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New restrictions on nucleus mass density of some short periodic comets

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Plan of report

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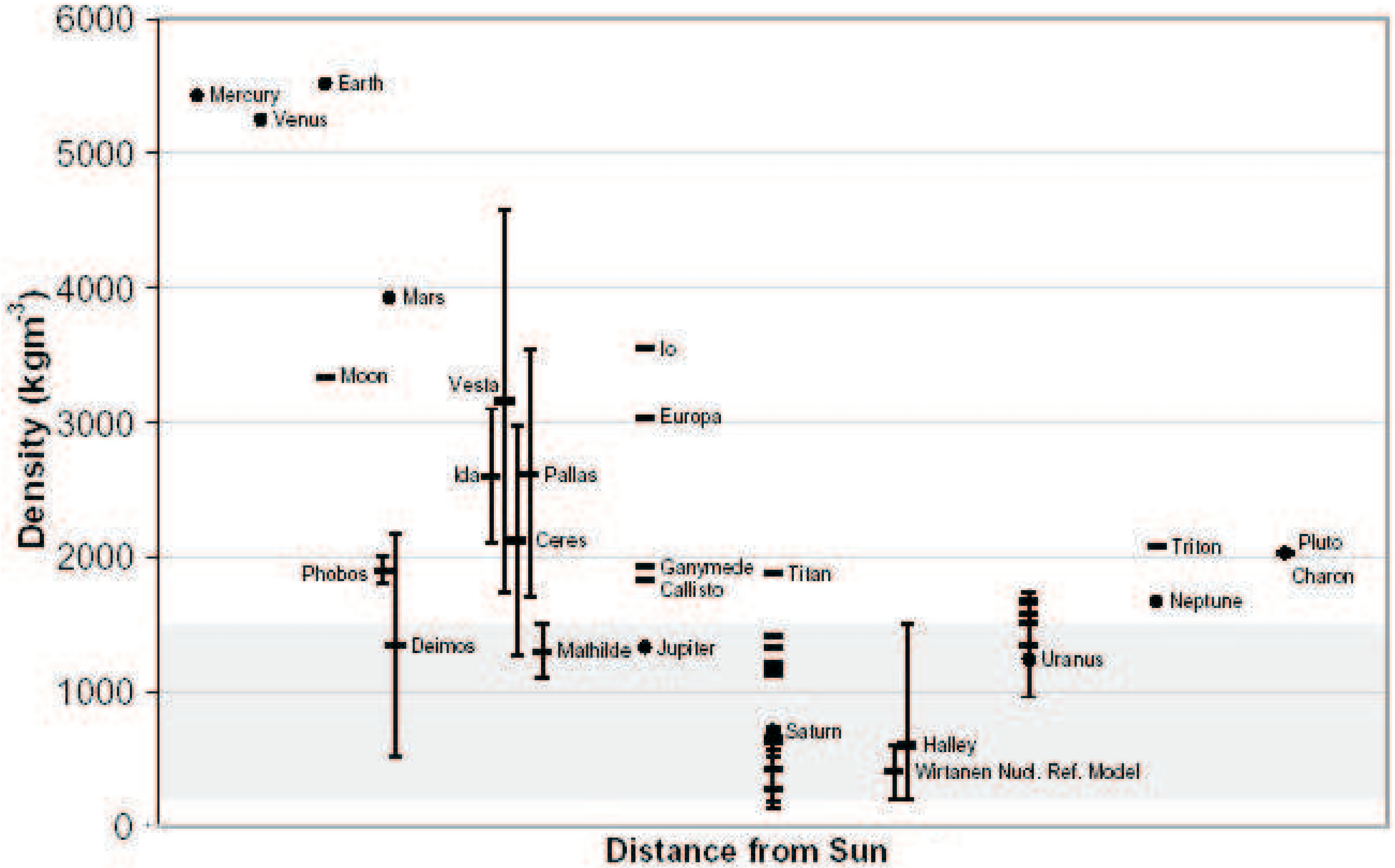
2. Definition of the model
3. New restrictions on nucleus mass density
4. Conclusion

1. Introduction

- √ In present time astronomers have a great interest to studying of physical properties of comets, approaching with Earth. It is necessary to know their mass, structure, mass density, sizes etc. for an estimation of consequences of probable collision such objects with the Earth. It is impossible to develop strategy of the Earth protection from collision with comet without knowledge of these properties.
- √ However comet nuclei are inaccessible for telescopic observations till now as they are veiled by luminous gas and dust environment.

- √ Today one of the main problems in research of the comet nature is a determination of mass density of comet nucleus. There is a set of serious difficulties on a way of definition of the given parameter. An estimation of nucleus mass is rather difficult task owing to the small effects of gravitational interaction of comets and planets. Determination of the nucleus size is also not easy task. The huge distances from a nucleus and dense comet coma interfere to decision of the last problem.
- √ For today it is derived some rather rough estimates of nucleus mass density for some comets characterized by wide intervals of admissible values (from 100 to 1500 kg/m^3)[a].

[a] Sagdeev R.Z., Elyasberg P.E., Moroz V.I. Is the nucleus of Comet Halley a low density body? // Nature, V. **331**, 240, 1988. P. 61.
Boss A.P. Tidal Disruption of Periodic Comet Shoemaker-Levy 9 and a Constraint on Its Mean Density // Icarus, V. **107**, 1994. P. 422-426.


6000
5000
4000
3000
2000
1000
0
Density (kgm^{-3})
Distance from Sun
Mercury
Earth
Venus
Mars
Moon
Vesta
Io
Europa
Ida
Pallas
Ceres
Phobos
Deimos
Mathilde
Ganymede
Callisto
Titan
Jupiter
Saturn
Halley
Wirtanen Nucl. Ref. Model
Uranus
Triton
Neptune
Pluto
Charon

$\surd$ According to the previous talk the main goal of present work is the construction of new algorithm of determination of mass density restrictions for comet nucleus.

$\surd$ The basic tasks of the work are

1. Determination of mean mass density and intervals of its probable values for nucleus of 17 short periodic comets with use of new algorithm.
2. Determination of nucleus mass for 17 short periodic comets with use of effective radius and mass density of comet nucleus.

2. Definition of the model

1. The comet nucleus is represented as a homogeneous sphere with smooth surface with radius R_N, mass density ρ_N, mass M_N, geometric albedo A_G and Bond albedo A_S.

2. Let's simulate the nucleus medium by mixture of 3 components in a solid phase with weight factors ν_i, $i = 1, \ldots, 3$. The shape of real nucleus considerably differs from the sphere, therefore we take into account presence of emptiness (fourth component with weight factor ν_4) for description unsphericity and porous structure of a nucleus.

3. Any small area dS of a nucleus surface can be submitted as superposition of areas dS_i, each of which is covered substance of i-th type with a refraction index n_i, thus $i = 4$ corresponds to a cavity, filled by gas with low concentration (mainly, water pairs).

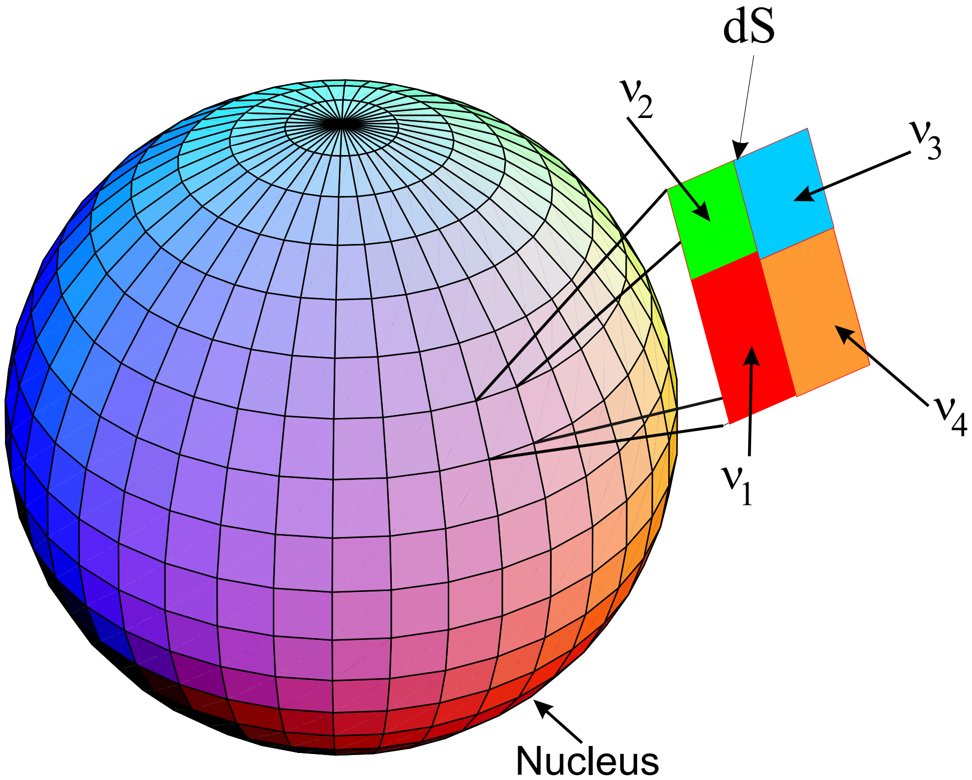


Рис. 1: to definition of point № 3

4. Let's suppose, that the given substances are regular distributed on volume of a nucleus owing to the small gravitational effects. Therefore we assume that the weight ν_i is constant on surface and volume of a sphere.
5. Taking into account a sublimation of substance from nucleus surface and demanding conservation of nucleus shape as a sphere, we guess that coma should contain specified components with the same weight factors ν_i (as a nucleus).

3. New restrictions on nucleus mass density

According to results of spectrometer researches of a comet 1P/Halley, derived with spacecraft GIOTTO, a nucleus consists of the following types of substances:

1) ices (leading part is water ice, $\eta_1 = 0.45$);

2) organic substances (dominating element – carbon, $\eta_2 = 0.27$);

3) inorganic substances (silicates, metals, $\eta_3 = 0.28$).

i	The basic types of substances	Dominating components	n $\lambda = 5 \cdot 10^{-7}$ (м)	ρ, $\times 10^3$ (кг/м^3)
1	ice	H_2O-ice	1.29	0.82
2	organic	C	1.35	1.2
3	inorganic	silicates, metals	1.65	3.2
4	emptiness + gas	H_2O-gas	1.0001	0.0

Table 1. The basic types of substances, making a nucleus, and their characteristics.

According to a point № 3 of the model, any small area dS can be submitted as a superposition of areas dS_i. Each of them is covered by substance of i-th type with a refraction index n_i, i.e.

$$dS = \sum_{i=1}^{4} dS_i, \;\; dS_i = \nu_i dS. \tag{1}$$

Then

$$\sum_{i=1}^{4} \nu_i = 1. \tag{2}$$

According to definition of Bond albedo

$$A_S = \sum_{i=1}^{4} \nu_i A_{S,i}, \text{ where } A_{S,i} = A_S(n_i).$$

Hence, Bond albedo of nucleus surface is the sum of Bond albedos of the spheres consisting only of one component, multiplied on corresponding weight factors. Experimental value of Bond albedo for a nucleus, $A_S^{(exp)}$ is determined from observations. Since we have next equation:

$$\sum_{i=1}^{4} \nu_i A_{S,i} = A_S^{(exp)}. \tag{3}$$

According to a point № 4 of the model, weight factors ν_i on volume and surface of a nucleus are constant. Hence, any small volume of nucleus dV contains the mass of a mix of 4 components, dm:

$$dm = \sum_{i=1}^{4} dm_i = \sum_{i=1}^{4} \rho_i dV_i = \sum_{i=1}^{4} \nu_i \rho_i dV.$$

Otherwise $dm = \rho_N dV$, hence

$$\sum_{i=1}^{4} \nu_i \rho_i = \rho_N. \tag{4}$$

Thus, we have derived the system of 3 equations (2), (3), (4). It is necessary to add this system by one more equation. This equation can be derived from the data of mass-spectrometer researches of nucleus coma, executed with the spacecraft (for example for comet 1P/Halley, investigated with spacecraft GIOTTO). Let's assume, that the estimation of a mass fraction of the first component has been derived:

$$\eta_1 = m_1 \Big/ \left[\sum_{j=1}^{3} m_j\right], \tag{5}$$

where m_1 – mass of the first component. Taking into account a point № 5 of the model, we derive the following equation:

$$\eta_1 = \rho_1 V_1 \Big/ \left[\sum_{j=1}^{3} \rho_j V_j\right] = \rho_1 \nu_1 \Big/ \left[\sum_{j=1}^{3} \rho_j \nu_j\right]. \tag{6}$$

At last we derive the system of 4 linear equations, which can be represented in matrix form:

$$M \cdot R = V, \text{ where} \tag{7}$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ A_{S,1} & A_{S,2} & A_{S,3} & A_{S,4} \\ \rho_1 & \rho_2 & \rho_3 & \rho_4 \\ (1-\eta_1)\rho_1 & -\eta_1\rho_2 & -\eta_1\rho_3 & 0 \end{bmatrix}, \; R = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{bmatrix}, \; V = \begin{bmatrix} 1 \\ A_S^{(exp)} \\ \rho_N \\ 0 \end{bmatrix} \tag{8}$$

The decision of the given system can be showed as

$$R = M^{-1}V. \tag{9}$$

In obvious form

$$
\begin{cases}
\nu_1 = \eta_1 \left[(\rho_2 - \rho_3)(A_S^{(exp)} \rho_4 - A_{S,4}\rho_N) + (\rho_4 - \rho_N)(A_{S,2}\rho_3 - A_{S,3}\rho_2) \right] / D; \\
\nu_2 = \Big[\eta_1 (\rho_1 - \rho_3)(A_{S,4}\rho_N - A_S^{(exp)} \rho_4) + \rho_1 (\rho_3 - \rho_4)(A_{S,4} - A_S^{(exp)}) - \\
\quad -(\rho_4 - \rho_N)(\eta_1 A_{S,1}\rho_3 + (1 - \eta_1) A_{S,3}\rho_1 - A_{S,4}\rho_1) \Big] / D; \\
\nu_3 = \Big[(\rho_1 - \rho_2)(\eta_1 A_S^{(exp)} \rho_4 - A_{S,4}\rho_N) + \rho_1 (\rho_2 - \rho_4)(A_S^{(exp)} - A_{S,4}) + \\
\quad +(\rho_4 - \rho_N)(\eta_1 A_{S,1}\rho_2 + (1 - \eta_1) A_{S,2}\rho_1 - A_{S,4}\rho_1) \Big] / D; \\
\nu_4 = \Big[(\rho_2 - \rho_3)(\eta_1 A_{S,1}\rho_N - A_S^{(exp)} \rho_1) - A_{S,2}(\eta_1 \rho_N (\rho_1 - \rho_3) + \rho_1 (\rho_3 - \rho_N)) + \\
\quad +A_{S,3}(\eta_1 \rho_N (\rho_1 - \rho_2) + \rho_1 (\rho_2 - \rho_N)) \Big] / D; \\
D = (\rho_2 - \rho_3)(\eta_1 A_{S,1}\rho_4 - A_{S,4}\rho_1) - A_{S,2}(\eta_1 \rho_4 (\rho_1 - \rho_3) + \rho_1 (\rho_3 - \rho_4)) + \\
\quad +A_{S,3}(\eta_1 \rho_4 (\rho_1 - \rho_2) + \rho_1 (\rho_2 - \rho_4)).
\end{cases}
\tag{10}
$$

The given results are functions only of one parameter ρ_N:

$$\nu_i = \nu_i(\rho_N), \quad i = 1, \ldots, 4.$$

The interval of allowable values of ρ_N can be determined by system of 4 conditions:

$$\nu_i \geq 0, \quad i = 1, \ldots, 4. \tag{11}$$

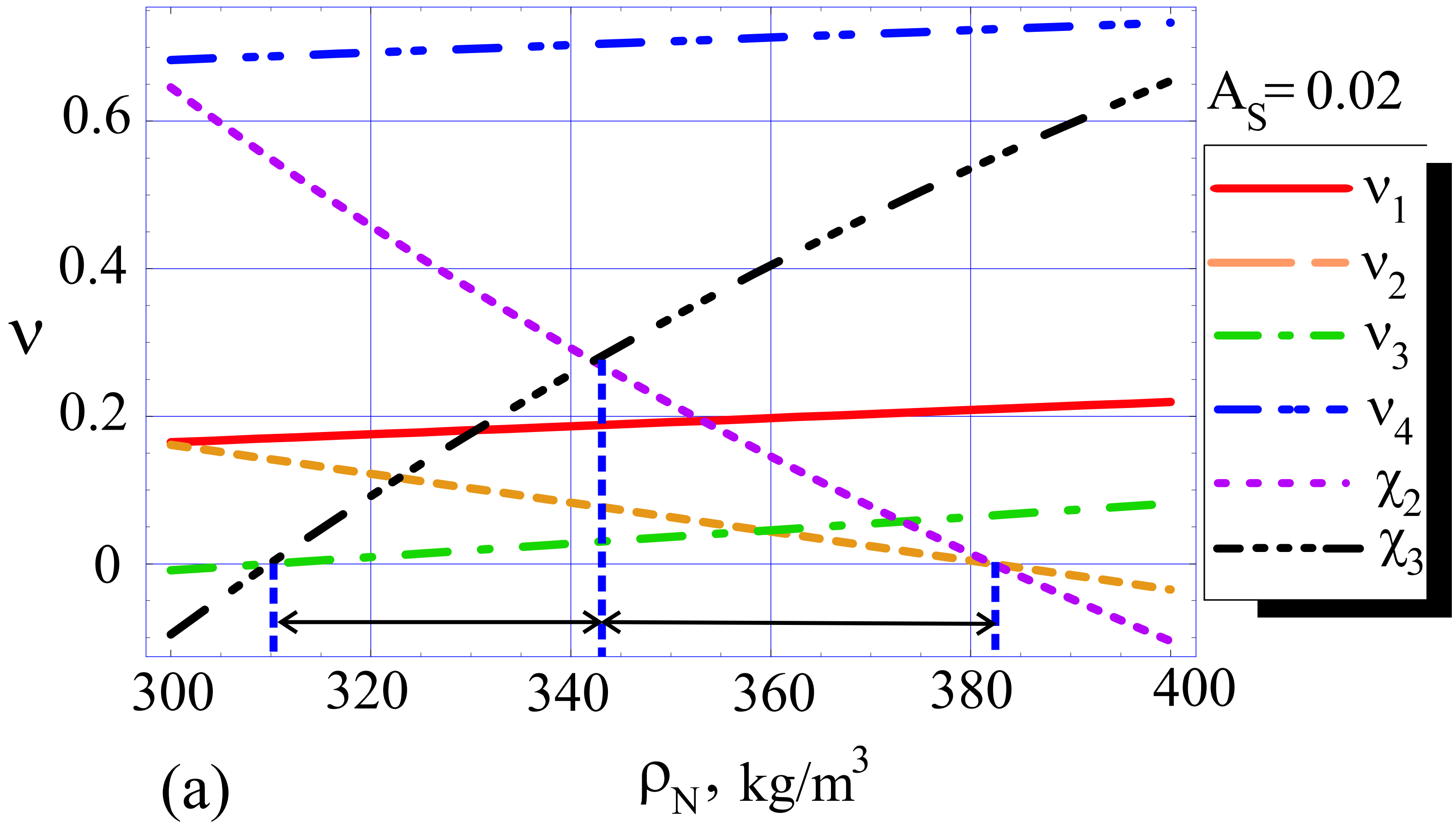
The inequalities (11) determine *the necessary condition* for definition of allowable values interval for ρ_N.

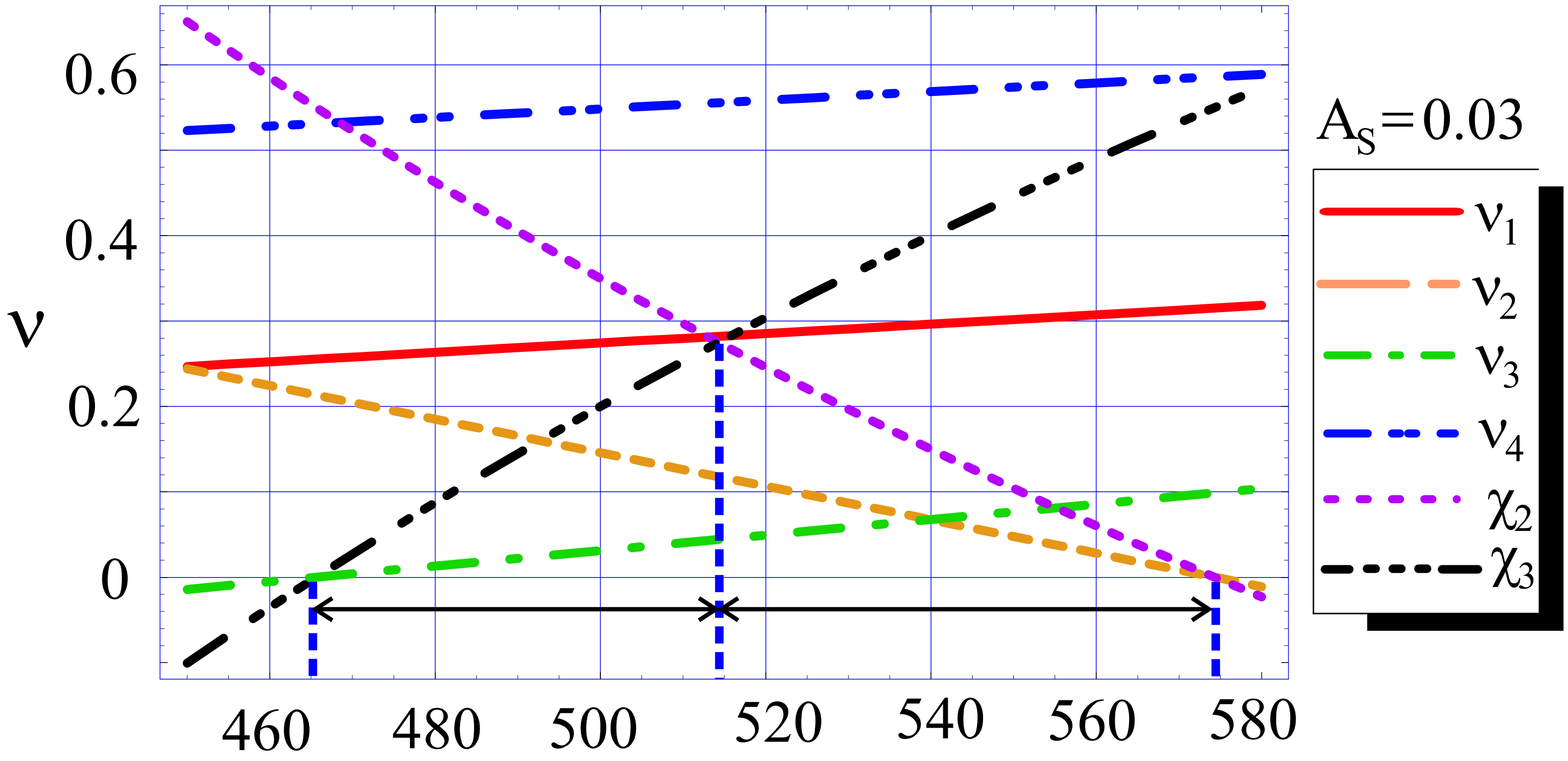
If we know the estimations of mass fractions for second and third components from experiment then, we can demand the performance of the following conditions:

$$\chi_i = \rho_i \nu_i \Bigg/ \left[\sum_{j=1}^{3} \rho_j \nu_j \right] \geq \eta_i, \quad i = 2, 3. \tag{12}$$

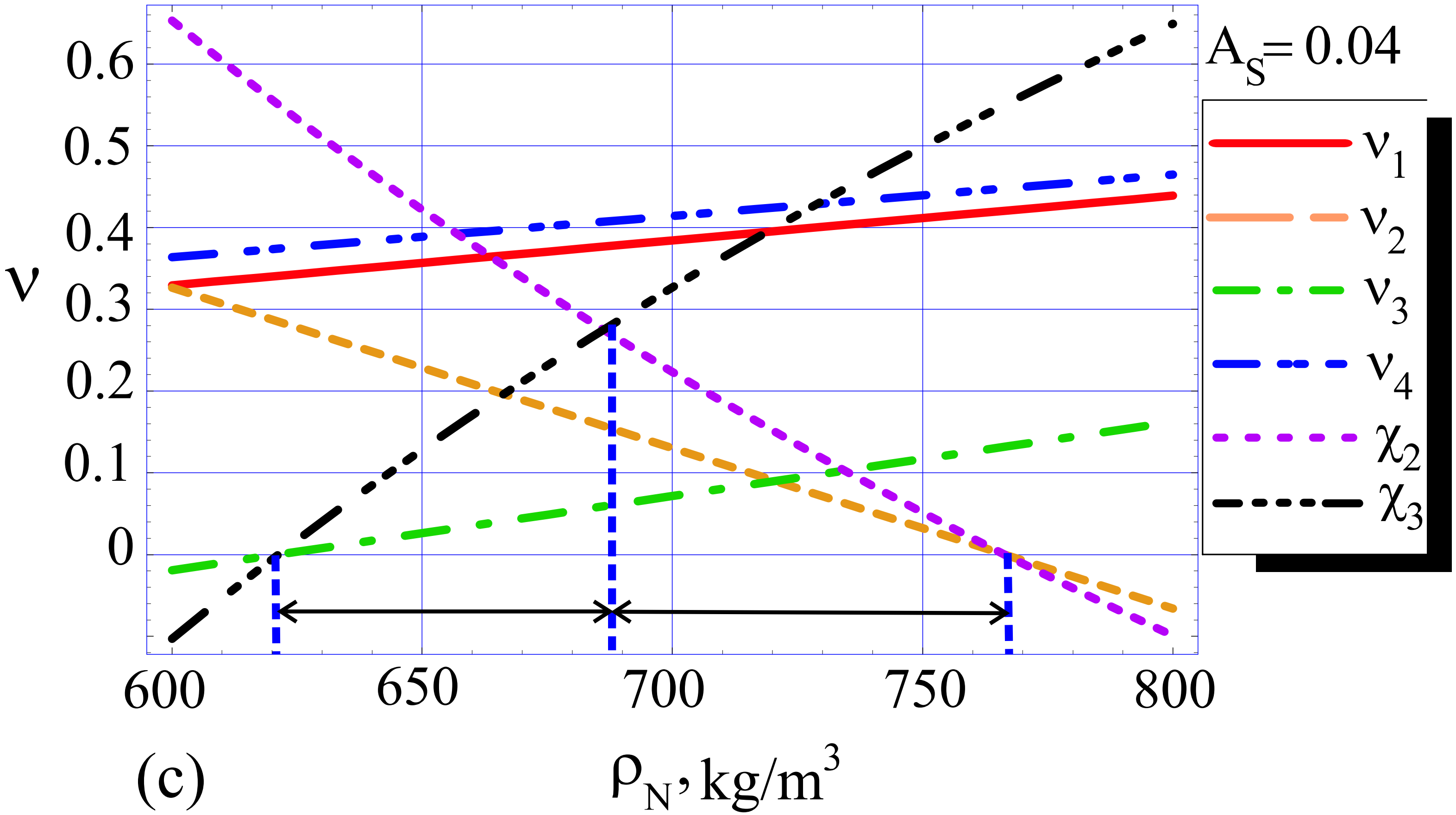
Here we take into account that cavities of nucleus can contain additional sources of the given components, which not sublimate.

The inequalities (12) determine *the sufficient condition* for definition of allowable values interval for ρ_N.


$A_S = 0.02$
ν_1
ν_2
ν_3
ν_4
χ_2
χ_3
0.6
0.4
0.2
0
ν
300
320
340
360
380
400
(a)
ρ_N, kg/m^3


0.6
0.4
0.2
0
ν
460
480
500
520
540
560
580
A_S=0.03
ν_1
ν_2
ν_3
ν_4
χ_2
χ_3

(b) ρ_N, kg/m^3


$A_S = 0.04$
ν
0.6
0.5
0.4
0.3
0.2
0.1
0
600
650
700
750
800
ν_1
ν_2
ν_3
ν_4
χ_2
χ_3
(c)
ρ_N, kg/m^3

At previous fig. the dependences of weight factors ν_i, $i = 1, \dots, 4$ and mass fractions χ_2, χ_3 from mass density of a nucleus ρ_N are submitted for three values A_S and a set of model parameters (they are shown in table 1).

On a basis of the derived results with use necessary (11) and sufficient (12) conditions we determine mean mass density and interval of its allowable values for three values A_S:

$$\rho_N = \left\{ \begin{array}{c} 343 \pm^{39}_{33} \,(\text{kg/m}^3) \text{ for } A_S = 0.02, \\ 515 \pm^{59}_{50} \,(\text{kg/m}^3) \text{ for } A_S = 0.03, \\ 688 \pm^{79}_{66} \,(\text{kg/m}^3) \text{ for } A_S = 0.04. \end{array} \right\} \qquad (13)$$

Using experimental values of Bond albedo for comets nuclei, we determine mean mass density and its interval of allowable values for 17 short periodic comets on a basis of the suggested algorithm.

According to a point № 1 of the model we can calculate nucleus mass M_N with use of effective radius R_N and mass density ρ_N:

$$M_N = \frac{4\pi}{3}\rho_N R_N^3. \qquad (14)$$

We also have calculated effective radius R_N with use of the following expression

$$R_N = a_0 \sqrt{\frac{10^{-0.4(m_{hel} - m_{Sun}^{[red]})}}{A_G}}. \tag{15}$$

We represent corresponding numerical results for 17 short periodic comets in the table 2.

The comet name	A_G	R_N, (km)	ρ_N, (kg/m^3)	M_N, $\times 10^{13}$ (kg)
1P/Halley	0.04	3.4	$688 \pm^{79}_{66}$	11
2P/Encke	0.04	1.7	$688 \pm^{79}_{66}$	1.4
4P/Faye	0.04	1.5	$688 \pm^{79}_{66}$	1
9P/Tempel 1	0.04	2.1	$688 \pm^{79}_{66}$	2.7
10P/Tempel 2	0.021	3.9	$360 \pm^{41}_{35}$	8.9
19P/Borrelly	0.029	2.1	$498 \pm^{57}_{48}$	1.9
22P/Kopff	0.042	1.4	$722 \pm^{83}_{70}$	0.8
28P/Neujmin 1	0.025	9.8	$429 \pm^{49}_{41}$	169

43P/Wolf-Harrington	0.04	1.7	$688\pm^{79}_{66}$	1.4
45P/Honda-Mrkos-Pajdušáková	0.04	0.3	$688\pm^{79}_{66}$	0.007
46P/Wirtanen	0.04	0.5	$688\pm^{79}_{66}$	0.03
49P/Arend-Rigaux	0.028	3.5	$481\pm^{55}_{46}$	8.6
67P/Churyumov-Gerasimenko	0.04	1.7	$688\pm^{79}_{66}$	1.4
73P/Schwassmann-Wachmann 2	0.04	0.8	$688\pm^{79}_{66}$	0.1
81P/Wild 2	0.03	1.8	$515\pm^{59}_{50}$	1.3
129P/Shoemaker-Levy 3	0.04	1.4	$688\pm^{79}_{66}$	0.8
143P/Kowal-Mrkos	0.04	3.7	$688\pm^{79}_{66}$	15

Table 2. Comet characteristics.

Heliocentric magnitudes for the given nuclei were taken from the next work:

Tancredi G., Fernández J.A., Rickman H., Licandro J. Nuclear Magnitudes and the Size Distribution of Jupiter Family Comets // Icarus, V. **182**, Issue **2**, 2006. P. 527-549.

4. Conclusion

- The basic points of the comet nucleus model are formulated for deriving new theoretical results for mass density and mass of nucleus.
- New more strong restrictions on allowable values of nucleus mass density for 17 short periodic comets are derived with use of the new algorithm. This algorithm is based on the assumption of 4-componental nucleus structure. It is shown, that new restrictions essentially depend from nucleus Bond albedo. It is important to note, that new intervals of allowable values for nucleus mass density are essentially less than the intervals derived by predecessors. The derived restrictions on nucleus mass density for 1P/Halley, 81P/Wild 2, 9P/Tempel 1 successfully coincide with the experimental data of space missions.
- Numerical values of mass for 17 comet nucleus are derived with use of results for radius and mass density in approximation of a spherical homogeneous nucleus. The given results are in good agreement with estimations of comet nucleus mass for 1P/Halley, 9P/Tempel 1, 19P/Borrelly, 67P/Churyumov-Gerasimenko.