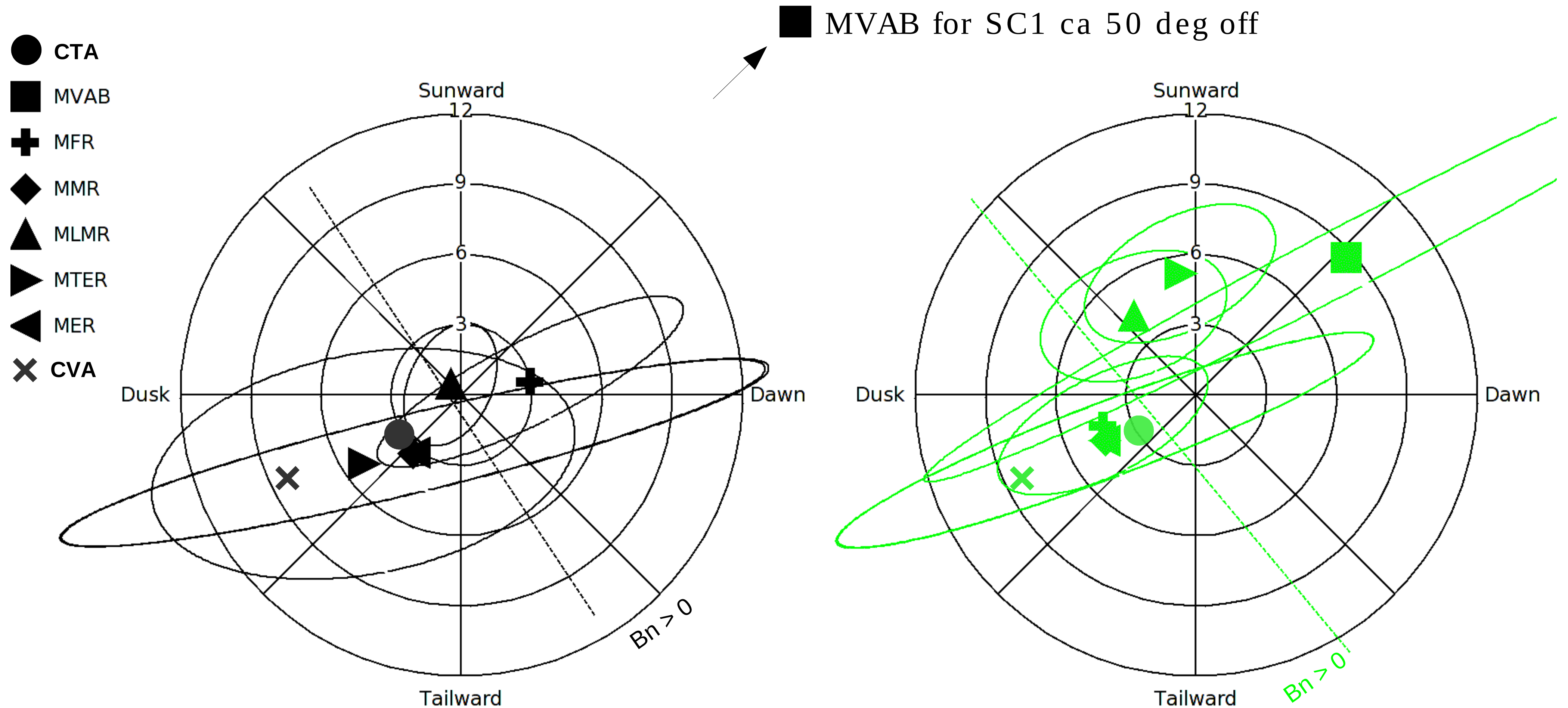


Flank crossing on 5 July 2001

1) tested with 25 samples a 4 sec

2) tested with nested segments

Magnetopause orientation



scale 12 deg

More fun with covariance matrices

1) Combing Q matrices from more spacecraft

$$\boldsymbol{Q}_{\boldsymbol{ALL}} = w_1 \boldsymbol{Q}_{\boldsymbol{1}} + w_2 \boldsymbol{Q}_{\boldsymbol{2}} + w_3 \boldsymbol{Q}_{\boldsymbol{3}} + w_4 \boldsymbol{Q}_{\boldsymbol{4}}$$

-> multi-SC method !

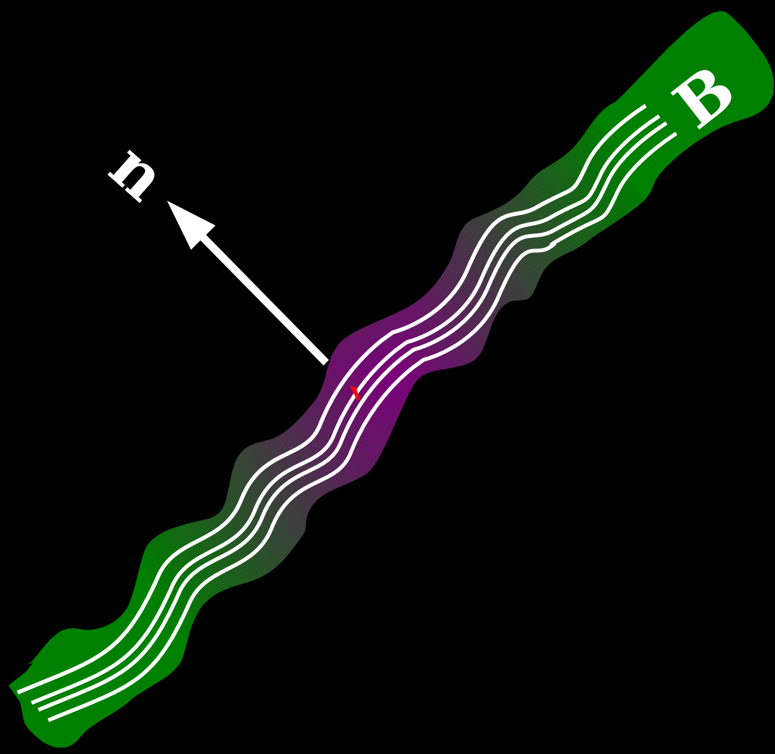
2) Combing Q matrices from several methods

$$\boldsymbol{Q}_{\boldsymbol{COM}} = \sum_{i=0}^{i=N} w_i \boldsymbol{Q}_{\boldsymbol{i}}, \qquad i = MVAB, MER, MVAJ....$$

3) Constrain Q

$$\boldsymbol{Q}_{\boldsymbol{C}} = \boldsymbol{P} \cdot \boldsymbol{Q} \cdot \boldsymbol{P}$$

Constraining the variance analysis



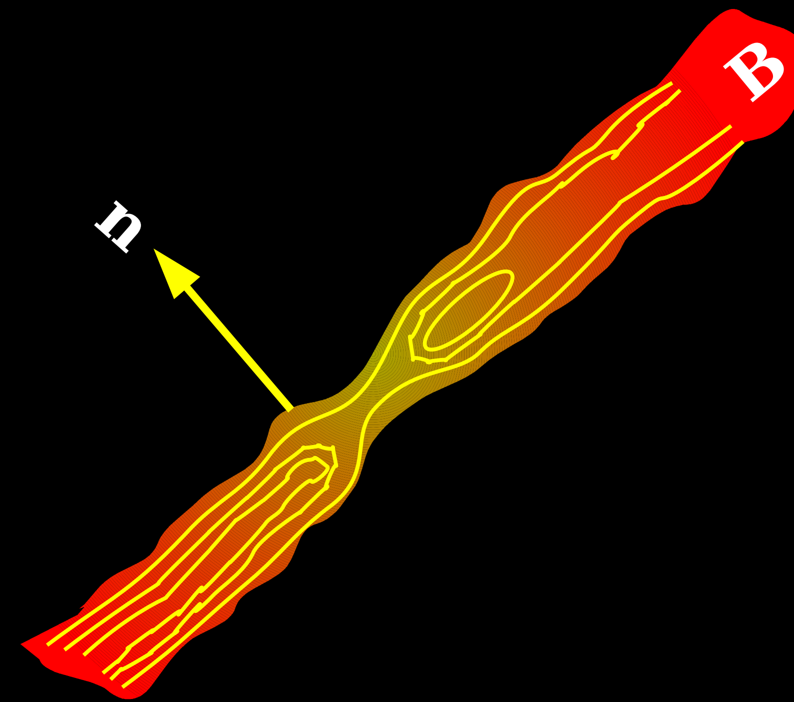
TD – tangential discontinuity

B•n = 0, **V•n** = 0, …

⬇

Constrain Q matrix so that

B•n == 0



RD – rotational discontinuity

|B•n| > 0,   **V•n** = e.g., V_A

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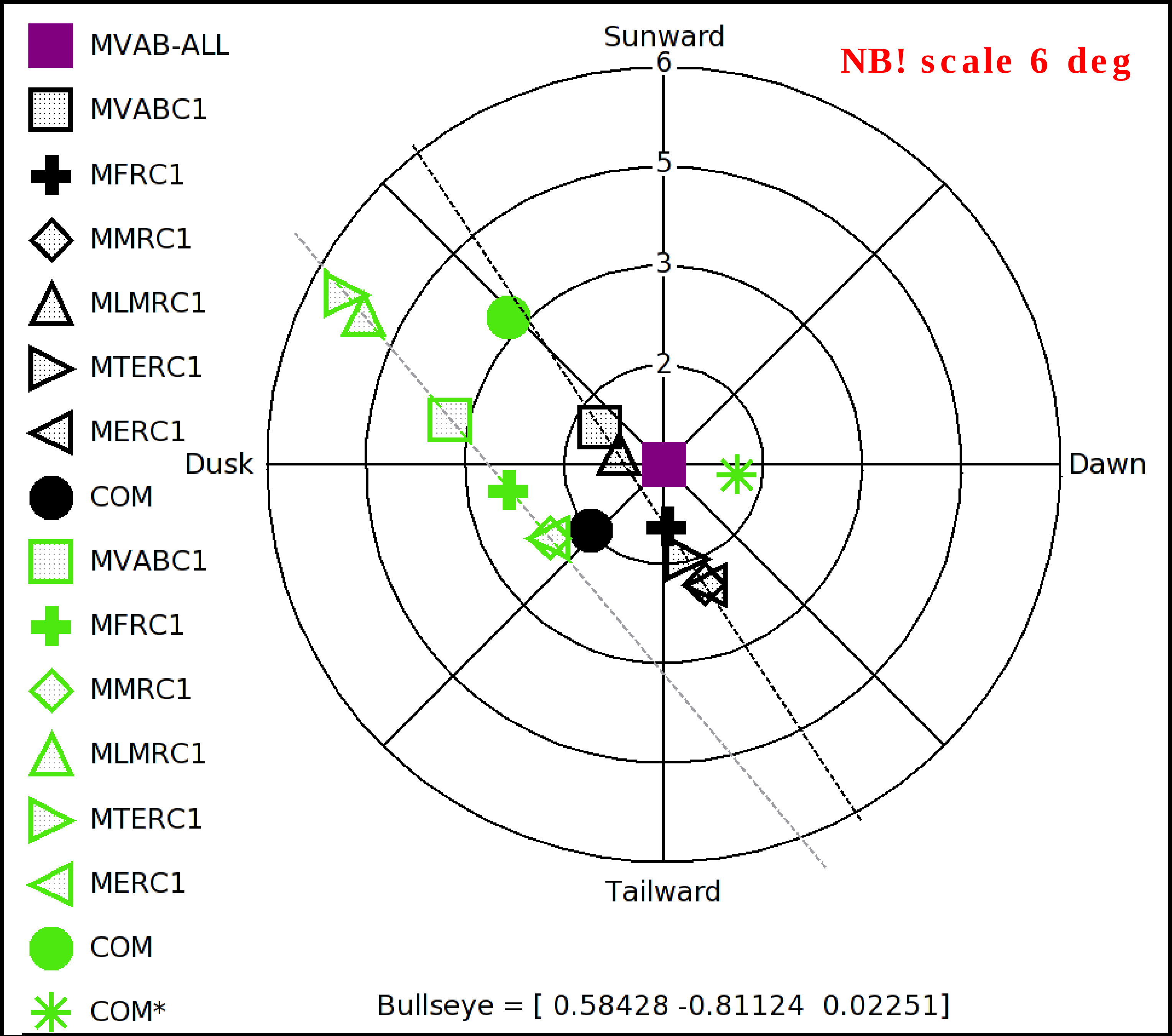
Constrain Q matrix so that

flow across is e.g., Alfvénic

Modified Q matrices - orientation



Summary / Conclusion

Cluster allows for much more precise determination of macroscopic boundary parameters

- Orientation
- Velocity, including acceleration
- Thickness/ dimension

Three basic techniques complement each others

- 4 SC timing (medium, small SC separation)
- 4 SC gradient (small SC separation)
- Single SC methods (any SC separation)

Single SC important for validation !