A TIME ESTIMATE FOR CONSOLIDATION AND DISINTEGRATION OF AN ASTEROID – RUBBLE PILE. THE SIMPLEST MODEL. A PRELIMINARY ANALYSIS.

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Abstract

The existence of asteroids - rubble piles is significant for estimation analysis of asteroidal danger to the Earth, because, as is shown in the present work, in the evolution of such asteroids there is a long period during which the asteroid is a dissipating (or consolidating) cluster of fragments. The vector of evolution is determined by the dynamic characteristics of the asteroid fragments. The effective cross-section of interaction of the Earth with such a cluster is much larger than for the interaction with individual fragments or consolidated asteroids.

The analysis of the simplest model shows that an asteroid – rubble pile evolves, depending on the parameter V^2d (where V is the average velocity of fragments and d is the average distance between fragments), either as a conglomerate of "independent mutually gravitating clusters" (when $V^2d < fm$, where f is the gravitational constant and m is the average mass of a fragment) or as a "receding cluster" (when $V^2d > fm$). In the latter case the recession energy is drawn from the gravitational energy of the cluster. Within the framework of the model considered, the characteristic consolidation time in the first ("elliptical") case is estimated to be within ~ ten million years; in the second ("hyperbolic") case, the doubling time for the average distance between the asteroid fragments lies within the limits of several hundred thousand to several million years. It should be noted that the actual consolidation time in the first case may be considerably smaller due to the presence of diffuse matter increasing kinetic energy loss. In the second case, the presence of diffuse matter will result in accelerated exchange of gravitational and kinetic energies and consequently in accelerated "recession" of the cluster of fragments. Thus the mechanism considered enables an asteroid – rubble pile to survive for a long time, and on the other hand, even without tidal effects, it prevents the transformation of the whole Asteroid belt into a structureless "cloud".

Leikin and Sanovich (2002, 2003) showed that a typical asteroid – rubble pile (ARP) within 10^7 – 10^8 s after its formation loses the fragments whose velocities about its center-of-mass are greater than ~10 m/s.

In this paper, the time of consolidation or disintegration of an ARP due to energy dissipation of its fragments is estimated using the simplest model. The model is not intended to be precise; its aim is to determine the direction of evolution of asteroids – rubble piles and to give a very rough estimate of the temporal scale of their evolution.

We consider an ARP to be a cluster of n identical bodies of radius r distributed at random in volume W with the average distance between the bodies equal to d $(d \approx (W/n)^{1/3})$. The bodies are assumed to be of mass m and density ρ and to have within the cluster velocities V with equal magnitudes and random orientations.

In this model, it is easy to estimate the free path length $\lambda \approx d^3/(4\pi r^2)$ and the time between collisions $\tau = \lambda/V$. Obviously, if the free path length is greater than the characteristic cluster diameter, the model is not applicable, – collisions are negligible, – but in that case an ARP will actually disintegrate approaching a planet for the first time due to tidal dissipation.

In modelling the collision process, the following considerations were used.

A body in a collision is perfectly elastic with respect to the normal velocity and completely inelastic with respect to the tangential (shearing) velocity. If the speed of propagation of longitudinal oscillations in the body is c, the duration of the collision is determined as $\tau_c = 2r/c$ (discharge time). The depth of the deformation zone over this period is $h = V_n \tau_c$, – or $h = (1/3)\cdot(2r/c)\cdot V$, as in the model under consideration we have $V_n = V/3$, – and the radius of the deformation zone is $r_c = 2r(V_n/c)^{1/2}$, as $V_n \ll c$; so the volume of the deformed spherical cap is $W_d = 2\pi r_c h/3$ or $W_d = (2\pi/9)\cdot(2r/c)V\cdot[(2r/c)\cdot(V/3)]^2 = (16/81)\cdot r^3 (V/c)^3$, with the mass $m_d = (16/81)\pi r^3 \rho (V/c)^3$.

In this situation, V_t (the tangential velocity) is equal to $(2/3)V$, and the deformed mass is given energy $m_d \times (2V/3)^2/2$, that is, $(2/9) m_d V^2$, or around $0{,}2\times10^6$ erg/g for $V \ll 10$ m/s, which for the heat capacity of rock equal to $\sim 10^7$ erg/(g·K) corresponds to the increase of the temperature by fractions of a degree and is unlikely to cause destruction of the rock unless the rock is weakened by jointing.

Thus the kinetic energy loss in a single collision is $\Delta K = (16/81)\pi r^3 \rho (V/c)^3 (2/9) V^2 =$

$= [(16\cdot 2\cdot \pi)/(81\cdot 9)]\cdot[(r^3 \rho V^5/c^3)]$. The number of such collisions in a unit of time is $(1/\tau) = [d^3/(4\pi r^2 V)]^{-1}$, hence the derivative of the kinetic energy and the parameters of the model are related by the following differential equation:

$$\frac{dK}{dt} = \frac{32\cdot 4\cdot \pi^2}{81\cdot 9} \cdot \frac{r^5 \cdot \rho \cdot V^6}{c^3 \cdot d^3} \qquad (A)$$

As the sound velocity in a solid is considerably greater than the velocity of the fragments, dK/dt is quite small compared to the kinetic energy of a fragment, which is mainly due to the short duration of the collision.

Because of the slow change of K with time we can use the virial theorem as the 2nd equation relating the velocity V and the distance d between fragments through gravitational forces. Here the virial theorem should be regarded as an osculating approximation. For our model, the simplest form of the virial theorem is $V^2d=2fm$, where f is the gravitational constant.

Taking this relation into account in the case $V^2d>2fm$ (hyperbolic motion) leads to the conclusion that the mutual distance between fragments grows with time. Then the collisional "transparency" of the cluster of fragments also grows, which increases the loss of high-velocity fragments. However, it should be noted that, depending on the spatial distribution of fragments in the cluster, the motion in the outer part of the cluster may become "elliptical", and for some time the cluster may have a rarefied "corona", which will be lost when approaching planets due to tidal effects.

It should be noted, however, that the presence of a diffuse component in the volume occupied by an ARP radically changes the situation. If during some time a fragment sweeps away an amount of matter comparable to its mass, its kinetic energy roughly halves in that time. Note however that the above considerations are only valid as long as the gravitational interaction between the particles doesn't make their orbits "elliptical" with the major axes less than d. In this case the contact interaction will take place within a dense group of gravitating fragments and will recur with the frequency corresponding to the gravitational period. It should be noted that this process, especially in the presence of a diffuse component, facilitates the consolidation of massive objects and may play a significant role in the consolidation of large objects, for example in the Kuiper Belt.

The analysis of the model shows that an asteroid – rubble pile evolves, depending on the parameter V^2d, either as a conglomerate of "independent mutually gravitating clusters" (when $V^2d<fm$) or as a "receding cluster" (when $V^2d>fm$). In the latter case the recession energy is drawn from the gravitational energy of the cluster. Within the framework of the model considered, the characteristic consolidation time in the first case (see Table 1) is estimated to be within ~ ten million years; in the second case, the doubling time for the average distance between the asteroid fragments (see Table 1) lies within the limits of several hundred thousand to several million years. It should be noted that the actual consolidation time in the first case may be considerably smaller due to the presence of diffuse matter increasing kinetic energy loss. In the second case, the presence of diffuse matter will result in accelerated exchange of gravitational and kinetic energies and consequently in accelerated "recession" of the cluster of fragments. Thus the mechanism considered enables an asteroid – rubble pile to survive for a long time, and on the other hand, even without tidal effects, it prevents the transformation of the whole Asteroid belt into a structureless "cloud".

The important point is that the process described makes an ARP's lifetime long enough for the asteroid to be observed in the ARP phase, and at the same time explains the possibility of multiple asteroids such as Ida.

Analysing the asteroid danger problem it must be keep in mind, that the asteroid can be not a single object, but a cluster of objects, thus hindered seriously the solving of the problem.

References

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Table 1. The parameters of the model and the characteristic evolution time of an ARP

m (grams)	ρ (g/cm^3)	*c* (cm/s)	*V* (m/s)	*d* (km)	*T* (years)
"elliptical" case					
$1{,}003\times10^{16}$– –$2{,}836\times10^{19}$	1 – 8	$1{,}0\times10^{5}$– .$5{,}1\times10^{5}$	1 – 10	1,340– – 37,840	~ 10^{7}
"hyperbolic" case					
$3{,}75\times10^{17}$- – $2{,}40\times10^{19}$	1 - 8	$1{,}0\times10^{5}$- – $5{,}1\times10^{5}$	1	20 – 40	~ 10^{5} – 10^{6}

m – the average mass of a fragment, ρ – the average density of a fragment, *c* – the speed of logitudinal wave in the body of the fragment, *V* – the root-mean-square velocity of a fragment. Table 1 also shows *T* – the characteristic doubling time *d* (hyperbolic case) and *T* – the characteristic consolidation time (elliptical case).