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LISA Pathfinder: Einstein's Geodesic Explorer

The Science Case for LISA Pathfinder

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1 EXECUTIVE SUMMARY

LISA Pathfinder is an experiment to demonstrate Einstein's geodesic motion in space more than two orders of magnitude better than any past, present, or planned experiment, except for LISA.

The concept that a particle falling under the influence of gravity alone follows a geodesic in spacetime is at the foundation of General Relativity, our best model of gravitation, yet.

Within General Relativity, gravity is not a force acting on material particles, but instead is identified with curvature in spacetime geometry. Particles, in the absence of forces, travel in the straightest possible way in curved spacetime: this path is called a *geodesic*. In the absence of gravity, spacetime is flat and geodesics are simply straight lines travelled at constant velocity.

All experiments aimed at directly measuring curvature caused by celestial bodies, like gravitational wave observatories, measurements of Post-Newtonian light deflection and time delays, and general relativistic dragging of reference frames, require particles in geodesic motion. In addition, all experiments aimed at probing the limits of General Relativity and the possibility of alternative theories of gravitation search for violations of geodesic motion.

Achieving high purity geodesic motion is made difficult by non-gravitational forces acting on masses, accelerating them away from the geodesic lines. As gravity is by far the weakest of all fundamental interactions, achieving the required extremely low level of non-gravitational acceleration implies the reduction and control of the disturbances produced by a wide range of physical phenomena. For most of these phenomena, this requires ground-breaking achievements within their relevant fields of science, including surface physics, cosmic ray research, and precision interferometry.

LISA Pathfinder's experiment concept is to prove geodesic motion by tracking two test-masses nominally in free-fall through laser interferometry with picometre distance resolution. LISA Pathfinder will show that the relative parasitic acceleration between the masses, at frequencies around 1 mHz, is at least two orders of magnitude smaller than the value demonstrated so far or to be demonstrated by any planned mission.

By using the geodesically moving test-masses as reference for a drag-free control system, also the spurious acceleration of the spacecraft will be suppressed more than three orders of magnitude better than for any other existing or planned mission. In combination with its high stability and low self-gravity design, the LISA Pathfinder spacecraft will then demonstrate the most perfect inertial laboratory available for Fundamental Physics experiments.

LISA Pathfinder will also realize a high precision differential dynamometer of unprecedented resolution, paving the way for a new generation of force experiments, like searches for $1/r^2$ law violations, spin-spin interactions, and spin-mass interactions, aimed at searching for new long-range interactions beyond the Standard Model.

The high resolution test-mass to test-mass tracking demonstrated by LISA Pathfinder is an essential step for enabling a similar tracking of test-masses even when they are located in different spacecraft, at large distances, and in interplanetary space, like in LISA, but also at short distances in low Earth orbit, like in future geodesy missions.

LISA Pathfinder hardware has been designed to be transferred directly to LISA. However, it is obvious that many other possibilities are opened by the results of LISA Pathfinder. LISA Pathfinder is indeed a mission both in General Relativity and in Precision Metrology and will open the ground for an entirely new generation of missions not just in General Relativity, but in Fundamental Physics at large and in Earth Observation.

In conclusion, the science case for LISA Pathfinder is overwhelming and the ground-breaking effect this mission will have on the whole future science programme of ESA cannot be overestimated!

2 INTRODUCTION: GEODESIC MOTION

LISA Pathfinder[1][2] is devoted to a realization of Einstein's geodesic motion more than two orders of magnitude better than any past, present, or planned experiment.

The concept that a particle falling under the influence of gravity alone follows a geodesic[a] in spacetime is at the foundation of General Relativity, our best model for gravity so far.

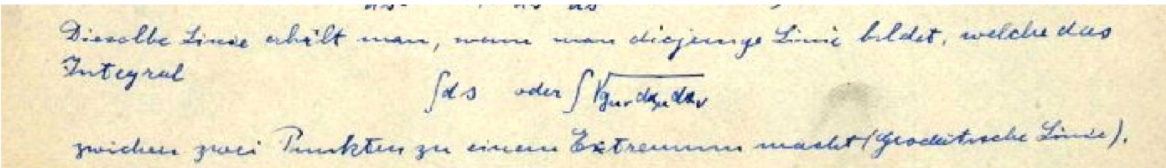

Figure 1: The definition of geodesics in Einstein's "The Meaning of Relativity" (Courtesy: The Hebrew University of Jerusalem)

Gravity influences the motion of free-falling particles by changing the geometry of spacetime and consequently modifying its geodesic lines. More specifically, gravity generates curvature, causing a relative acceleration between nearby geodesics that, in the absence of gravity, would just be straight lines.

This is the reason why almost all of the experiments in General Relativity and Gravitation face the need for using particles that are, to varying accuracy, in geodesic motion.

Certainly all experiments aimed at directly measuring curvature caused by celestial bodies and their motion share this need. Viking[3] and Cassini[3] measurements of Shapiro delay, the proposed high accuracy measurements of Post-Newtonian parameters with LATOR[5] and ASTROD[6] and drag-free satellites at L1[7], the test of frame-dragging with GP-B[8] or Hyper[9], and finally LISA[1], LIGO[10], GEO600[11] VIRGO[12] and all gravitational wave experiments are obvious examples of this. In all these experiments curvature is studied through its effect on some aspect of motion of particles along geodesics.

In addition, experiments investigating the foundations of General Relativity, like all those aimed at a test of the Equivalence Principle (Microscope[13], STEP[14], GG) or in general devoted to the search for new long range interactions (many are under study like GAUGE [18], and in preparation for future windows of opportunity), almost invariably search for violations of geodesic motion.

Many alternative theories of gravity, including those that seem to follow naturally from unifying gravity with the other fundamental interactions via string theory, predict non-geodesic motion at some level of accuracy.

Finally, gravity has now entered into other fields of human investigation. The gravitational field of the Earth or some other celestial body is a powerful tool to reconstruct the mass distribution and motion within the body or on its surface. Earth geodesy space missions (Grace[16],

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[a] In the language of General Relativity (Figure 1) a geodesic in spacetime for a massive particle is the line joining two events (the points of space-time) P_1 and P_2 which has extremal value of

$$\int_{P_1}^{P_2} ds$$

with ds the spacetime line element. In the absence of gravitation, geodesics are straight lines travelled at constant velocity. Also light rays follow geodesics, in this case called null geodesics.

GOCE[17]) measure its gravitational field by relying on geodesic motion of test-bodies in order to perform their measurements.

Forces of all sorts compete with space-time geometry to set particles into motion, perturbing them away from their geodesic lines. As gravity is by far the weakest of all fundamental interactions, achieving the required extremely low level of non-gravitational acceleration implies the mastering of many physical phenomena. For most of these phenomena this requires ground-breaking achievements within their relevant fields of science, including surface physics, cosmic ray research, and precision interferometry. The science of precision metrology has a history of producing breakthroughs in physics by pushing the measurement accuracy beyond existing limits, as witnessed by many Nobel Prizes in Physics.

LISA Pathfinder's experiment concept (see sect. 3) is to demonstrate geodesic motion by 1) tracking, using picometre resolution laser interferometry, two test-masses nominally in free fall, and by 2) showing that their relative parasitic acceleration, at frequencies around 1 mHz, is at least two orders of magnitude smaller than anything demonstrated or planned so far, except obviously for LISA. To implement such a concept, the key elements are the suppression of force disturbances on the test-masses and the use of picometre resolution interferometry (see sect. 4).

The test-masses on LISA Pathfinder will define the best ever Local Lorentz Frame, i.e. a locally flat inertial reference frame in which free-falling particles near each other move with no relative acceleration. The existence of such a frame is a cornerstone assumption in General Relativity. The availability of such a frame will also make the LISA Pathfinder spacecraft the most perfect inertial orbiting laboratory available for Fundamental Physics experiments. Thus, even though LISA Pathfinder is aimed at demonstrating geodesic motion of its test masses, it will also improve drag-free performance, i.e. the lack of acceleration of the spacecraft relative to a local inertial frame, by more than two orders of magnitude compared to any other flight mission.

To reach its goals, LISA Pathfinder will have to achieve many "firsts" simultaneously (see sect. 5 and 6). Its test-masses will be the first large-mass high-purity metal test-bodies flown freely in space at a distance of several mm from their immediate surroundings and with no mechanical contact to them. With its test-mass to test-mass and test-mass to spacecraft interferometric motion readout, it will realize the first laser interferometric tracking of orbiting bodies in space. With its nanometre spacecraft to test-mass control and its picometre test-mass to test-mass control, it will realize the first nano- and sub-nanometre formation flight of bodies in orbit. With its sub nano-g self-gravity suppression at both test-masses locations, it will be the first high-quality orbiting gravitational laboratory for Fundamental Physics missions.

LISA Pathfinder is at the same time a mission in General Relativity and in Precision Metrology and opens new ground (see sect. 7) for an entire new generation of missions in General Relativity, in Fundamental Physics at large, and in Earth Observation.

The front runner of all these possible missions, for its outstanding scientific case and a huge return in Astronomy, Cosmology, Fundamental Physics and Precision Metrology, remains obviously LISA, the first space-borne observatory to open the observational window of low-frequency gravitational radiation.

3 LISA PATHFINDER EXPERIMENT CONCEPT

The LISA Pathfinder experiment concept is based on tracking two test-masses in free fall, i.e. in geodesic motion, (Figure 2), by high precision laser interferometry.

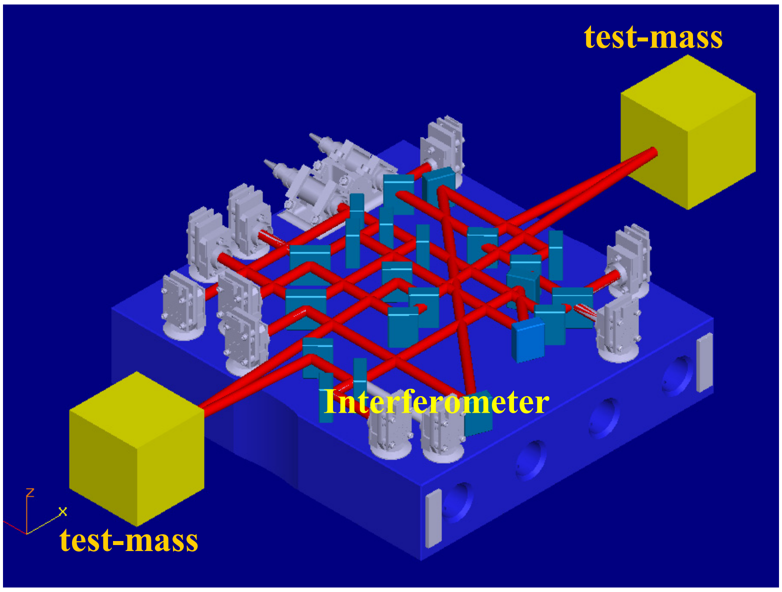


Figure 2: The concept of the LISA Technology Package (LTP) on board of LISA Pathfinder: two test-masses freely float within a single spacecraft with no mechanical contact to their surroundings. A laser interferometer reads out the test-masses' relative displacement with picometre resolution. Test-masses nominally follow two parallel geodesics. The spacecraft follows the test-masses with nanometre resolution to avoid disturbing them away from their geodesics. Violation of geodesic motion manifests itself as a relative acceleration of test-masses as measured by the interferometer.

This is the real-world realization of the basic "Gedanken" experiment that almost all General Relativity text-books use in order to explain the concept of curvature! It is illustrated in Figure 3: two free-falling particles continuously exchanging a light-ray that over time sweeps the spacetime in between the two geodesics.

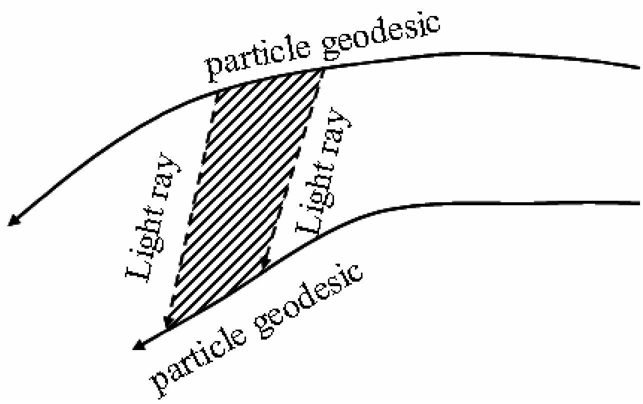


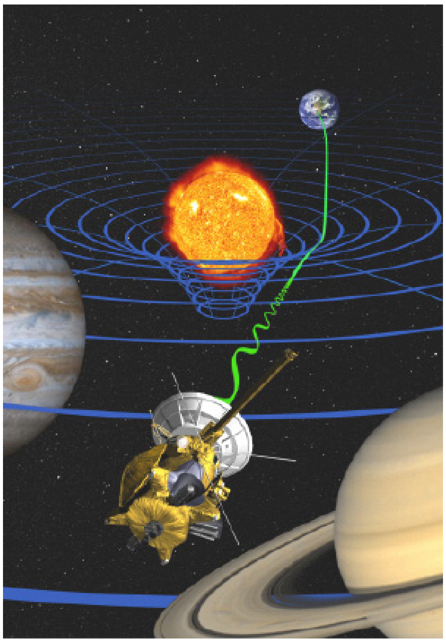
Figure 3: The "curvature diagram"

Curvature, if present over the shadowed area, manifests itself in various ways. For our purpose what is most relevant is that it modifies the frequency of light. For instance, one can calculate that the receiving particle, in the limit of nearby geodesics and low velocities, observes the frequency ω_{light} changing at a rate:

$$\frac{1}{\omega_{light}} \frac{d\omega_{light}}{dt} \approx c\, R^{1}{}_{010} \Delta x^{1},$$

where Δx^1 is the separation between the particles (taken along x^1) and $R^{\mu}{}_{\kappa\lambda\nu}$ is the Riemann curvature tensor, representing the real effect of matter-energy on space-time geometry.

A similar effect of curvature also exists at larger distances. Thus, when Cassini's multi-band radio beam starts grazing the Sun (Figure 4), the frequency of the received wave starts changing with time, due to the relatively high curvature zone crossed by the beam (Figure 4). The same happens when the laser beams exchanged by the LISA spacecraft are crossed by a gravitational wave. The curvature transported by the wave makes the frequency oscillate and the differential frequency shift between the arms reveals the gravitational wave signal.



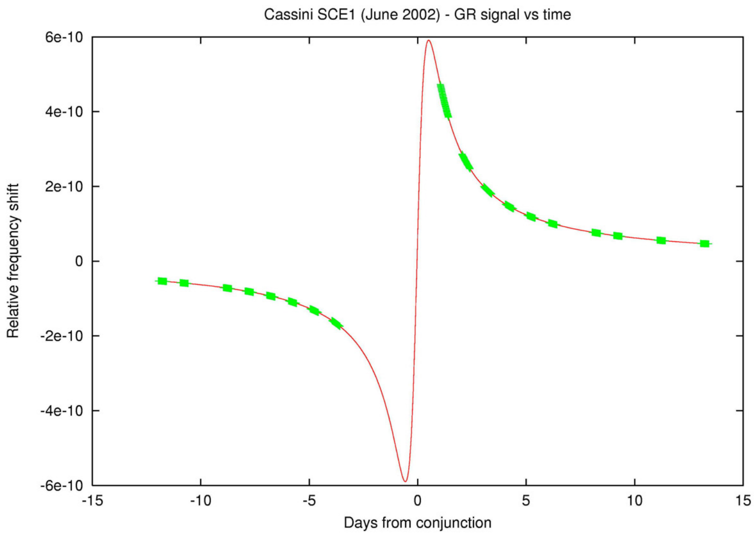


Figure 4: The Cassini General Relativity experiment concept. Left: a pictorial from the Mission web site. Right: schematic of the frequency shift signal as a function of time around conjunction (B. Bertotti et al., *Nature*, **425**, (2003) 374 [4]).

Unfortunately, the frequency also changes for other reasons. Secular variations due to orbital motion Doppler shifts are always present. However, these are usually at frequencies much lower than those at which the curvature signal is observed (days in the example of Cassini and hours to minutes for LISA).

Much worse is the effect of non-gravitational forces that push the particles away from their geodesics. For slow particles these induce a frequency variation with time of

$$\frac{1}{\omega_o}\frac{d\omega}{dt} \approx \frac{1}{c}\left[\left(\frac{\vec{F}}{m}\right)_{em} - \left(\frac{\vec{F}}{m}\right)_{rec}\right]\cdot \hat{n}_{ray}$$

where $\vec{F}$ is the ordinary, non-gravitational force, m is the mass of the particle, "em" stands for the particle emitting the light ray, "rec" for the one receiving it and finally $\hat{n}_{ray}$ is the unit vector in the ray's direction. Thus stray forces, or accelerations, mimic curvature signals.

Obviously this is not the only source of error when measuring curvature. Any instrumental frequency fluctuation directly spoils the measurement by mimicking a relative acceleration. This is the reason that normally null measurements are performed by comparing the change to a reference in an interferometer or taking the difference between two arms.

Thus, in order to measure curvature, two essential ingredients are required:

- free-falling test-mass pairs with very low relative acceleration of non-gravitational origin,

- the ability of tracking these test-masses with light-beams with very small instrumental fluctuations.

LISA Pathfinder aims at demonstrating both of these at an unprecedented level. In particular, LISA Pathfinder will demonstrate immunity from relative accelerations of non-gravitational origin to

$$\Delta a \leq 3\times 10^{-14} \frac{m}{s^2\sqrt{Hz}} \quad (1)$$

at frequencies around 1 mHz and above.

In addition, LISA Pathfinder will demonstrate the ability of tracking free-floating test-masses by laser interferometry with a resolution of

$$\Delta x \leq 10^{-11} \frac{m}{\sqrt{Hz}} \quad (2)$$

down to 3 mHz over a dynamic range of a millimetre.

Here the main challenge is the low frequency at which this performance is required. Achieving low stray accelerations and low displacement noise becomes increasingly difficult at the lowest frequencies. Resolutions of better than 10^{-18}m/√Hz are routinely achieved by the LIGO, Virgo and GEO600 ground-based interferometers[10][11][12], but at frequencies above 100 Hz. However, all effects of interest for space missions are comparatively slow processes as they involve the motion of large bodies. Thus, gravitational waves from super-massive cosmological Black Holes are emitted at frequencies below 1 mHz, conjunctions of spacecraft and the Sun happen on scales of days, and Equivalence Principle and other Fundamental Physics experiments work mostly around a low Earth orbit frequency in the mHz range.

The demonstration of geodesic motion to some level of accuracy requires the comparison of at least two test-masses moving free of non-gravitational forces to that same level of accuracy. Indeed, in order to assess if a test-mass is falling geodesically, one needs to set up a local inertial reference frame and compare the motion of the test-mass to it. The second test-mass of LISA Pathfinder's configuration defines the simplest local inertial reference frame that allows measurement of the motion of the first test-mass, at least along one degree of freedom.

Thus, as an added bonus of the demonstration of geodesic motion to the level of accuracy in Equation 1, LISA Pathfinder will also demonstrate the possibility of defining local inertial frames at the same level of accuracy. This will demonstrate the cornerstone assumption in General Relativity that at any point in spacetime a Local Lorentz Frame can be constructed, i.e. a locally flat inertial reference frame in which free-falling particles near each other move with no relative acceleration.

4 ACHIEVING GEODESIC MOTION

LISA Pathfinder is not the only mission that implicitly or explicitly compares the motion of nominally free-falling bodies. Actually, for many experiments that do not require high-quality free-fall, standard spacecraft in interplanetary orbits achieve a good enough approximation of it. The free-fall quality of the Cassini spacecraft, or even the surface of Mars in the pioneering experiment with Viking, was good enough to allow achievement of their fundamental results. The second body in these experiments is the Earth and the pair falls geodesically at $\approx 10^{-9}$ m s^{-2} (Cassini).

A few other missions plan to use or have used tracking of the relative motion of nearly free-falling artificial bodies:

GRACE is a satellite-to-satellite tracking mission for Earth gravity field mapping. Its test-masses are a pair of accelerometers within satellites in low Earth orbit subject to residual air drag. Tracking by means of a radio link between the spacecraft delivers several μm resolution. The acceleration of each spacecraft relative to its own accelerometer test-mass is then measured to 10^{-9}m s^{-2} to correct the data for it, thus basically realizing test-mass to test-mass tracking.

Both GOCE, again a gravity mission, and Microscope, a test for violations of the Equivalence Principle, measure relative accelerations, inferred from a capacitive position read-out, of test-masses sitting within the same satellite. The spacecraft is in low Earth orbit, but a drag-free loop takes care of the disturbance due to the interaction with the atmosphere. In a drag-free loop the displacement of a test-mass from a fiducial position inside the spacecraft is measured and the spacecraft is forced, by micro- or milli-Newton thrusters, to move and cancel the displacement. Test-mass motion in these missions is detected with high resolution capacitive sensing. However, the price for good capacitive position resolution is the need for large voltages and narrow gaps between sensing electrodes and the test-mass. Also, the severe charging environment of an Earth orbit forces the use of a tiny grounding wire for the test-mass, which invariably causes excess noise.

A comparison of the expected performance of these missions is shown in Figure 5. LISA Pathfinder is clearly a major step forward in defining geodesic motion and local inertial frames.

This big performance leap of LISA Pathfinder is built on several major features:

- it is the first free-fall, drag-free mission in interplanetary orbit,
- it is the first mission to perform picometre laser interferometer tracking of free-falling test-masses,
- it is the first mission to fly a contact-free, kilogramme-size test mass surrounded by gaps of many mm.

It is important to stress that LISA Pathfinder is not a mission mainly aimed at demonstrating drag-free control. Drag-free control is just one of the many tools used to achieve test-mass geodesic motion. The main difference is that *geodesic motion is the lack of relative acceleration between free test-masses other than due to spacetime curvature, while drag-free motion is the lack of acceleration of the spacecraft relative to a local inertial frame.*

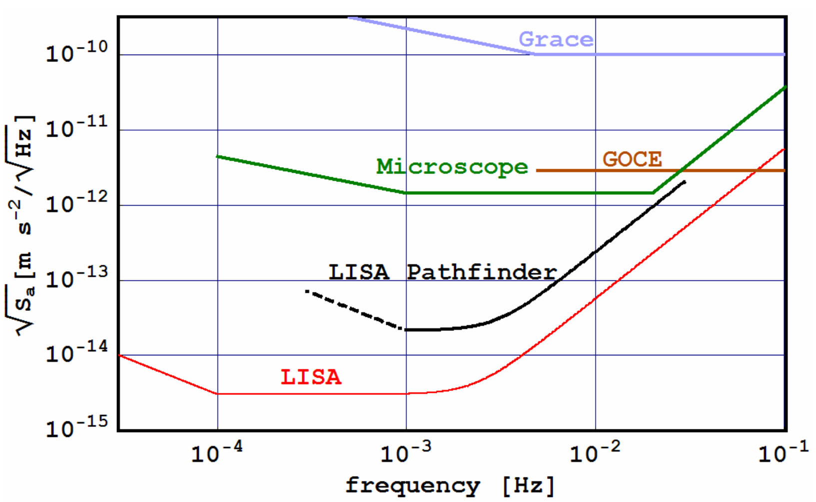


Figure 5: Comparison of required performances of various missions for relative geodesic deviation of test-mass pairs. For LISA Pathfinder the mission requirements are reported as the solid line. The dashed tail is a design goal. Actual projected performance is discussed later.

All that said, however, LISA Pathfinder also improves drag-free performance by more than two orders of magnitude relative to any other flight mission except LISA (Table 1).

In addition, the proof of geodesic motion assesses the level of parasitic force on the test-masses and thus defines the quality of the local inertial frame defined by the test-masses. As LISA Pathfinder carries an independent picometre optical readout that measures the displacement of the spacecraft relative to one of the test-masses, it also allows measurement of the residual acceleration of the spacecraft relative to this local inertial frame with a resolution of better than $10^{-14} ms^{-2}/\sqrt{Hz}$, allowing for further post-processing corrections of effects of the spacecraft on any experiment carried on board.

Table 1. Comparison of main features of missions requiring geodesic motion

Mission	Test-masses	Test-mass environment	Tracking method	Orbit	Test-masses geodesic motion performance ($m\ s^{-2}/\sqrt{Hz}$ at 1 mHz)	Drag-free (Residual spacecraft acceleration) ($m\ s^{-2}/\sqrt{Hz}$ at 1 mHz)
GRACE	Accelerometer test-masses (<100 g)	< 200 μm gaps from electrodes. Mechanical contact via grounding wire	Radio-link plus capacitive sensing	Low Earth Orbit	$\approx 10^{-10}$	No
Microscope	Differential accelerometer test-masses (< 0.5 kg)	≈ 200 μm gaps from electrodes. Mechanical contact via grounding wire	Capacitive sensing relative to S/C	Low Earth Orbit. Drag-free	2×10^{-12} ([a])	3×10^{-10}
GOCE	Accelerometer test-masses (320g)	≈ 300 μm gaps from electrodes. Mechanical contact via grounding wire	Capacitive sensing relative to S/C	Low Earth Orbit. Drag-free	3×10^{-12} ([b])	3×10^{-8}
LISA Pathfinder	Gravity Reference Sensor test-masses (Au-Pt 2 kg)	No mechanical contact. 4 mm gaps	High resolution TM-TM interferometry	Interplanetary (L1), drag-free	3×10^{-14}	3×10^{-13}

[a] D. Hudson, P. Touboul, M. Rodrigues, Proceedings 9th ICATPP Conference, Villa Olmo 2005 (World Scientific 2006)
[b] Single test-mass acceleration specified at $\approx 2\times10^{-12}$ above 5 mHz (Source: GOCE Project)

5 THE SCIENCE OF MEASUREMENT

In LISA Pathfinder, the traditional distinction between spacecraft and payload disappears. The instrument really involves the entire spacecraft. It is fair to state that LISA Pathfinder implements a "formation flight" of three orbiting bodies, namely the spacecraft and the pair of test-masses (see Figure 6 for the configuration concept). This formation flight is implemented using one of several variants of the basic "drag-free" control scheme: One of the test-masses is in pure free-fall in all translational degrees of freedom (x, y, and z in Figure 6) and no force is purposely applied to it. The spacecraft follows this first test-mass within a standard drag-free control scheme in all translational degrees of freedom. The second test-mass is free along y and z and the spacecraft can follow this by using rotation around z and y, respectively.

Unfortunately, the second test-mass cannot be entirely free along x also. Mostly because of the existence of tiny unbalanced static gravitational forces on board the spacecraft (discussed in more detail later on), this test-mass would slowly drift away. These dc-forces are compensated by highly stable electrical forces. Care must be taken, however, to make these forces vanishingly small in the mHz frequency range in which geodesic motion is pursued. Static torques on both test-masses are electrically compensated as well.

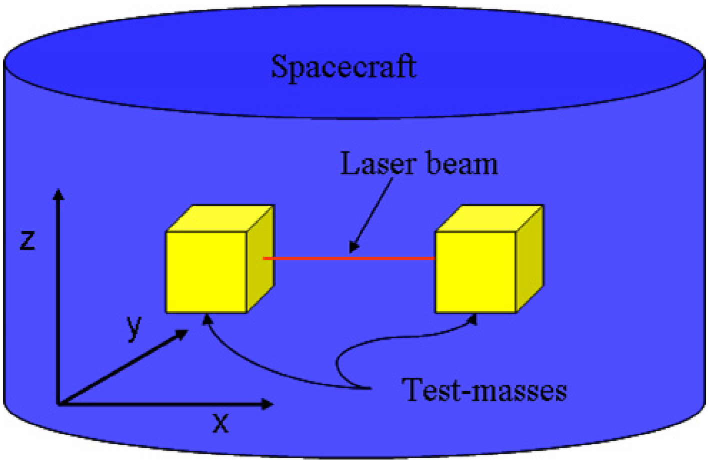


Figure 6: Concept of LISA Pathfinder configuration

The relative displacement of test-masses is measured by a laser interferometer with picometre resolution in order to infer their residual relative acceleration. In addition, the displacement of the spacecraft relative to one of the test-masses is measured with the same resolution in the same direction.

The basic elements to achieve and prove geodesic motion are the following:

- free floating test-masses equipped with motion sensors in all degrees of freedom and free of dynamical disturbances ($< 3\times10^{-14}\ \mathrm{m/s^2\sqrt{Hz}}\ \mathrm{at}\ 1\,\mathrm{mHz}$),
- low-thrust (≈10 µN), low-noise ($\leq 0.1\,\mu N/\sqrt{Hz}$) proportional thrusters to push the spacecraft to follow the test-masses,
- a high-resolution $\left(<10^{-11}\ \mathrm{m/\sqrt{Hz}}\right)$ laser interferometer to measure test-mass relative displacement,
- 18-degree of freedom dynamical control laws,
- gravitationally "flat" (< 5×10^{-11}g) and gravitationally stable $\left(<1\times10^{-14}\ \mathrm{m/s^2\sqrt{Hz}}\right)$ spacecraft to host the test-masses.

Three enabling technologies needed new advances to make this realization possible: low-noise test-mass charge control, a precision test-mass release device, and micro-Newton thrusters.

- First; a non-contacting test-mass in vacuum will be charged by cosmic rays and will never discharge on its own. Charge rapidly produces large forces incompatible with geodesic motion.

A strategy for measuring the charge and for removing it without touching the test-mass is then mandatory. This can be achieved by photo-electron emission controlled by an ultra-violet (UV) light managing system for cosmic-ray ray induced charge ($<10^6$ e).

- Second; injecting a kg-size test-mass into orbit, many mm away from the next solid body, with almost zero momentum and with μm accuracy after having blocked it during launch, turned out to be a subtle problem in mechanics. This is achieved by a test-mass zero-momentum $\left(<10^{-5}\,\mathrm{kg\,m\,s^{-1}}\right)$ release device.
- Third; the innovative potential of μN field emission electrical propulsion is a fascinating engineering problem, in the field of space science. It constitutes an obvious enabling technology for many different future missions.

5.1 Test-masses and the Gravity Reference Sensor

A free-falling test-mass and its immediate surroundings, i.e. the set of electrodes and other accessories that surround it, are usually nicknamed "Gravity Reference Sensor"[19][20][21] (GRS).

The electrodes are used to sense the motion of the test-mass via the change of the intervening capacitance. This information constitutes the driving signal for the control loops that suppress the relative motion of spacecraft and test-mass. Electrodes are also used to apply the very tiny forces required to counteract static gravitation.

A picture of an Engineering Model of the GRS for LISA Pathfinder, is shown in Figure 7.

While the GRS may resemble space-borne electrostatic accelerometers like those developed by ONERA in France, the GRS *is not at all an accelerometer*. Though the instrument configuration is similar, the design drivers are rather different from those of an accelerometer. An accelerometer is trying to accurately measure the acceleration of the test-mass, but a GRS is trying to enable the test-mass to move in undisturbed free fall!

This means that a GRS only needs a motion sensing resolution good enough to keep the test-mass and spacecraft centred. This comparatively low resolution makes it easier to avoid perturbing the test-mass along the sensitive axis away from its geodesic motion.

This is illustrated by comparing the acceleration of the test-mass relative to a local inertial frame to that of the spacecraft when the motion sensor output signal is used to drive the drag-free control loop making the spacecraft follow the test-mass by using its micro-thrusters:

$$\frac{a_{TM}}{a_{S/C}} \approx \frac{\dfrac{f_{TM}}{m_{TM}} + \omega_p^2\left(\dfrac{f_{SC}}{\omega_{df}^2 m_{SC}} + x_n\right)}{\dfrac{f_{TM}}{m_{TM}} + \left(\omega_p^2 - \omega^2\right)\left(\dfrac{f_{SC}}{\omega_{df}^2 m_{SC}} + x_n\right)} \qquad (3)$$

Here "TM" and "SC" indicate test-mass and spacecraft respectively, m is a mass and f is a disturbance force. x_n is the readout noise and $\omega_{df}^2 m_{SC}$ is the gain of the drag-free loop, a frequency dependent quantity here assumed to be much larger than $m_{SC}\omega^2$ with ω the angular frequency. Just for the sake of argument, here all disturbances are treated as "signals" in the frequency domain.

The most relevant parameter is the residual spring-like coupling stiffness constant $m_{TM}\omega_p^2$ that expresses the space derivative of any steady-state force acting on the test-mass, like gravitation or electric field. This number can in principle be made in absolute value much smaller than $m_{TM}\omega^2$ for all frequencies of relevance, thus keeping the effect of f_{SC} and x_n on test-mass acceleration smaller or comparable to that of the intrinsic noisy forces f_{TM} acting directly on the test-mass. This is not true for the spacecraft acceleration, where the same terms are multiplied by ω^2 and quickly dominate the error budget upon increasing frequency.

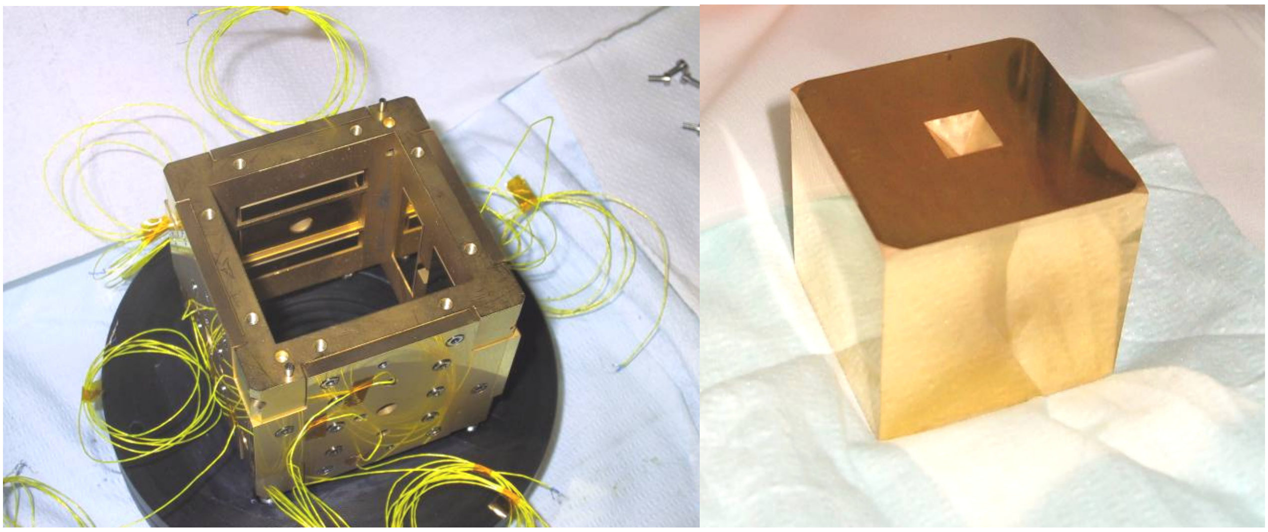


Figure 7: Left: electrode housing and electrodes of the GRS prototype to be flown on LISA Pathfinder. Electrodes are the shiny metal plates inside the housing. Top and bottom covers are not shown. Right: the gold-platinum, 2 kg test-mass that fits into the central volume of the electrode housing. Once integrated with the top and bottom covers the electrode housing completely surrounds the test-mass. In flight the test-mass has no mechanical contact with its surroundings.

Since the GRS constituents are so close to the test-mass, they are the likely source of most of the parasitic forces that disturb the test-mass geodesic motion.

This has the following consequences for the design:

- In the GRS the gaps between the test-masses and the surrounding electrodes are kept as large as possible, compatible with achieving sufficient position sensitivity $\left(\le 2\,\text{nm}/\sqrt{\text{Hz}}\right)$. Large gaps readily suppress disturbances due to uncontrolled potentials and to the possibility of developing pressure gradients across the test-mass. Thus GRS works with 4 mm gaps, compared to the 10-100 μm gaps of accelerometers.
- In the GRS, the voltages used to sense the test-mass motion are kept again as low as possible as voltages create stiffness much more rapidly than they increase sensitivity. Overall the stiffness due to sensing and due to self-gravity are maintained at $\left|\omega_p^2\right| \le 4\times10^{-7}\,\text{s}^{-2}$
- No mechanical contacts of any sort, like for instance the μm size wires used in accelerometers for electrical grounding, are allowed in the GRS as they create both stiffness and, even worse, noise in the form of Brownian forces.
- No dc-voltages are allowed on any test-mass and electrode surfaces as these potentials multiply low frequency voltage noise to produce low frequency fluctuating forces. Thus any electrical forces that need to be exerted by control loops have to be applied via ac voltage carriers, as these forces are proportional to the *square* of the voltage,.

- As for all forces, except the gravitational ones, acceleration is inversely proportional to the mass value of the test-mass, this is chosen to be the largest possible one compatible with the engineering conditions, 2 kg against the 200-300 g of an accelerometer.
- Large mass value with limited geometrical dimensions implies high density and then the use of metals. Achieving a dense non-magnetic ($|\chi|\,\square\,10^{-5}$) metallic test-mass is the reason for the special gold-platinum monophasic alloy, obtained by a rapid quenching technique, used for LISA Pathfinder test-masses.

This radically new design has several consequences:

As no contact is allowed to the test-mass, the charge accumulated on it must be taken away by some other means. This problem has already been encountered for the gyroscopes of GP-B [8]. The method used here is similar to that used by GP-B, namely illumination by UV-light in order to extract photo-electrons from a metal surface. However, GP-B only cares about parasitic forces at a level of $\approx 10^{-9}\ \mathrm{ms}^{-2}/\sqrt{\mathrm{Hz}}$. Thus, in GP-B, once electrons have been extracted from some properly chosen surface, they are steered toward the test-mass or away form it by use of an applied electrical potential. LISA Pathfinder cannot use dc-voltages. Thus charge mastering is obtained by regulating the illumination that hits the test-mass and the one that hits some facing surface. Photo-electron currents can be regulated in this way to keep the test-mass charge neutral.

A much-underestimated consequence of the need for flying a comparatively heavy test-mass in the centre of a wide empty cavity is that this requires a challenging procedure for its release in orbit, almost a new launch campaign within the orbiting spacecraft. This is different from all previous or planned missions (Microscope, Goce, GP-B) that fly lighter masses within micron-size gaps, with 1000 times less kinetic energy, and can then let them shake freely during launch.

The need for blocking the test-mass during launch requires the test-mass to pass from the blocking forces of a few kN, progressively down to forces lower than those that can be mastered by the electrostatic forces provided by the electrodes, i.e. μN at most due to the large gaps. Residual adhesion at metal surfaces provides forces that can be much larger than these values so that finally inertia must be used to break residual adhesion. Addressing this question has required advances in the understanding of the basic mechanism of adhesion in metals and the development of new techniques to measure the transfer of momentum between separating bodies.

5.2 Picometre, low frequency interferometry

LISA Pathfinder will realize the first laser interferometric tracking of orbiting bodies in space. Precision interferometric tracking of suspended bodies is now routinely done on the ground at frequencies >100 Hz, with resolutions better than 10^{-18}m/√Hz. At these high frequencies, sensitivity is mainly limited by photon shot noise. Due to the short arms of ground-based interferometers, resonant optical cavities can be used in the arms to enhance the light power and sensitivity. These high finesse optical resonators can also be used as high-gain discriminators to stabilize the laser frequency using the average arm length as a reference.

Interferometry at mHz frequencies and low power, as required here, is a completely new domain of instrumentation that needed new investigations [23][24][25]. Interferometer sensitivity at these low frequencies is not usually limited by photon shot noise (except for LISA in part of its measurement band), but instead by thermal expansion, drift of opto-mechanics, beam alignment, and electronic noise.

A slow drift of the mechanical mounting of optical components has already faced by the designers of the GP-B mission and to overcome this for their telescope, they pioneered the use of silicate bonding, where natural silica bonds are re-formed at the interface between joining optical devices with the assistance of hydroxide-catalysis. This technology has been perfected for the LISA Pathfinder optical bench and will be directly transferable to any future mission relying on high-precision optical measurements (Figure 8).

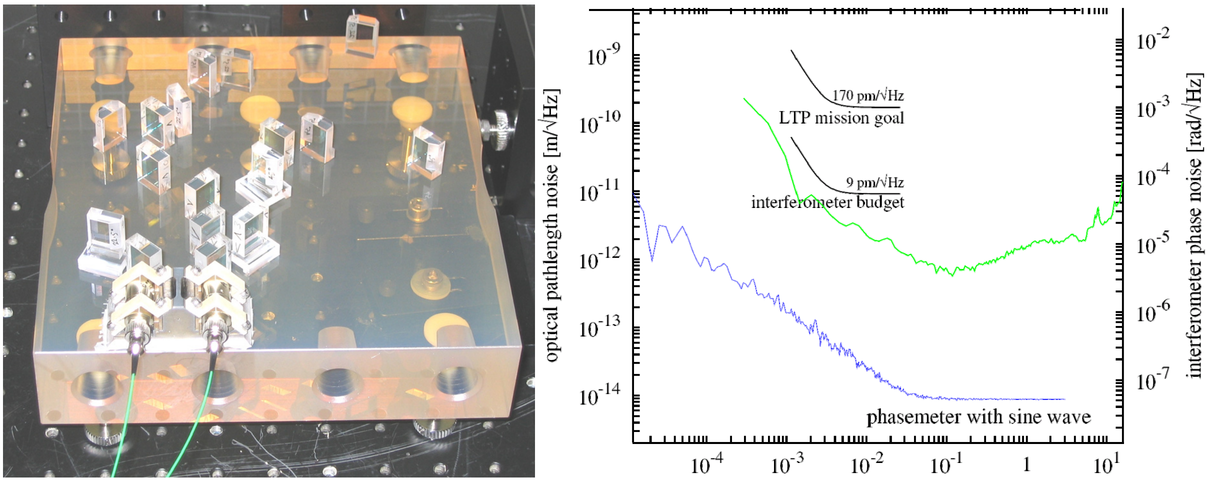


Figure 8: Left: engineering model of the monolithic optical bench of the LISA Pathfinder interferometer. Right: performance of the interferometer engineering model and of its phasemeter electronics during ground testing.

During the interferometer selection process for LISA Pathfinder, many different types of interferometers have been studied and compared. The outcome of the comparison is the present baseline design, a non-polarizing heterodyne Mach-Zehnder interferometer. It measures:

- the distance between both test-masses;
- the position of one test mass with respect to the optical bench;
- the differential alignment of both test masses w.r.t. the optical bench, as it appears in the angular fluctuations of the reflected beams;
- the alignment of one test mass w.r.t. the optical bench.

The laser is a Nd:YAG laser emitting light at 1064 nm, with a power of approximately 25mW arriving at the end of an optical fibre. After verification on LISA Pathfinder, this laser will be available as the standard master laser for many future missions.

The laser is fibre-coupled through the modulation bench onto the optical bench. On the modulation bench, the beam is split into two parts. Each of them is frequency-shifted by an acousto-optical modulator by an amount f_{het} of a few kHz. On the optical bench, the two beams are made to interfere after travelling along different paths. The photocurrent of the interfering beams is measured with photodiodes. These signals contain the heterodyne beat frequency f_{het}. Variations in the differential path-length between the two beams result in a phase shift of that beat frequency. The phase shift is measured electronically by comparison with a similar beat note obtained from an auxiliary interferometer that is not subject to the path-length variations. The optical bench contains four separate interferometers providing, besides the measurement discussed above, also the reference phase for the measurements and the laser frequency fluctuations.

Achieving picometre resolution over a large dynamic range of millimetres was a special challenge for LISA Pathfinder. It required heterodyne interferometry and the development of dedicated electronics that needs to achieve high stability at low frequency. Laboratory testing has demonstrated the performance on the ground (Figure 8). After verification on Pathfinder, it will be available for many future missions requiring internal interferometric metrology, like Darwin, LISA, or TPF.

The LISA Pathfinder interferometer also includes a channel that measures, with the same picometre resolution, the position and two orientation angles of one test-mass relative to the spacecraft. This can be used as an input signal for the drag-free control system to keep the spacecraft quiet and centred relative to the drag-free reference test-mass with picometre resolution. Considering the very low level of parasitic forces of a GRS, this is potentially a key step forward for missions requiring very low parasitic acceleration of the spacecraft.

5.3 LISA Pathfinder control laws: nanometre and picometre formation flight

LISA Pathfinder will realize the first sub-micron formation flight of orbiting bodies:

- the relative residual jitter of the spacecraft relative to both test-masses is constrained by the control laws to be <5 nm/√Hz
- the jitter in the relative separation of test-masses is constrained to ≈150 pm/√Hz at 1 mHz going down to 5 pm/√Hz at 10 mHz

The synthesis of an 18-degree of freedom control law to enable formation flight of three test-bodies also breaks new ground in its own discipline. Indeed this formation flight must be achieved without losing track of the main objective of the mission, i.e. that the test-masses must follow geodesics. Thus, the control loop acting on one of the test-masses to keep it from drifting away from the other one, must exert quite strong forces at dc, but should avoid applying noisy forces in excess of the mission goal within the measurement bandwidth and should not supply any extra coupling of the test-masses to the spacecraft.

In addition, in a mission dedicated to establish highly accurate coordinate frames, control loops must avoid mixing motion and forces along different degrees of freedom. This has required a design synthesis where transfer functions may be "orthogonalised" in flight to refine and optimize coordinate definitions.

5.4 The spacecraft as a gravitational laboratory

Any gravitational field locally generated by the spacecraft will disturb the free geodesic motion of the test-masses in the external curvature of spacetime. This has profound consequences for the spacecraft design, particularly with regard to self-gravity:

- A residual static difference of the self-gravity field between the test-masses needs to be compensated to keep them from drifting apart. Doing this by applying an electric field causes problems because any field fluctuations make it a source of force noise. In addition, the use of electrical fields produces position-dependent forces that couple the test-mass to the spacecraft. In order to suppress the self-gravity effect in LISA Pathfinder, the static difference of self-gravitational fields at the test-mass location is compensated by a proper distribution of masses down to $\approx 5\times10^{-11}$ g.
- A residual field gradient couples the test-mass to the spacecraft. Thus the local gradient at the test-mass position must be flattened, along the x-direction, down to $\approx 2\times10^{-7}$ s^{-2}.
- Finally, thermo-mechanical distortion of the spacecraft induces gravitational noise by slowly moving the mass distribution around the test-masses.

Mastering these three problems implies a new concept of gravitational control on board the spacecraft. This control consists of the following measures:

- Mass distribution on board the spacecraft must be kept under control to unprecedented levels by measuring masses and positions of all elements with cm-like resolution at the spacecraft outer rim and sub-millimetre resolution within the LTP.

Compensation masses machined with ≈5 μm accuracy must be manufactured at the very last possible moment (system CDR) to flatten the field as accurately as one can with the minimum amount of mass. Residual compensation is finally performed once the system has been entirely assembled (Figure 9):

- Time-resolved high-resolution thermo-elastic modelling of the spacecraft coupled with gravitational field calculation tools is also required, an entirely new analysis technique (Figure 10).

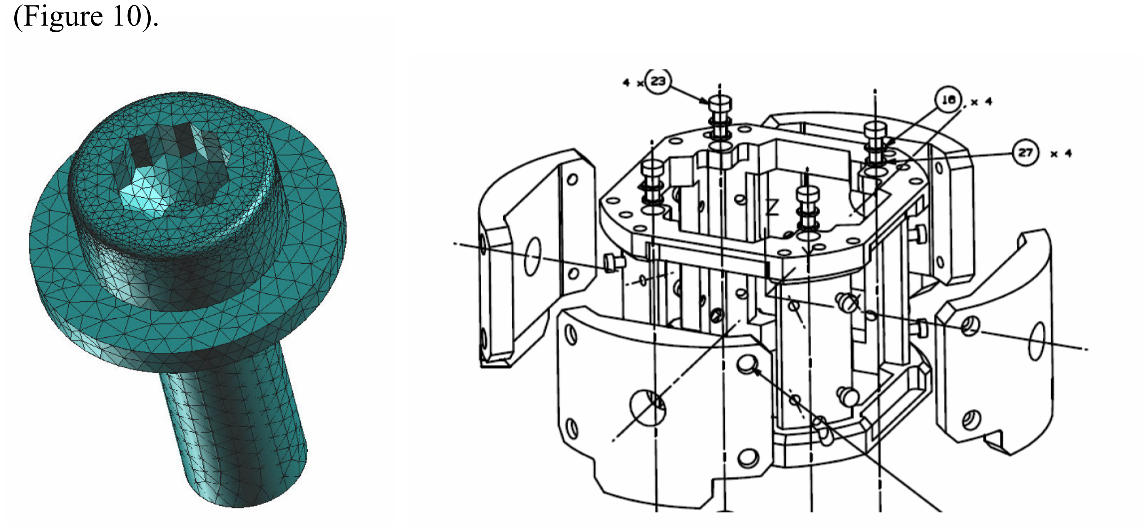


Figure 9: Gravitational compensation. Left: examples of fine meshing on one of the GRS fasteners used for gravitational calculations. Right: exploded view of electrode housing supporting structure and of the gravitational compensation masses, the 4 curved bodies on the outside.

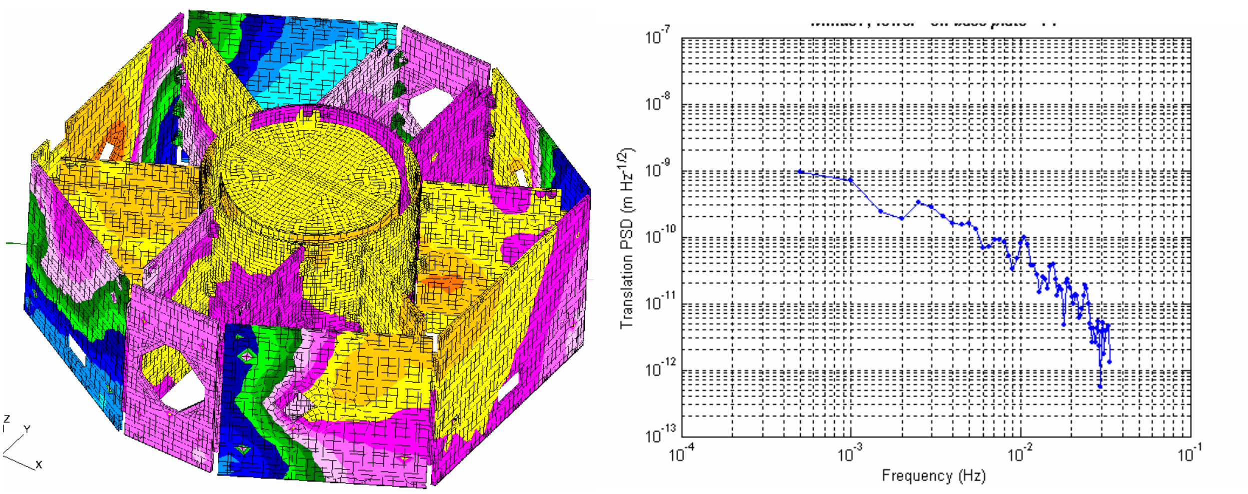


Figure 10: The time-resolved high resolution thermo-elastic modelling. Left: the fine meshing of temperature modelling. Right: the calculated power spectral density displacement of one of the attachment points of the LTP.

6 THE LISA PATHFINDER OBJECTIVE: A PHYSICAL MODEL OF GEODESIC MOTION

The starting point of almost any textbook in General Relativity is the statement that a particle that is not subject to any (non-gravitational) forces will follow a geodesic.

The practical implementation of such an experiment, however, is not at all trivial. The only possible implementation is to identify all known interactions that the particle might encounter and capture them in a physical model that lends itself to an extrapolation to other parameter ranges and environments. This model then needs to be validated by analysis and by dedicated experiments in an environment not too different from the foreseen operating conditions.

LISA Pathfinder will verify one by one the topmost entries in the model and demonstrate the validity and scalability for the residual subtle forces that disturb a nominally free-falling particle.

Identifying the list of physical noise processes is a programme started decades ago with the first experiments in gravitational physics, namely torsion pendulums, and continued with gravitational wave detectors. A list of the expected effects for the test-masses of LISA Pathfinder is represented in the excerpt from the Science Requirements Document [29] in Figure 11.

Discussing the origin and the details of each entry goes beyond the scope of this document. However, it is worth illustrating a few examples of some of the most subtle effects.

6.1 Patch fields

Stray voltage fluctuations are one of the most difficult items in precision metrology. The surface of conductors is nominally at the same constant potential everywhere. However, what needs to be constant is the chemical potential, the sum of the electrical potential and the work function, and not just the electrical potential. The consequence of this is that spatial variations of the work function from point to point entail redistribution of charges to compensate for these variations with electrical potential. Work function variations, even for nominally uniform composition, are due to different crystallographic grains or to oxide or contaminant atomic layers. These charge patches are commonly detected in gravitational experiments on the ground [30] and can create voltages of many tens of mV (Figure 12).

Temporal stability of these charge patches is a critical item for many experiments in Fundamental Physics and a subject of current investigations in Surface Physics. The development effort for LISA Pathfinder is already pushing forward these investigations, producing an assessment of the phenomenon to be finally confirmed in flight.

6.2 Brownian motion

The fluctuation-dissipation theorem states that whenever the motion of a body is subject to damping with coefficient β, the body will be subject to a Brownian random force with spectral density $4k_BT\beta$. In ground-based gravitational experiments, Brownian motion is an intrinsic limiting factor and originates inside the fused silica test masses (mirrors) and the mechanical suspensions isolating the mirrors from micro-seismic noise in a 1g-environment. For a freely falling particle, one would expect damping just from the residual gas surrounding the test-mass. This is kept at a low enough pressure (10^{-5} Pa for LISA Pathfinder) to make the effect negligible.

Source	Allocated noise	Total per group
	$\left(\times 10^{-15}\frac{\mathrm{m}}{\mathrm{s}^2\sqrt{\mathrm{Hz}}}\right)\left[1+\left(\frac{f}{3\mathrm{mHz}}\right)^2\right]$	
Uncorrelated part of readout back-action	0.3	**0.3**
Correlated part of readout back-action	0.02	**0.02**
Stray-voltage fluctuations within MBW	1.2	**1.2**
Noise from actuation	10	**10.0**
Thermal effects		
Radiometer effect	1.5	
Radiation pressure asymmetry	2.5	
Asymmetric outgassing	2.5	
Thermal distortion of sensors	0.01	
Inertial sensor self-gravity noise	0.1	
Total, linear sum		**6.6**
Brownian Noise		
Dielectric losses	1	
Residual gas damping	1	
Magnetic damping	1	
Magnetic losses in impurities	0.1	
Total, quadratic sum		**1.7**
Cross talk		
TM that drives drag-free control		
y	2	
z	2	
ϕ	2.5	
η	1	
θ	1	
Total, quadratic sum		**4.0**
TM subject to suspension control		
y	3	
z	3	
ϕ	2	
η	4	
θ	2	
Total, quadratic sum		**6.5**
Magnetic field fluctuation		
Sources on board the S/C	12	
Interplanetary		
Coupling to TM moment	0.5	
Coupling to charge	0.01	
Total	0.51	
Total, quadratic sum		**12.0**
Random charging	1.5	**1.5**
Laser radiation pressure	0.6	**0.6**
Self gravity Noise	3	**3.0**
Total		**19.04**
Margin (incoherent)[a]		**20.53**
Margin (coherent)[a]		**8.96**
Total		**28.0**

Figure 11: The noise allocation to various sources of force disturbances in the Science Requirements Document [29] of LISA Pathfinder. To these a value of 1×10^{-14}m $s^{-2}/\sqrt{\mathrm{Hz}}$ must be incoherently added for the effect of coupling of test-mass to spacecraft.

However, the studies in connection with the development of LISA Pathfinder [31][32][33] have shown that even non-contacting bodies can dissipate via electrical losses occurring at their surface. These so-called "surface dielectric losses" are a sort of thermodynamic equilibrium counterpart of charge patches and have recently been measured on the ground with the torsion pendulum and found to be compatible with the requirements.

6.3 Radiometer effects and thermal gradients

Thermal gradients may translate into a force on the test-mass in a few different ways. For instance, molecules coming from the hot side carry more momentum and can then exchange higher momentum with the test-mass. This effect, known as the radiometer or thermal transpiration effect is one of the most discussed and studied in dynamical experiments in fundamental physics and is predicted to exert on the test-mass a force proportional, for a given gradient, both to pressure and inverse temperature.

In addition, thermal photons emitted by a hot electrode will have a larger average momentum than those coming from a colder surface. By being reflected by the test-mass, these photons will exert a net force on it independent of any gas pressure.

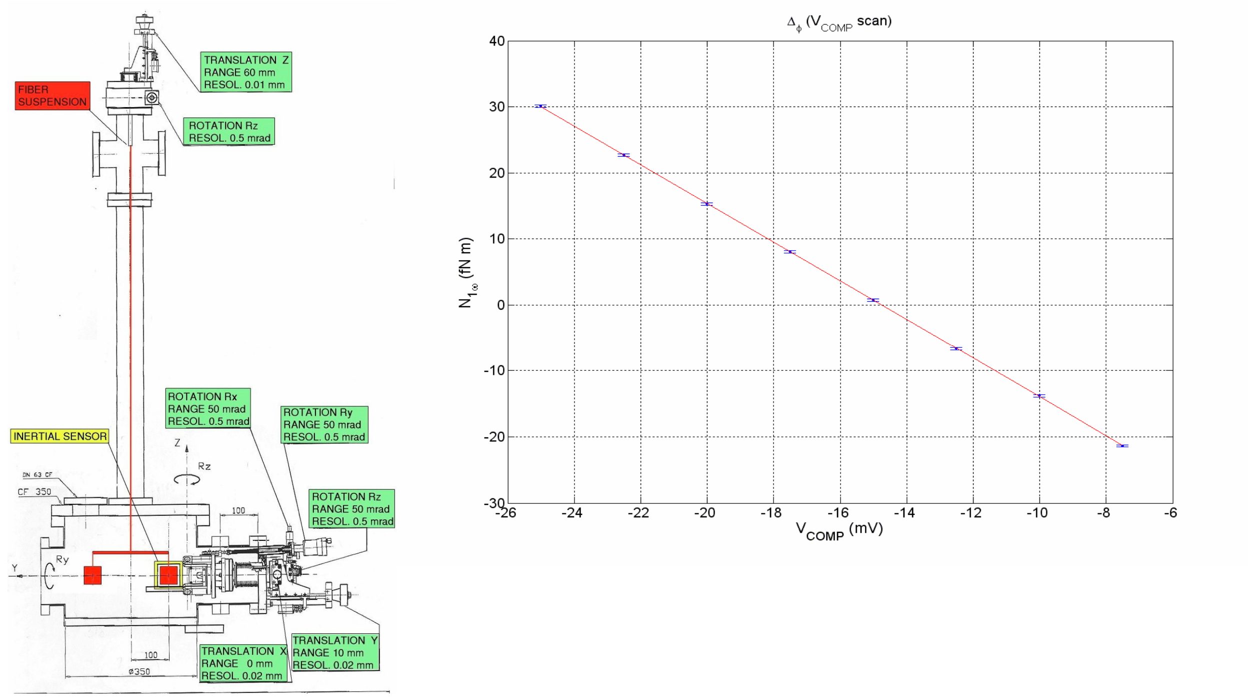


Figure 12: Measurement of "charge patches" with a torsion pendulum on a replica of LISA Pathfinder test-masses. Left: the torsion pendulum. Test-masses hang from a very tiny torsion fibre and move in the horizontal plane under high vacuum and surrounded by a faithful replica of the electrode housing. The device is a very sensitive dynamometer for disturbances at mHz. Right: peak torque on the pendulum in response to an oscillating modulation of the test-mass potential. The torque is due to the interaction of the modulated test-mass potential with the charge patches. By applying some compensating voltage on the electrode, shown on the horizontal axis, the torque can be suppressed. This measures the amount of charge patches around the test-mass

The overall effect of thermal gradients is being actively addressed in connection with LISA Pathfinder development. Extensive studies [34] with torsion pendulums with purposely applied large gradients have confirmed to quite a good extent the prediction of the ideal model (Figure 13). The flight test will verify if the model is correct also for the residual small fluctuating gradients present in the ideal flight conditions.

6.4 Cosmic rays

Mastering and understanding test-mass charging is of paramount importance. The first key scientific issue in this field is the prediction of the test-mass-charging rate from cosmic rays. This is not a straightforward question as most of the charging is predicted to be due to secondary showers caused by protons hitting the spacecraft. These studies have greatly advanced our understanding of the limitation of flying really untouched test-masses in interplanetary space. LISA Pathfinder will give a final confirmation of the accuracy of this understanding.

The prediction of charging rate entails the detailed simulation [35] with the most sophisticated Monte Carlo codes, namely Geant 4, of the dynamics of cosmic rays hitting the spacecraft.

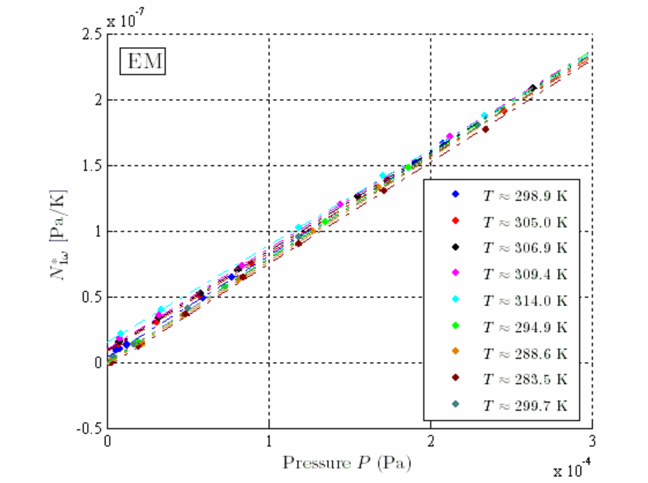


Figure 13: Torsion pendulum results for thermal-gradient-induced forces. Forces normalized for applied temperature gradient as a function of the measured residual gas pressure. The linear pressure dependence, with a slope which decreases with increasing temperature, is as expected for the radiometric effect. The zero-pressure intercept is compatible with negligible outgassing phenomena.

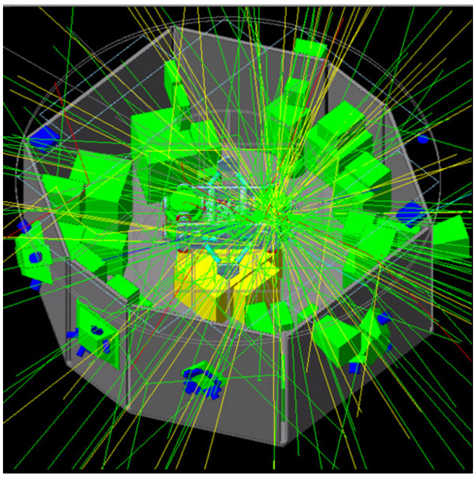


Figure 14: Particle tracing with Geant 4 for the calculation of the charging rate and the charging noise in LISA Pathfinder

6.5 Mission goal: the physical model

The final objective of LISA Pathfinder is to confirm the overall physical model of the forces that act on a test-mass in interplanetary space. To fulfil this programme, the mission is not going to just make a measurement of acceleration, but will implement a full menu of measurements:

- Measurement of acceleration noise between 0.0001 and 1 Hz.
- Measurement of dc-forces.
- Measurement of force gradients.
- Calibration of control-loop transfer functions.
- Characterization of thrust and thrust noise of micro-thrusters.
- Measurement of interferometer performance and interferometer cross-talk.
- Measurement of all cross-talk coefficients among different degrees of freedom.
- Test of continuous charge measurement.

- Test of continuous discharging and of discharging induced noise.
- Test of magnetically induced noise.
- Test of thermally induced acceleration noise.
- Characterization of charging environment.

At the end of this set of measurements, the noise model will be verified down to painstaking detail. The best example of this is constituted by the effect of the magnetic field fluctuations that exert parasitic forces both via the coupling to the test-mass remnant magnetization and to its susceptibility. LISA Pathfinder carries four three-axis magnetometers and two coils to generate magnetic fields at both test-mass locations. By purposely applying a calibration magnetic field, the transfer function from magnetic field to acceleration can be measured. During an acceleration noise measurement, the magnetic field fluctuations can be simultaneously recorded, and their expected contribution can be calculated at any given time by using the already measured transfer function. This projected time series can be subtracted from the data and the residual noise estimated by standard power spectral density analysis. If the procedure is applied correctly, the resulting power spectral density indeed decreases. Figure 15 illustrates the point with a real example taken from torsion pendulum data and with simulated LISA Pathfinder data.

The current estimate of the error budget (Figure 11) predicts the sensitivity to be somewhat better than the requirement at $\approx 2\times10^{-14}\left(\mathrm{m}/\mathrm{s}^2\sqrt{\mathrm{Hz}}\right)$, with the largest contributions being the cross-talk from motion of the test-masses along degrees of freedom other than the measurement ones, the magnetic field fluctuations, and the residual coupling to the spacecraft via the gravitational and electrical field gradients. This last coupling, which will be enhanced to allow measurement of the effect, can also be balanced between the test-masses in order to suppress the influence of the spacecraft coupling on their *relative* motion[a].

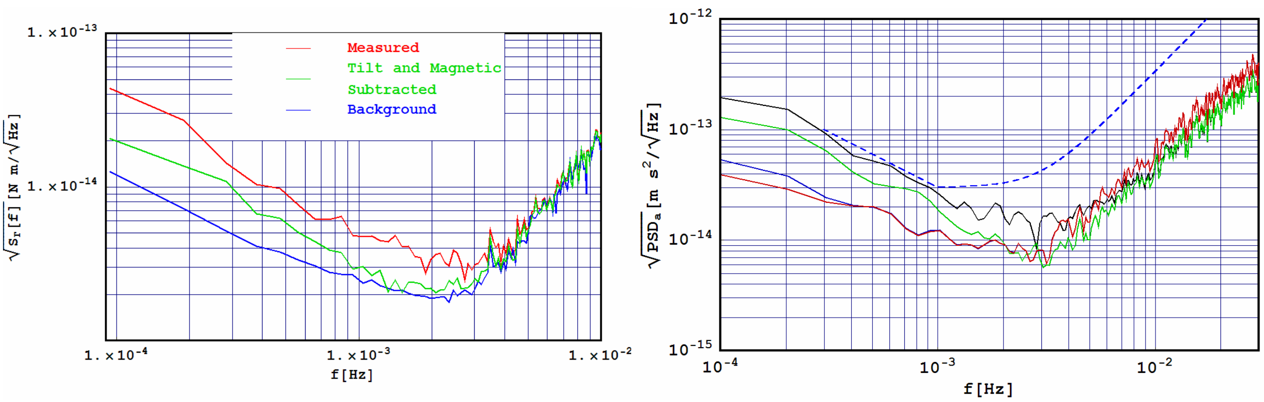


Figure 15: Examples of noise projection. Left: torque power spectral density data from a torsion pendulum. Coupling of pendulum to laboratory ground tilt and magnetic field has been subtracted by measuring the corresponding time series and by multiplying them by the measured transfer functions. The calculated torque time series is subtracted form original torque data before calculating the power spectral density. Right: a simulation for LISA Pathfinder data. Cross-talk (green), magnetic field (blue) and temperature effects (red) have been subtracted from the original data (black) before estimating the reported differential acceleration power spectral density. Notice that pendulum data refer to *torque* while the LISA Pathfinder simulation refers to acceleration. *Vertical scales should not be compared directly*

[a] This is standard practice with gradiometers: matching the stiffness of the coupling of test-masses to the spacecraft, the motion of this one exerts equal forces on both test-masses and thus has no influence on their relative acceleration. Here the matching is only needed at ≈10% level

LISA Pathfinder also carries thermometers so that thermal-gradient-effect correction will be possible, although it probably only entails a small correction.

After the largest entries are suppressed in this way, a comparison with the prediction of the noise model can be performed to assess if any unpredicted non-modelled disturbance is perturbing the test-masses out of their geodesics.

Taking into account the uncertainty in the estimate of the various model entries, LISA Pathfinder may be able to put an upper limit on non-modelled disturbances of *the relative acceleration of test-masses* at:

$$\Delta a_{\text{non-modelled}} \approx 7\times10^{-15}\left(\mathrm{m}/\mathrm{s}^2\sqrt{\mathrm{Hz}}\right) \text{ @ 1mHz} \tag{4}$$

with some slight improvement up to ≈3 mHz.

Above this frequency the laser readout noise is expected to limit the sensitivity, while below 1 mHz noise degradation is expected to show a $1/f^2$ dependence limited by the noise due to the electrical actuation required by the compensation of residual static gravitational field.

7 FINDING THE PATH: FUTURE MISSIONS EXPLOITING LISA PATHFINDER RESULTS

LISA Pathfinder will achieve a series of critical objectives in the field of precision measurements.

- LISA Pathfinder will demonstrate that electromagnetic and locally generated gravitational fields can be suppressed to a level that allows geodesic motion better than $\Delta a_{\text{non-modelled}} \approx 7\times10^{-15}\left(\mathrm{m}/\mathrm{s}^2\sqrt{\mathrm{Hz}}\right)$ @ 1mHz. This is just 1.5 times the level required by LISA to reach its full scientific goals[a].
- It will demonstrate that a spacecraft can be flown with a residual acceleration relative to a local inertial frame, as defined by the drag-free reference mass, of $a_{SC} \approx \sqrt{\Delta a^2/2 + \omega^4 \Delta x}$. Here Δa is the relative acceleration noise of the test-masses which, for the sake of the argument, we divide by two, assuming that stray forces on different test-masses add up incoherently. Δx is the residual jitter of the spacecraft relative to the test-mass, which is set by requirements at 5 nm/√Hz. This formula gives $a_{SC} \approx 2\times10^{-13}\,\mathrm{m\,s^{-2}}/\sqrt{\mathrm{Hz}}$. An inertial platform free of spurious acceleration is essential for many experiments in Fundamental Physics. Platform acceleration is a limiting factor for cold atom clocks and may set the base effective temperature in ultra-cold Bose Einstein condensates. The LISA Pathfinder spacecraft will be the most perfect inertial laboratory ever realized at 1 mHz for Fundamental Physics Experiments.
- Due to the picometre local readout of the test-mass position relative to the spacecraft, the acceleration of the spacecraft relative to the local inertial frame can be *measured* to better than $a_{SC} \approx \sqrt{\Delta a^2/2 + \omega^4 \Delta x_{ifo}} \approx \sqrt{\Delta a^2/2} \approx 5\times10^{-15}\,\mathrm{ms^{-2}}/\sqrt{\mathrm{Hz}}$, with Δx_{ifo} the interferometer displacement noise, and its effect then possibly subtracted from the data of any gravitational experiment on board. Thus the extremely low residual acceleration of the LISA Pathfinder inertial laboratory will be further measured with percent resolution for post-processing correction of its effects.

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[a] LISA requires the *relative acceleration of a pair of its test-masses* to be $<\sqrt{2}\times3\ 10^{-15}\mathrm{ms^{-2}}/\sqrt{\mathrm{Hz}}$

- The LISA Pathfinder measurement scheme, with the separate high-resolution optical readout of test-mass motion relative to the spacecraft, allows test-mass to test-mass tracking with accuracy unspoiled by the spacecraft motion even for test-masses located in different spacecraft. Indeed, as in LISA, one can track one test-mass relative to its hosting spacecraft and then one spacecraft relative to the other one and then reconstruct the test-masses' relative motion by adding up these three measurements. Thus LISA Pathfinder demonstrates the possibility of undertaking high resolution geodesy with test-mass to test-mass tracking.
- As a differential dynamometer, LISA Pathfinder will demonstrate the possibility of measuring oscillating forces at, say, 1 mHz, with amplitude uncertainty on the order of $\Delta a/\sqrt{T} \approx 2\times10^{-17}$ N for an integration time T≈5 days. This is less than 10^{-7} times the oscillating gravitational force due to a kilogramme mass oscillating by 1 mm at a distance of 10 cm from one of the test-masses. This allows for high resolution force experiments.

  Included in this class of experiments are tests of violations of the $1/r^2$ law, searches for violations of the Equivalence Principle, searches for spin-spin and mass-spin interactions, etc. As a comparison, this resolution, if maintained in low Earth orbit, would allow for the detection of a violation of the Equivalence Principle of 1 part in 10^{18}.
- Again as a differential dynamometer, LISA Pathfinder will have a noise energy at 1 mHz of $a_{SC} \approx \omega \Delta a \Delta x_{\substack{\text{interfe-}\\ \text{rometer}}} \approx 8\times10^{-28}\,\text{J}$. This is several orders of magnitude better than any foreseeable ground-based dynamometer at the same frequencies. A low noise energy gives the possibility of resolving comparatively fast phenomena like the momentum transfer of a particle being stopped within the test-mass. In terms of linear momentum resolution, LISA Pathfinder may resolve a momentum of 3×10^{-13} kg m s^{-1}.
- Finally, as a gradiometer, LISA Pathfinder in interplanetary orbit will have a resolution of $\approx 1.5\times10^{-14}\,s^{-2}/\sqrt{Hz} \approx 15\mu E/\sqrt{Hz}$. Obviously this resolution, to be interesting for geodesy, should be maintained in the large Earth gradient of a low Earth orbit. The gravitational compensation installed in LISA Pathfinder is aimed at cancelling a gradient of order $\approx 10^{-6}\ s^{-2}$, which is of the same order as that of the Earth's. Thus the association of LISA Pathfinder gravitational compensation techniques and the very high resolution gradiometer may open the way to a new class of high-resolution geodesy missions.

LISA Pathfinder hardware has been designed to be transferred directly to LISA. It is an easy prediction that, given the outstanding science case, LISA will remain the front runner of the new generation of missions using geodesic motion enabled by LISA Pathfinder results.

However, it is obvious that many other possibilities will be opened up by the ground-breaking results from LISA Pathfinder:

For instance, in the field of mapping the curvature within the Solar System to test the foundations of General Relativity, some of the proposed missions, like those in Refs. [6][7] already plan to use laser tracking of geodesic test-masses. It is interesting to note that both proposals use LISA Pathfinder as the benchmark to demonstrate feasibility [36]. It has also recently been explicitly proposed to re-use the LISA Pathfinder platform and orbit besides its core instrument.

LISA Pathfinder is also the benchmark for missions in Fundamental Physics aimed at going beyond the standard model of fundamental interactions. These missions search for new long range interactions that manifest themselves as additional forces often connected to some special features. Classical examples are the search for extra forces between polarized bodies, in some models associated with the existence of an axion, the search for violations of the $1/r^2$ law of gravitational forces due to (compactified) extra dimensions, and finally the search for apparent violation of the

Equivalence Principle due to dilatonic coupling, often evoked by string theories. It is not by chance then that in a recent proposal LISA Pathfinder is again the proposed heritage for the mission [18].

There have also been proposals [37] to use LISA Pathfinder directly to investigate violations of General Relativity predicted by the most recent formulation of the so-called Modified Newtonian Dynamics theories, where bodies deviate from Newtonian dynamics when their gravitational accelerations are very small. These theories in their initial formulation were very controversial but explained the rotation curves of galaxies surprisingly well. More recent re-formulations have lost most of their non-Newtonian character and basically predict a non-linear behaviour of gravitational potential that might be tested in orbit by LISA Pathfinder. The team is considering this proposal. In general, LISA Pathfinder will put strong constraints on any theory of gravity that modifies General Relativity by introducing acceleration-dependent gravitational couplings.

Cold atoms in space have been proposed for a variety of applications. A good example is Hyper, the study leading the way towards the use of cold-atom-based interferometers for a variety of measurements, including the mapping of the gravito-magnetic frame-dragging field predicted by General Relativity around a spinning body. Hyper and all the concepts derived from it need a very good inertial laboratory for their experiment. Again, it is not by chance that Hyper refers to LISA Pathfinder as the mission paving the way in defining the inertial laboratory [9]. The cold-atom community is currently preparing several proposals to be submitted when ESA makes the first call for Cosmic Vision proposals, and it can be expected that all will rely on LISA Pathfinder to demonstrate the feasibility of a sufficiently good quality inertial laboratory. Indeed, the implementation of many of these proposals may be based on the LISA Pathfinder platform.

All future gravity-field mapping missions will be relying on geodesic motion of free-flying test masses. ESA is already issuing invitations to tender for studies into future geodesy satellite missions with laser interferometric read-out! They may pave the way for truly gravitational imaging of the Earth.

Thus, in conclusion, the science case for LISA Pathfinder is overwhelming and the ground-breaking effect this mission will have on the whole future science programme of ESA cannot be overestimated!

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