

Fundamental Cosmology with EUCLID

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**The reconstruction of space-time geometry with
Euclid**

Dark matter and inflation with Euclid

Synergy of Euclid with Plank, LHC, Gaia, EELT

Cosmic degeneracy

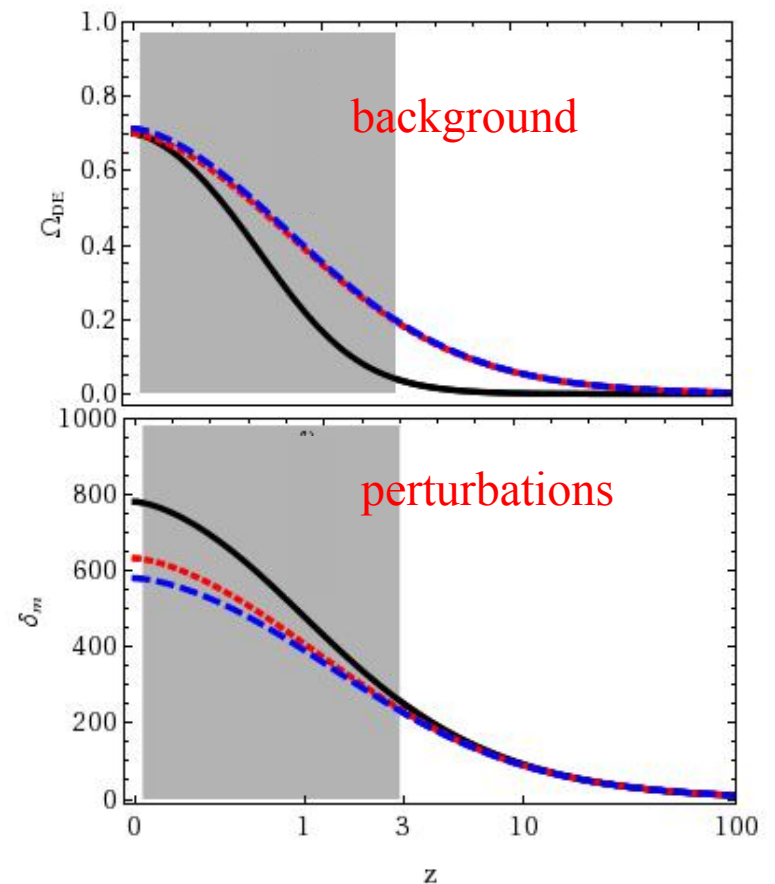
The background expansion
only probes

$$H(z)$$

The matter clustering growth
probes

$$\delta(k, z)$$

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta' - \frac{3}{2}\Omega_m \delta \neq 0$$



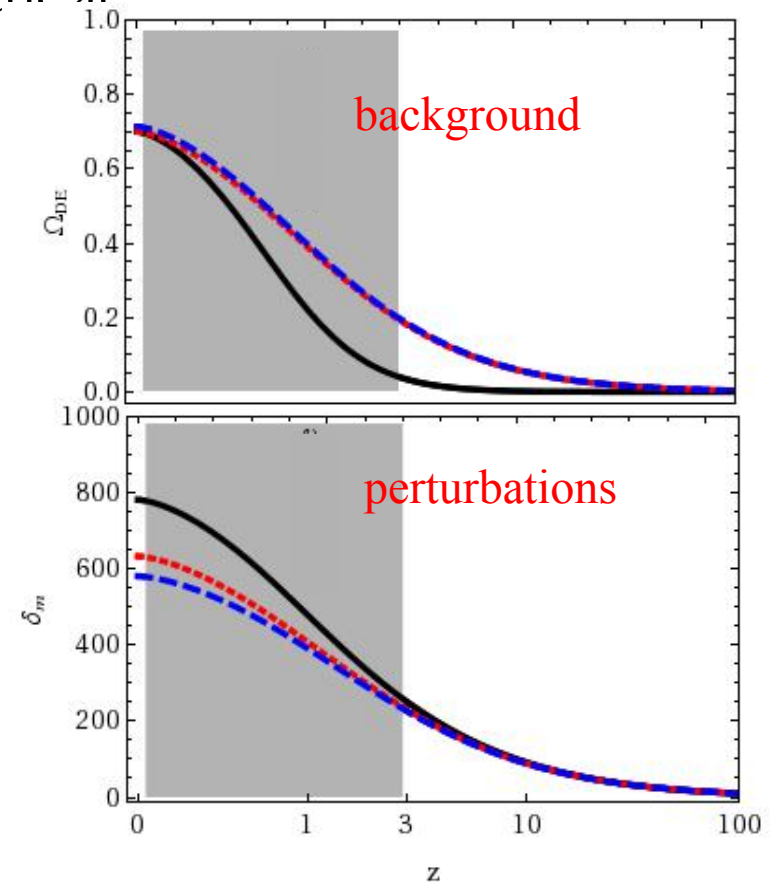
What background hides perturbations reveal

The most general (linear, scalar) metric at first-order

$$ds^2 = a^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)]$$

Full metric reconstruction at first order requires 3 functions

$$H(z) \quad \Phi(k, z) \quad \Psi(k, z)$$



Two free functions

$$ds^2 = a^2 [(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)]$$

At the linear perturbation level and sub-horizon scales

▪ modified Poisson's equation $k^2\Psi = -4\pi Ga^2 Q(k, a)\rho_m\delta_m$

▪ non-zero anisotropic stress $\eta(k, a) = \frac{\Phi + \Psi}{\Psi}$

Modified Gravity at the linear level

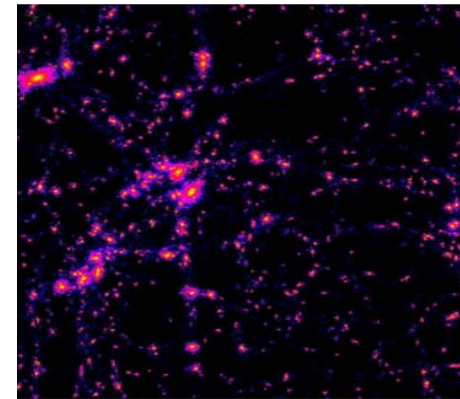
<ul style="list-style-type: none"> standard gravity 	$Q(k, a) = 1$ $\eta(k, a) = 0$	
<ul style="list-style-type: none"> scalar-tensor models 	$Q(a) = \frac{G^*}{FG_{cav,0}} \frac{2(F + F'^2)}{2F + 3F'^2}$ $\eta(a) = \frac{F'^2}{F + F'^2}$	Boisseau et al. 2000 Acquaviva et al. 2004 Schimd et al. 2004 L.A., Kunz & Sapone 2007
<ul style="list-style-type: none"> f(R) 	$Q(a) = \frac{G^*}{FG_{cav,0}} \frac{1 + 4m \frac{k^2}{a^2 R}}{1 + 3m \frac{k^2}{a^2 R}}, \quad \eta(a) = \frac{m \frac{k^2}{a^2 R}}{1 + 2m \frac{k^2}{a^2 R}}$	Bean et al. 2006 Hu et al. 2006 Tsujikawa 2007
<ul style="list-style-type: none"> DGP 	$Q(a) = 1 - \frac{1}{3\beta}; \quad \beta = 1 + 2Hr_c w_{DE}$ $\eta(a) = \frac{2}{3\beta - 1}$	Lue et al. 2004; Koyama et al. 2006
<ul style="list-style-type: none"> coupled Gauss-Bonnet 	$Q(a) = \dots$ $\eta(a) = \dots$	see L. A., C. Charmousis, S. Davis 2006

Reconstruction of the metric

$$ds^2 = a^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)]$$

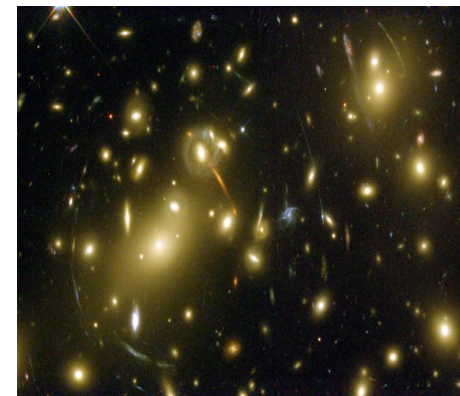
massive particles respond to Ψ

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta = \frac{k^2}{a^2}\Psi$$



massless particles respond to $\Phi - \Psi$

$$\alpha = \int \nabla_{\text{perp}} (\Psi - \Phi) dz$$

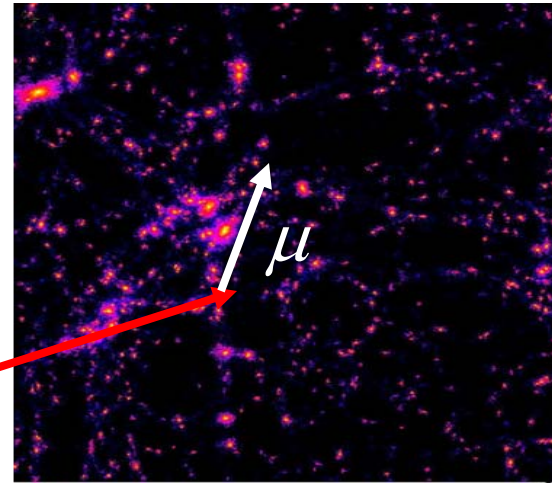


Reconstruction of the metric

Correlation of galaxy positions:
galaxy clustering

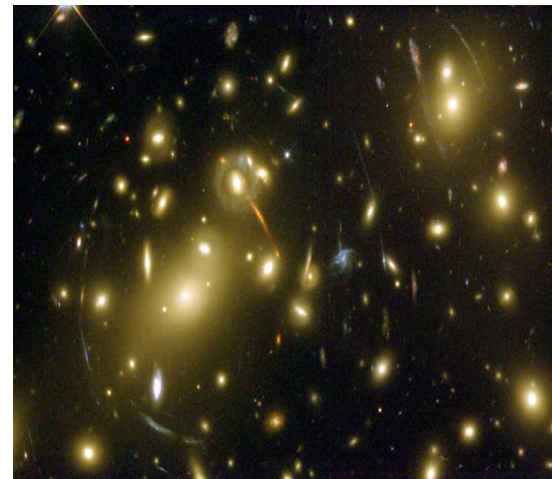
$$P_{gal}(k, z, \mu) = (1 + \beta\mu^2)^2 b^2 \delta^2(k, z)$$

$$\beta \equiv \frac{\delta'}{\delta b}$$



Correlation of galaxy ellipticities:
galaxy weak lensing

$$P_{ellipt}(k, z) \propto (\Phi - \Psi)^2$$



The Euclid theorem



Observables:

$$b\delta = \sqrt{P(k, z, transv)}$$

$$\frac{\delta'}{\delta b} = \sqrt{P(k, z, rad) / P(k, z, transv)} - 1$$

$$P_{ellipt}(k, z) = \int_0^z dz' K(z') (\Phi - \Psi)^2$$

5 unknown variables: $b(k, z), \delta(k, z), \theta(k, z), \Psi(k, z), \Phi(k, z)$

Conservation equations:

$$\delta' = 3\Phi' - \frac{\theta}{Ha}$$

$$\theta' = -\theta + \frac{k^2}{Ha} \Psi$$

We can measure 3 combinations and we have 2 theoretical relations...

Theorem: lensing+galaxy clustering allows to measure all
(total matter) perturbation variables at first order
without assuming any specific gravity theory

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(total matter) perturbation variables at first order
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An example: DGP

(Dvali, Gabadadze, Porrati 2000)


$$S = \int d^5x \sqrt{-g^{(5)}} R^{(5)} + L \int d^4x \sqrt{-g} R$$

$$H^2 - \frac{H}{L} = \frac{8\pi G}{3} \rho$$

L = crossover scale:

$$r \ll L \Rightarrow V \propto \frac{1}{r}$$
$$r \gg L \Rightarrow V \propto \frac{1}{r^2}$$

brane

5D Minkowski
bulk:

infinite volume
extra dimension



gravity
leakage

- 5D gravity dominates at low energy/late times/large scales
- 4D gravity recovered at high energy/early times/small scales

Modified Gravity predictions

$$\delta_k'' + \left(1 + \frac{H'}{H}\right) \delta_k' - \frac{3}{2} Q(k, a) \Omega_m \delta_k = 0$$

$$Q \approx 1 - \frac{1 - \Omega_m}{3}$$

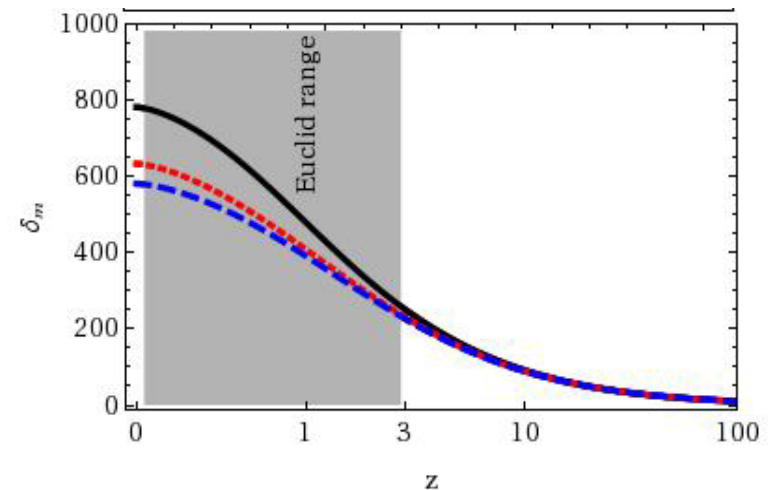
$$\frac{d \log \delta}{d \log a} = \Omega_m (a)^\gamma$$

$$\gamma_s = \frac{3(1-w)}{6w-5} \approx 0.55$$

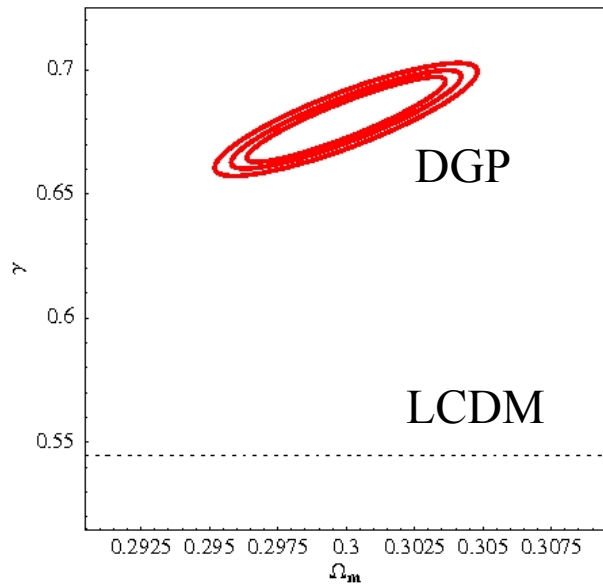
DGP $\gamma \approx \gamma_s \left(1 + \frac{1-Q}{(1-w)(1-\Omega_m)}\right) \approx 0.65 - 0.70$

f(R) $\gamma \approx \gamma_s \left(1 + \frac{k^2 a^2}{M(f)^2 + k^2 a^2}\right) \approx 0.40 - 0.55$

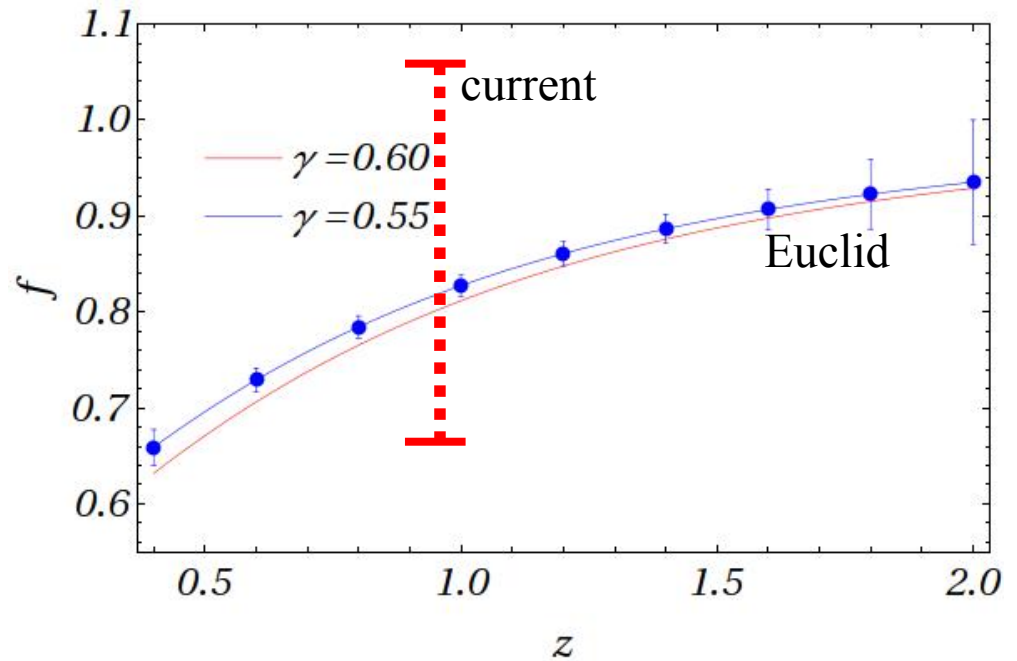
clustered DE $\gamma \approx \gamma_s \left[1 + \frac{(1+w)a^{-3w}}{(1-w) \left(1 - 3w + \frac{k^2 c_s^2 a^2}{H_0^2 \Omega_m}\right)} \right]$



Euclid vs. gamma




tomographic weak lensing



galaxy power spectrum
(redshift distortions)

ancient questions for Euclid



**what is the sky
made of?**

**where do we
come from?**

Euclid vs. dark matter

**dark matter halos from weak lensing maps
of clusters**

**dark matter abundance from power spectrum
shape**

dark matter clustering from cluster abundance

neutrino mass from power spectrum shape

Euclid cosmological bounty

Neutrino mass error $m_\nu \approx 0.03 \text{ eV}$
 error $N_\nu \approx 0.3$

Inflationary spectrum error $n_s \approx 0.005$
 error $\alpha \approx 0.05$

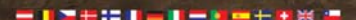
Primordial non gaussianity error $f_{NL} < 5$

Euro-Synergy

Planck 2009
LHC 2009
Gaia 2011
EELT 2020



Very Large Telescope



LHC (beyond the Higgs)

Standard model is complete but.....

no unification of forces and particles

no consistent inclusion of gravity

Answer: supersymmetry

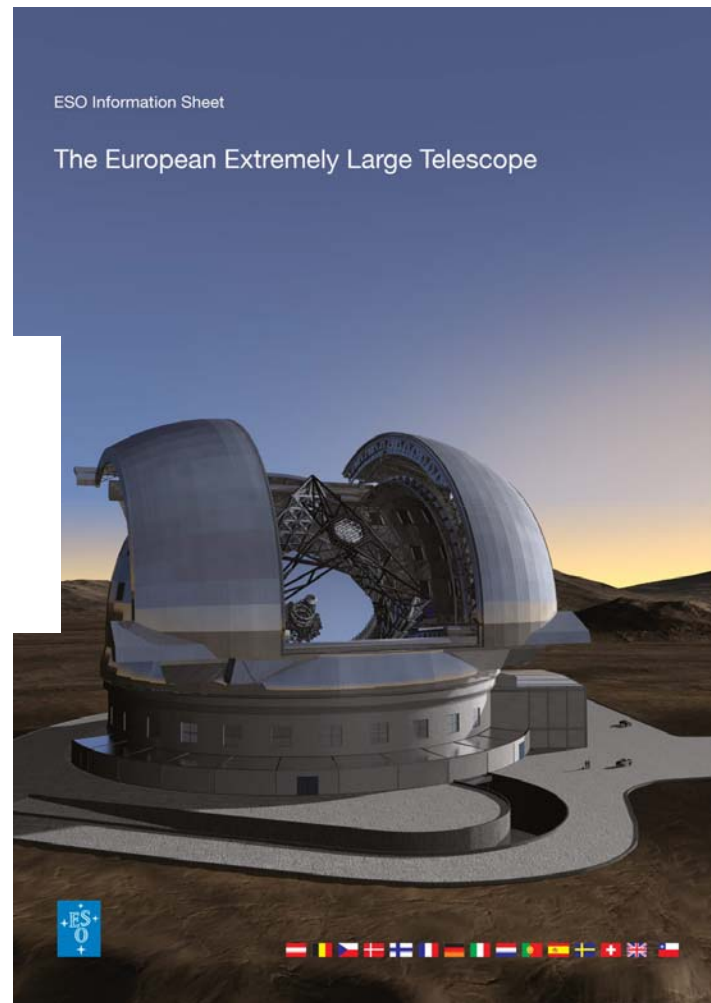
Answer: string theory

Consequence for cosmology:
the lightest
supersymmetric particle is a
perfect cold dark matter candidate

Consequence for cosmology:
the gravity leakage
into the extra-dimensions required by
strings could explain acceleration

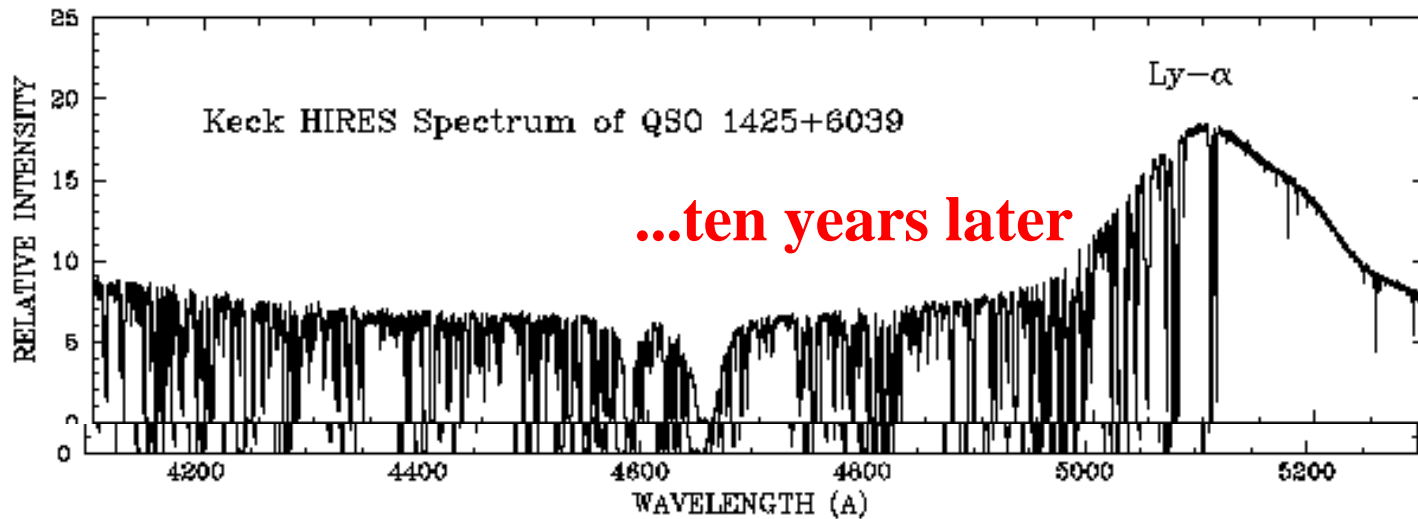
EELT

THE EXTREMELY LARGE TELESCOPE
IS THE ESSENTIAL NEXT STEP IN MANKIND'S DIRECT
OBSERVATION OF THE NATURE OF THE UNIVERSE.
IT WILL PROVIDE THE DESCRIPTION OF REALITY WHICH WILL UNDERLIE
OUR DEVELOPING UNDERSTANDING OF ITS NATURE.



ESTEC 2009

EELT



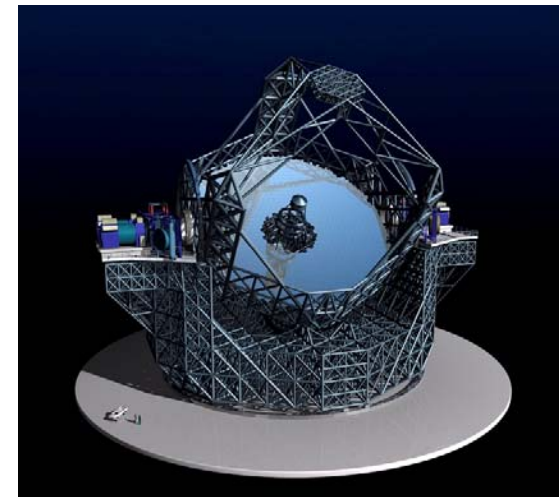
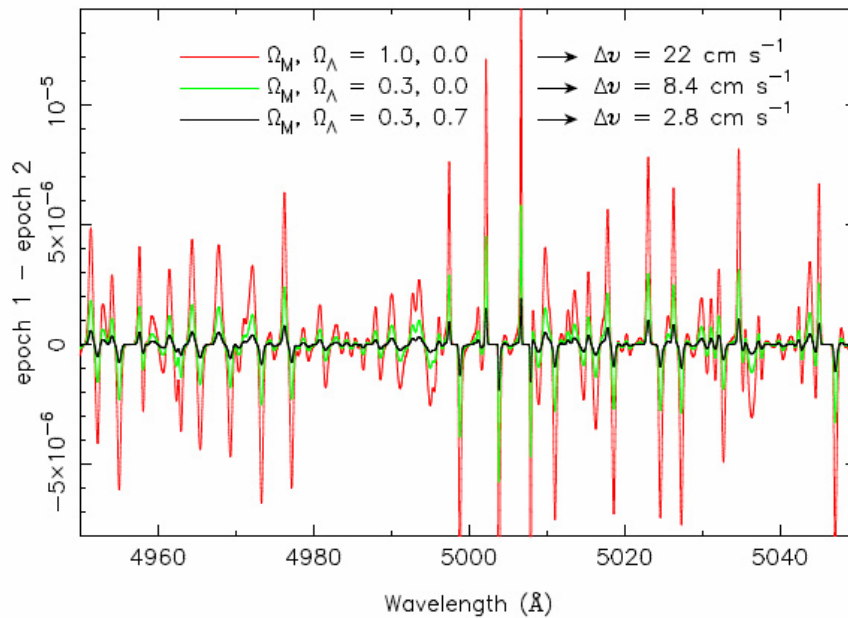
Sandage effect

$$\Delta z \approx \frac{a(t_0 + \Delta t_0)}{a(t_s + \Delta t_s)} - \frac{a(t_0)}{a(t_s)}$$

$$\Delta z = H_0 \Delta t_0 \left(1 + z - \frac{H(z)}{H_0} \right)$$

$$\Delta v = \frac{c \Delta z}{1 + z} \Big|_{1 \text{ yr}} \approx 1 \text{ cm/sec}$$

CODEX at EELT



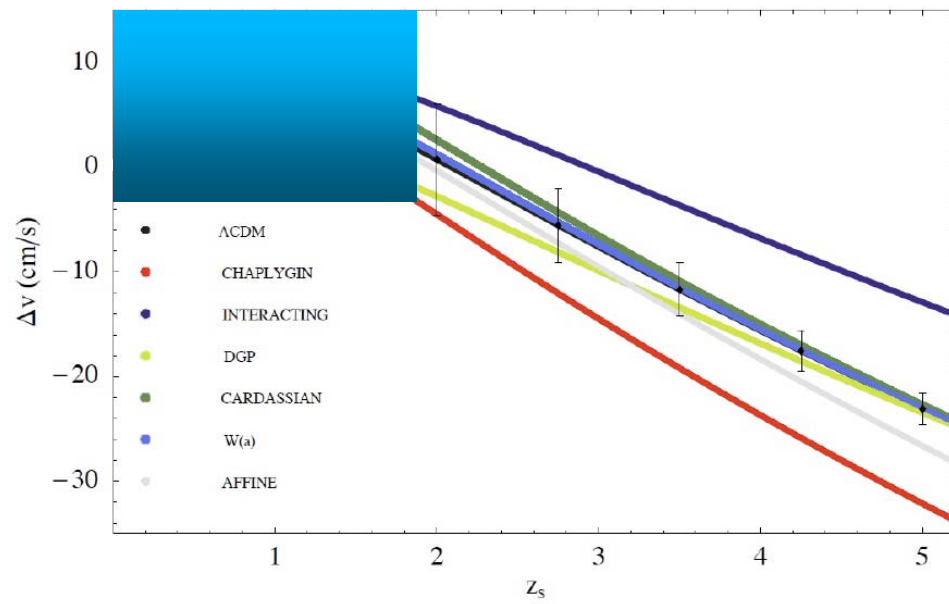
- large collecting area
- high resolution spectrographs
- stable, low-peculiar motion targets: Lyman-alpha lines

$$\sigma = 2 \left(\frac{2350}{S/N} \right) \left(\frac{30}{N_{QSO}} \right)^{1/2} \left(\frac{5}{1+z} \right)^{1.8} \text{ cm/s}$$

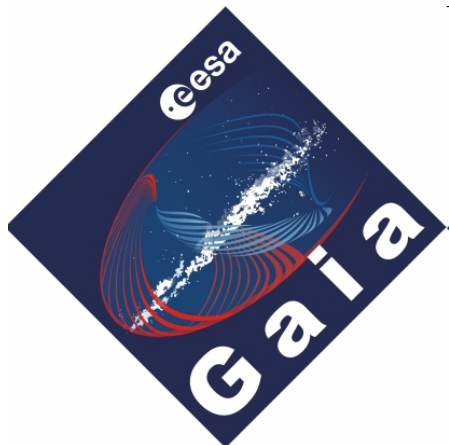
Liske et al. 2008

ESTEC 2009

Euclid range



Balbi & Quercellini 2007

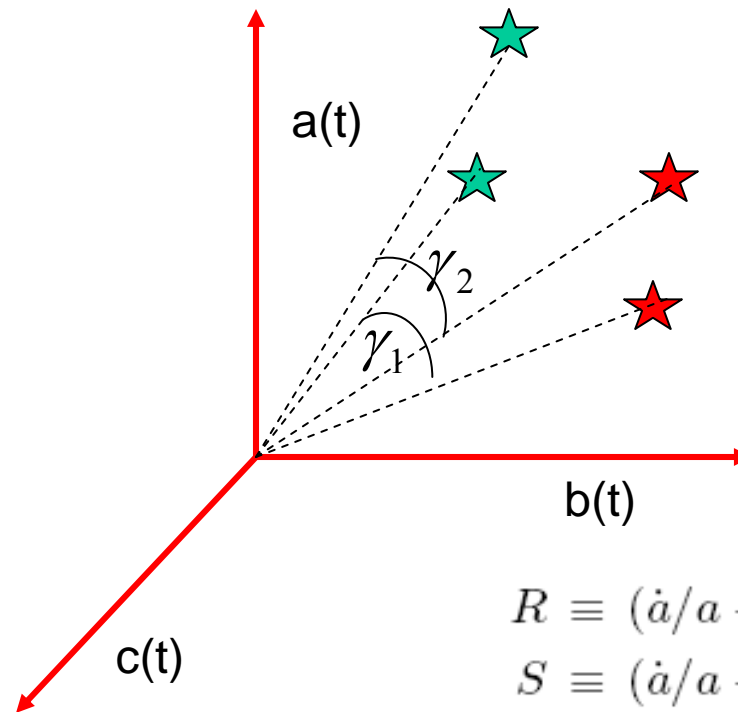


Gaia: Complete, Faint, Accurate

	Hipparcos	Gaia
Magnitude limit	12	20 mag
Completeness	7.3 – 9.0	20 mag
Bright limit	0	6 mag
Number of objects	120 000	26 million to V = 15 250 million to V = 18 1000 million to V = 20
Effective distance	1 kpc	50 kpc
Galaxies	None	$10^6 - 10^7$
Accuracy	1 milliarcsec	7 μ arcsec at V = 10 10-25 μ arcsec at V = 15 300 μ arcsec at V = 20
Photometry	2-colour (B and V)	Low-res. spectra to V = 20
Radial velocity	None	15 km/s to V = 16-17
Observing	Pre-selected	Complete and unbiased

The cosmic reference frame

Bianchi I



$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y,$$
$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y.$$

Anisotropic dark energy

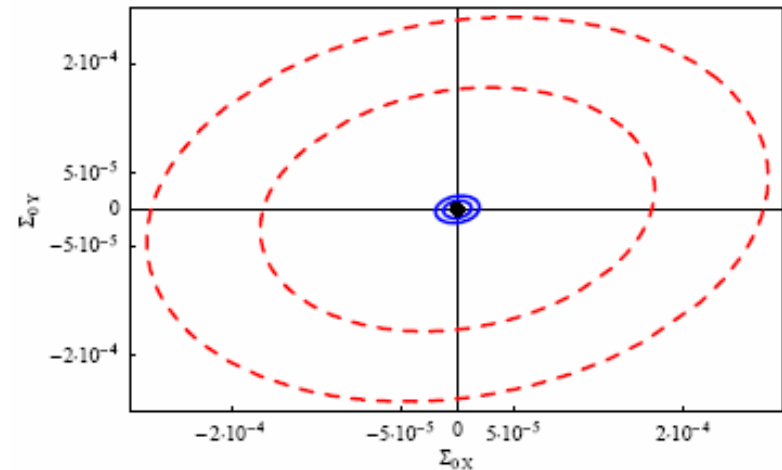
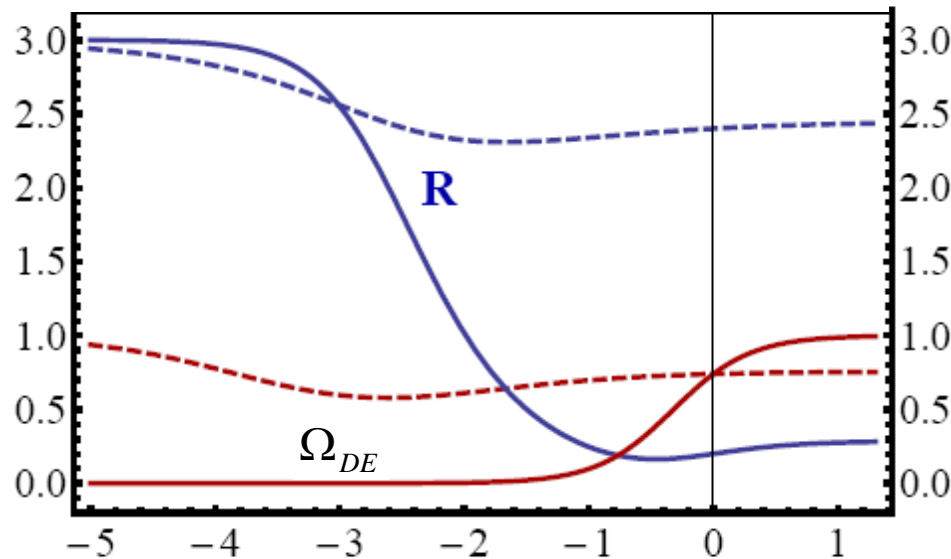
Mota & Koivisto 2008,
Barrow, Saha, Bruni, Rodrigues and many others..

$$T_{(\text{DE})\nu}^{\mu} = \text{diag}(-1, w, w + 3\delta, w + 3\gamma)\rho_{\text{DE}},$$

$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y,$$

$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y.$$

Experiment	N_s	σ_{acc}	Δt
Gaia	500,000	$10\mu\text{as}$	5yrs
Gaia+	1,000,000	$1\mu\text{as}$	10yrs



C. Quercellini, P. Cabella, L.A.,
M. Quartin, A. Balbi 2009

ESTEC 2009

$$R = \frac{\Delta H}{H} \leq 10^{-4}, \text{ at any } z.$$

A European Dream Team

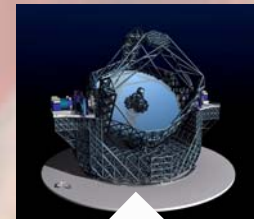
- The combination of weak lensing and clustering allows the full reconstruction of the space-time geometry
- In addition, Euclid will provide unique new constraints on dark matter and initial conditions
- Together with Planck, LHC, Gaia, EELT, Euclid will draw a new 3D atlas of the universe



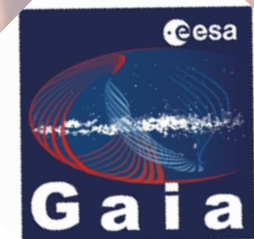
Euclid



LHC



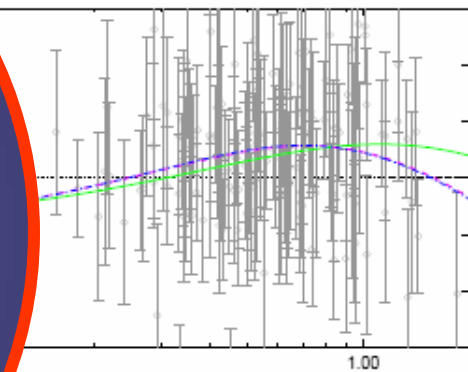
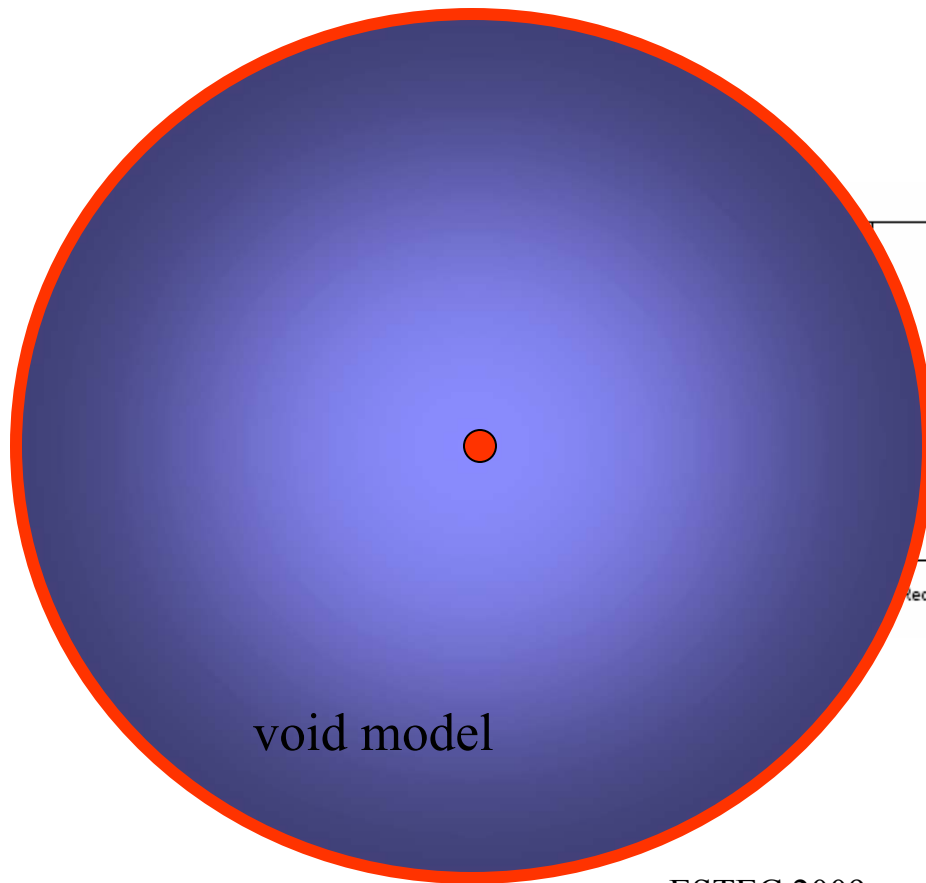
EELT



Gaia

Cosmic Degeneracy 3

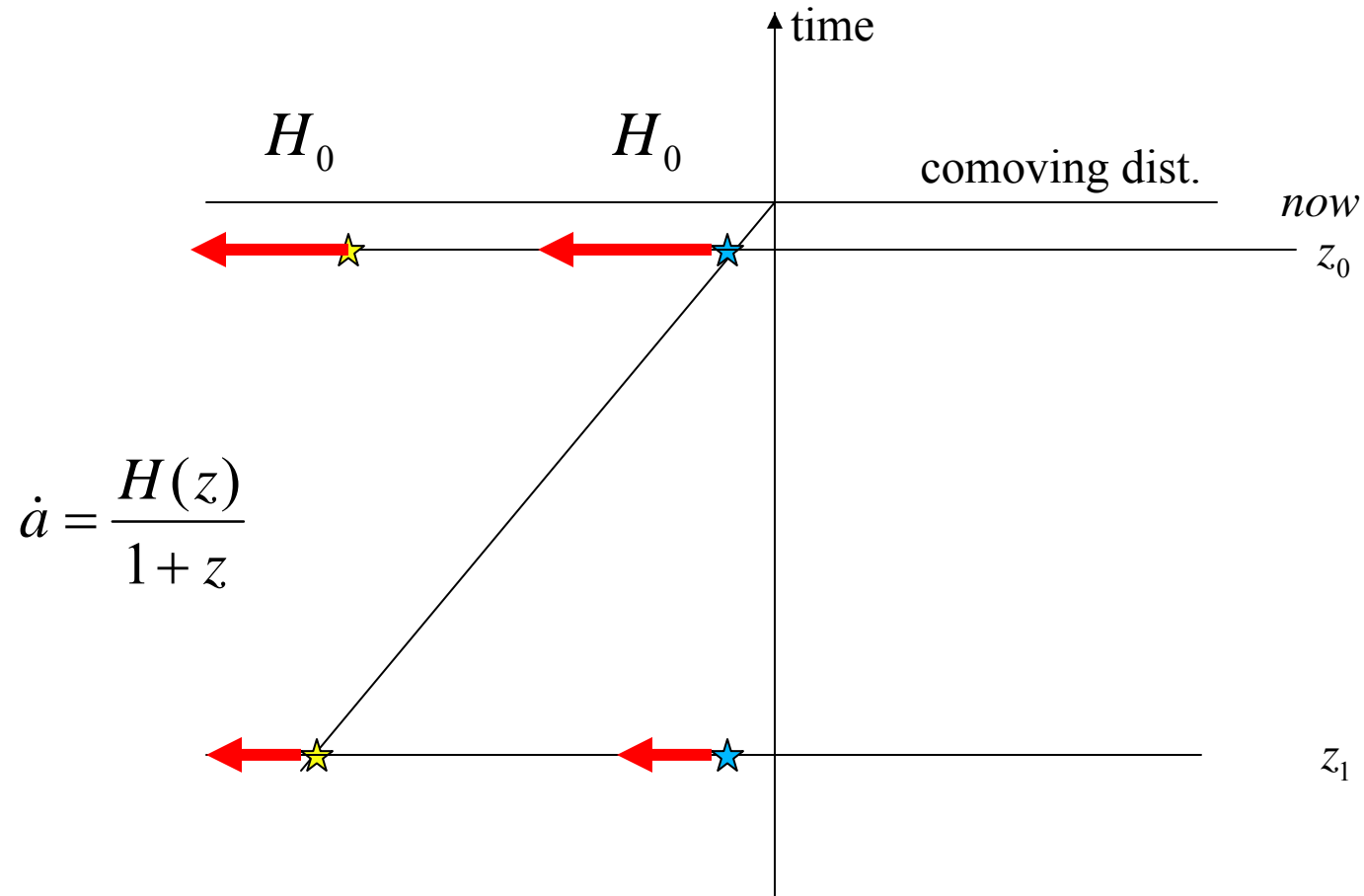
Tomita 2001
Celerier 2001
Alnes & Amarzguioi 2006,07
Bassett et al. 07
Clifton et al. 08
Notari et al. 2005-08
Marra et al. 08
Garcia-Bellido & Haugbolle 2008



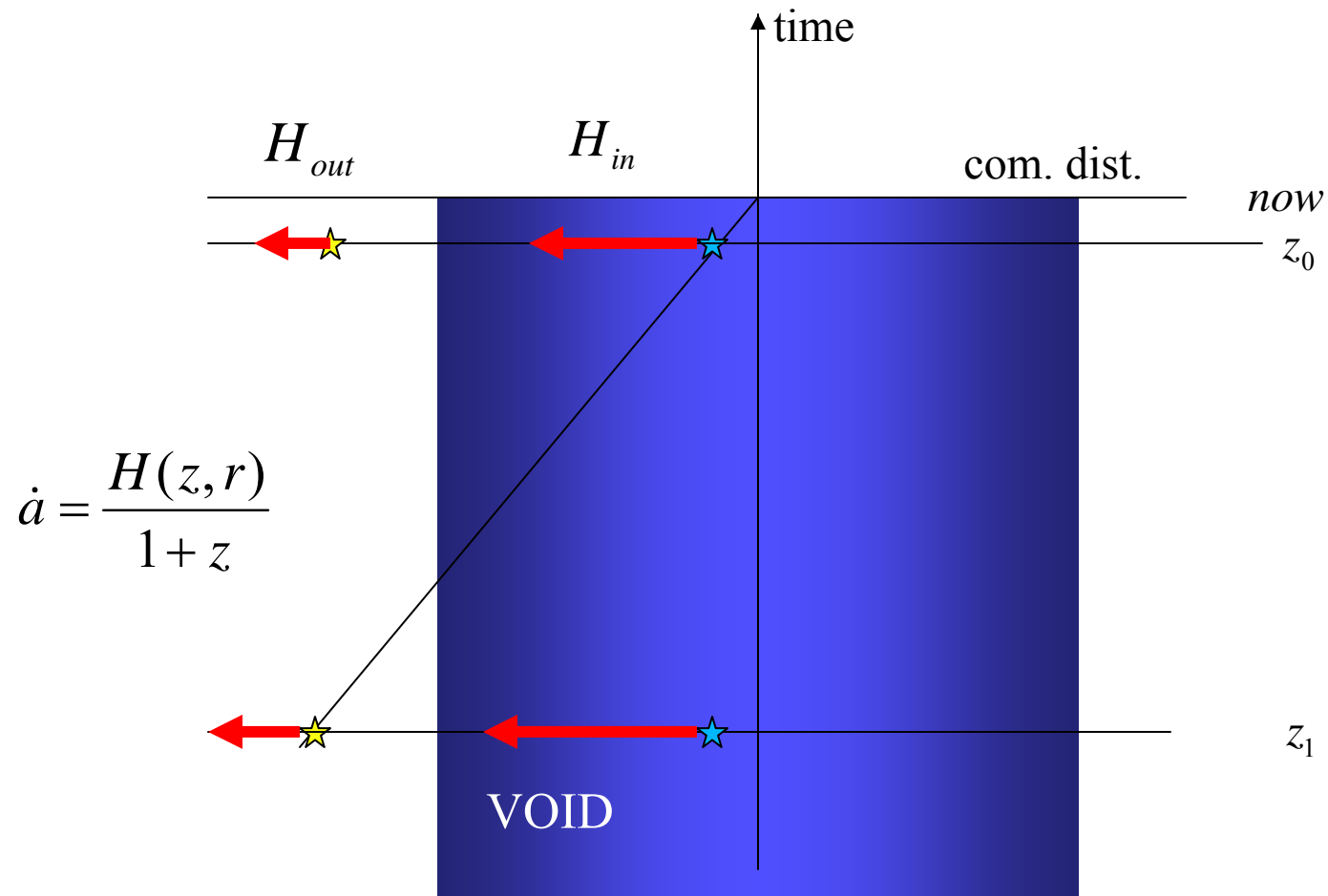
Garcia-Bellido & Haugbolle 2008

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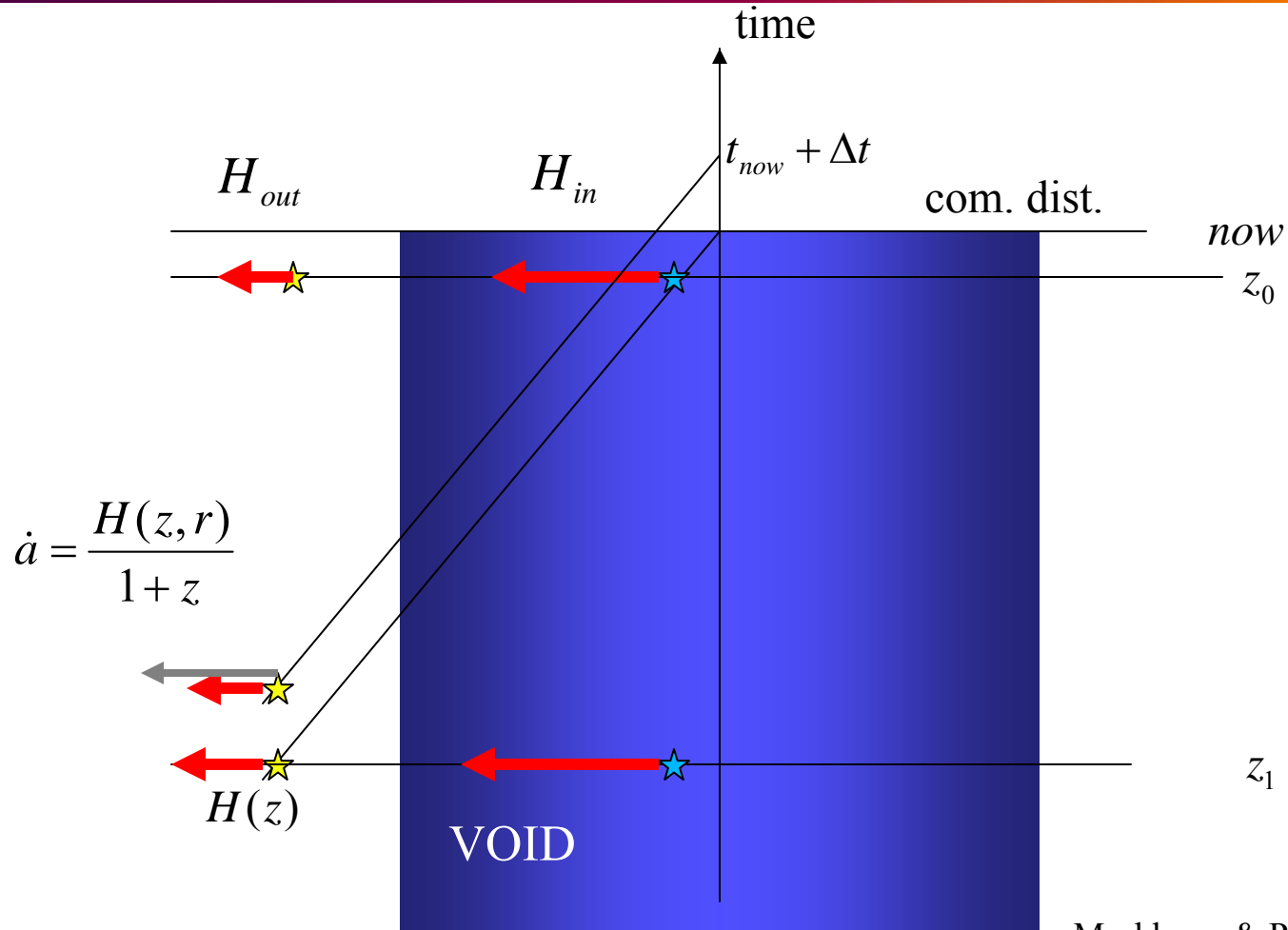
One null cone



One null cone



Two null cones are better than one!



ESTEC 2009

Mashhoon & Partovi 1985
 Uzan, Clarkson & Ellis 2007
 Quartin, Quercellini, L.A. 2009

Cosmology and modified gravity



in laboratory



in the solar system



at astrophysical scales



at cosmological scales



very limited time/space/energy scales;
only baryons

complicated by non-linear/non-gravitational effects

unlimited scales; mostly linear processes;
baryons, dark matter, dark energy !



VOLUME 136

SEPTEMBER 1962

NUMBER 2



$H(z_1)$



$H(z_2)$

THE CHANGE OF REDSHIFT AND APPARENT LUMINOSITY OF GALAXIES DUE TO THE DECELERATION OF SELECTED EXPANDING UNIVERSES

ALLAN SANDAGE

Mount Wilson and Palomar Observatories

Carnegie Institution of Washington, California Institute of Technology

(With an Appendix by G. C. McVITTIE, University of Illinois Observatory, Urbana)

Received February 2, 1962; revised April 13, 1962

ABSTRACT

The redshift and apparent luminosity of any given galaxy are not constant with time for most models of the expanding universe. Redshifts decrease with time because of the braking action of the gravitational field in all exploding models, except for the one where the matter density is zero. Apparent luminosities decrease with time, except for the oscillating model in the contracting phase and for galaxies with very large $\Delta\lambda/\lambda_0$ values, because the distances between galaxies are increasing. Redshifts increase with time for every galaxy in the steady-state model.

The theory and numerical results of the deceleration are presented for four selected world models. For a galaxy with redshift $z = \Delta\lambda/\lambda_0 = 0.4$ at the present epoch, the change of redshift with time is found to be $dz/dt = -11 \times 10^{-8}$ km/sec year for the oscillating model in the expanding phase at $q_0 = +1$; $dz/dt = -5.9 \times 10^{-8}$ km/sec year for the Euclidean model; $dz/dt = -4.3 \times 10^{-8}$ km/sec year for the hyperbolic model at $q_0 = 0.3516$; and $dz/dt = +9.2 \times 10^{-8}$ km/sec year for the steady-state model. These all assume that $H^{-1} = 13 \times 10^9$ years at the present epoch. With present optical techniques

$$\Delta v \approx \pm 1 \text{ cm/sec/year}$$

$$\Delta z \approx \frac{a(t_0 + \Delta t_0)}{a(t_s + \Delta t_s)} - \frac{a(t_0)}{a(t_s)}$$

$$\Delta z = H_0 \Delta t_0 \left(1 + z - \frac{H(z)}{H_0} \right)$$

$$\Delta v = \frac{c \Delta z}{1 + z} \Big|_{1 \text{ yr}} \approx 1 \text{ cm/sec}$$

IV. CONCLUSION

1. The foregoing considerations show that an "ideal" deceleration test exists between the exploding and the steady-state models in the sense that the sign of the effect is reversed. However, for the test to be useful, it would seem that a precision redshift catalogue must be stored away for the order of 10^7 years before an answer can be found because the decelerations are so small by terrestrial standards.

2. For all models, except the oscillating case, it will become more and more difficult to obtain observational information from the universe because the apparent luminosities of galaxies decrease with time. Indeed, if the oscillating case is excluded, there will be a time in the very distant future when most galaxies will recede beyond the limit of easy observation and when data for extragalactic astronomy must be collected from ancient literature.

κiv:astro-ph/9802122 v1 11 Feb 1998

Direct Measurement of Cosmological Parameters from the Cosmic Deceleration of Extragalactic Objects

Abraham Loeb

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

ABSTRACT

The redshift of all cosmological sources drifts by a systematic velocity of order a few m s^{-1} over a century due to the deceleration of the Universe. The specific functional dependence of the predicted velocity shift on the source redshift can be used to verify its cosmic origin, and to measure directly the values of cosmological parameters, such as the density parameters of matter and vacuum, Ω_M and Ω_Λ , and the Hubble constant H_0 . For example, an existing spectroscopic technique, which was recently employed in planet searches, is capable of uncovering velocity shifts of this magnitude. The cosmic deceleration signal might be marginally detectable through two observations of $\sim 10^2$ quasars set a decade apart, with the HIRES instrument on the Keck 10 meter telescope. The signal would appear as a global redshift change in the Ly α forest templates imprinted on the quasar spectra by the intergalactic medium. The deceleration amplitude should be isotropic across the sky. Contamination of the cosmic signal by peculiar accelerations or local effects is likely to be negligible.

Subject headings: cosmology: theory

submitted to *ApJ Letters*, Feb. 10th, 1998

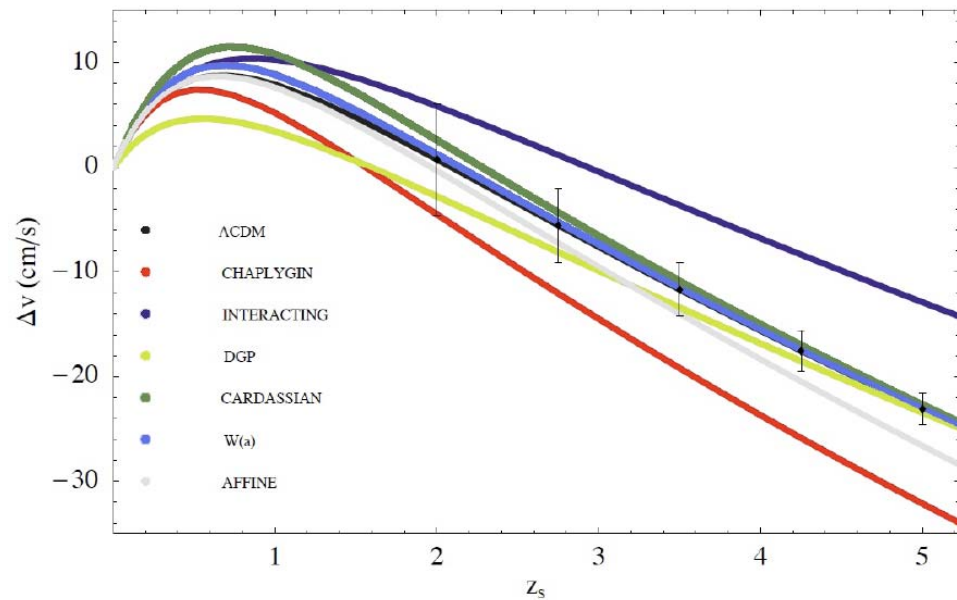
$$H_0 \Delta t \approx 10^{-9} \quad (10 \text{ yrs})$$

$$10^{-9} c \approx 30 \text{ cm/sec}$$

$$\Delta z \approx \frac{a(t_0 + \Delta t_0)}{a(t_s + \Delta t_s)} - \frac{a(t_0)}{a(t_s)}$$

$$\Delta z = H_0 \Delta t_0 \left(1 + z - \frac{H(z)}{H_0}\right)$$

$$\Delta v = \frac{c \Delta z}{1 + z} \Big|_{1 \text{ yr}} \approx 1 \text{ cm/sec}$$



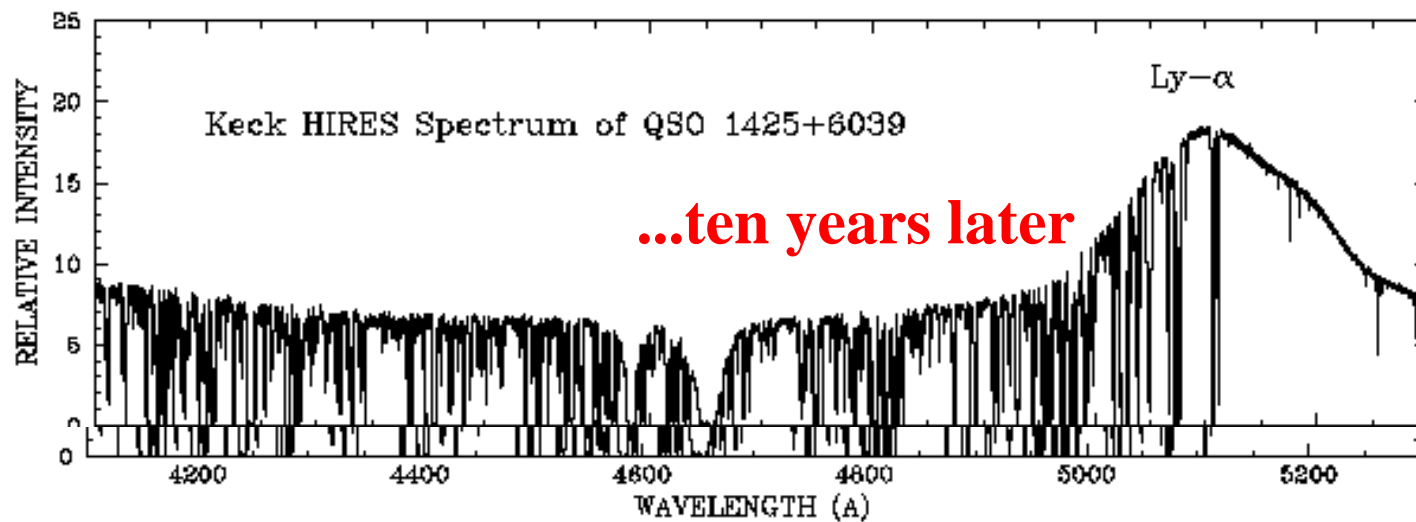
Corasaniti, Huterer, Melchiorri 2007
Balbi & Quercellini 2007

THE EXTREMELY LARGE TELESCOPE

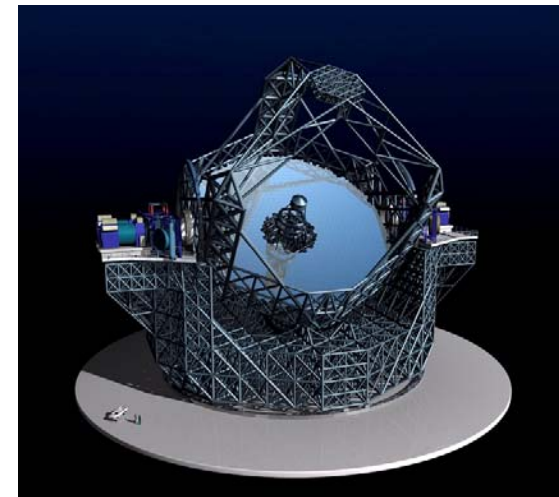
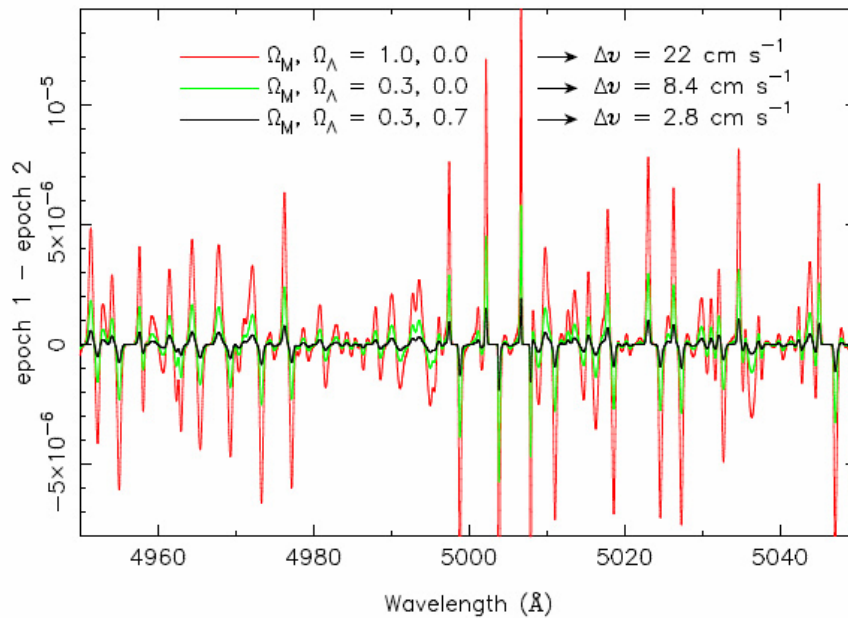
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CODEX at EELT



CODEX at EELT



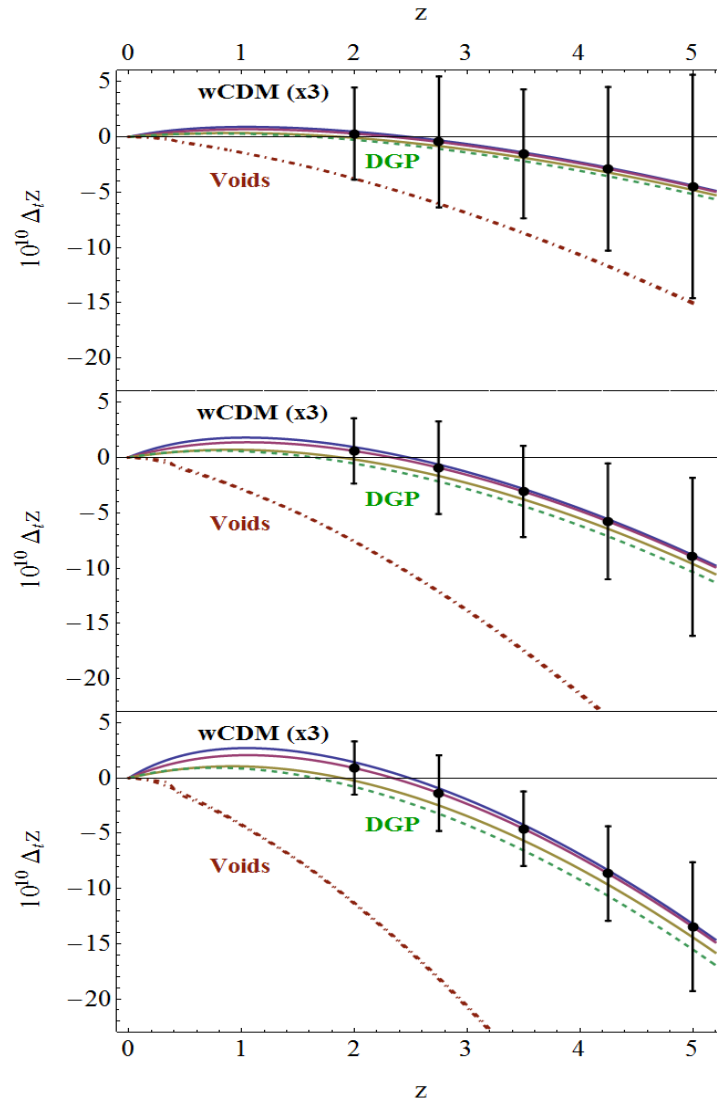
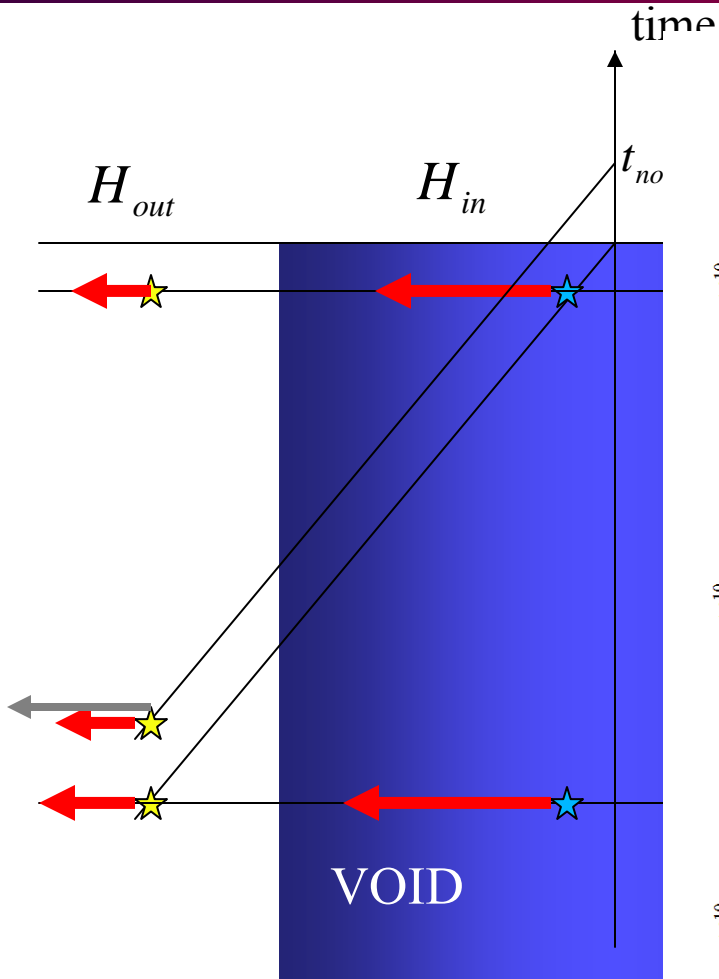
- large collecting area
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Liske et al. 2008

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Two null cones are better than one!



M. Quartin & L. A. 2009

Evolution

Rest of the Universe

Us



Ptolemaic system, I century

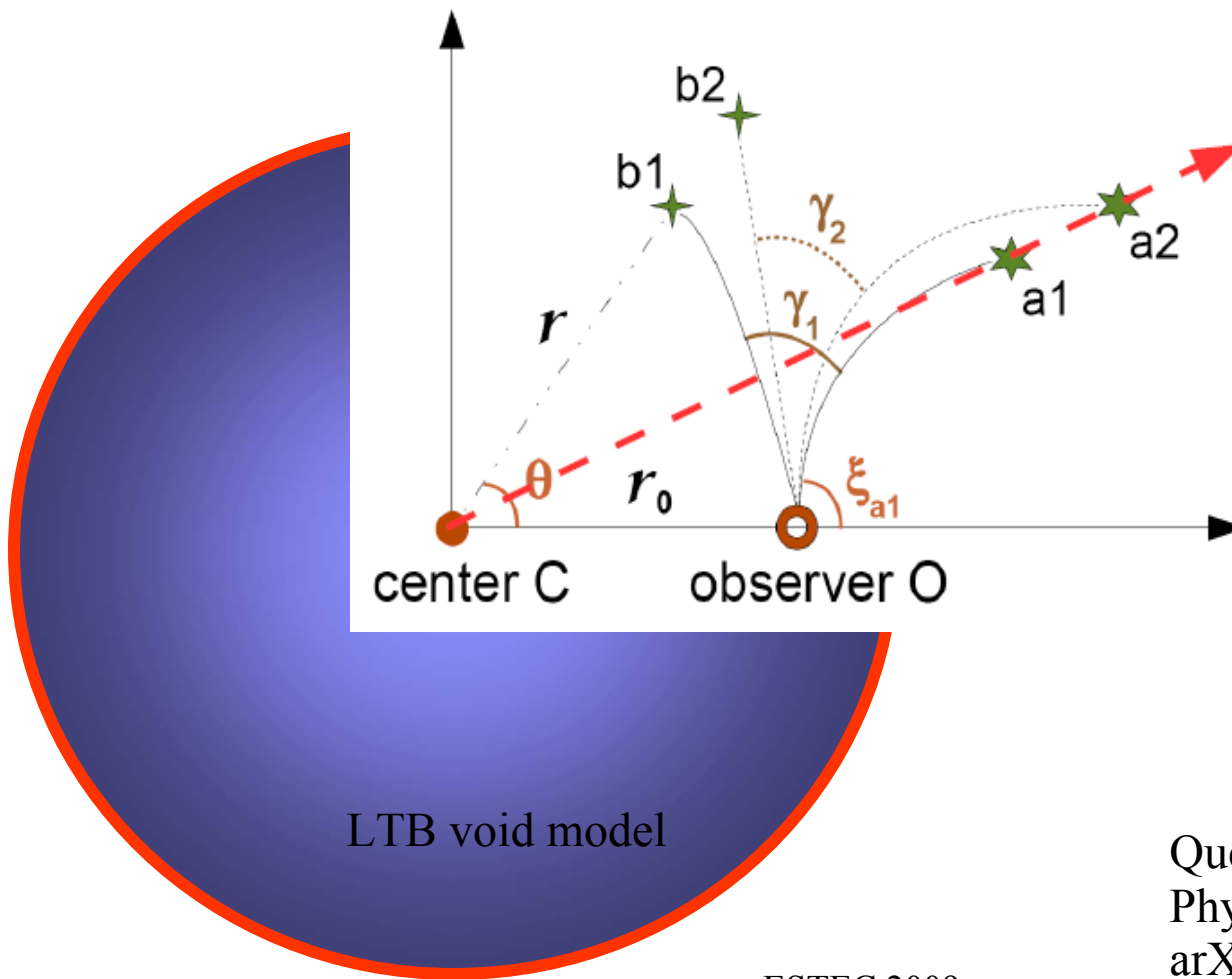
Rest of the Universe

Us



LTB void model, XXI century

Cosmic Parallax



$$H_0 \Delta t \approx 10^{-9}$$

$$10^{-9} \text{ rad} \approx 200 \mu\text{as}$$

astrometric satellites
GAIA, SIM, Jasmine etc:
1-100 μas

Quercellini, Quartin & LA,
Phys. Rev. Lett. 2009
arXiv 0809.3675

Lemaître-Tolman-Bondi models

- LTB metrics describe **void models**

$$R' \equiv \frac{\partial R}{\partial r}$$

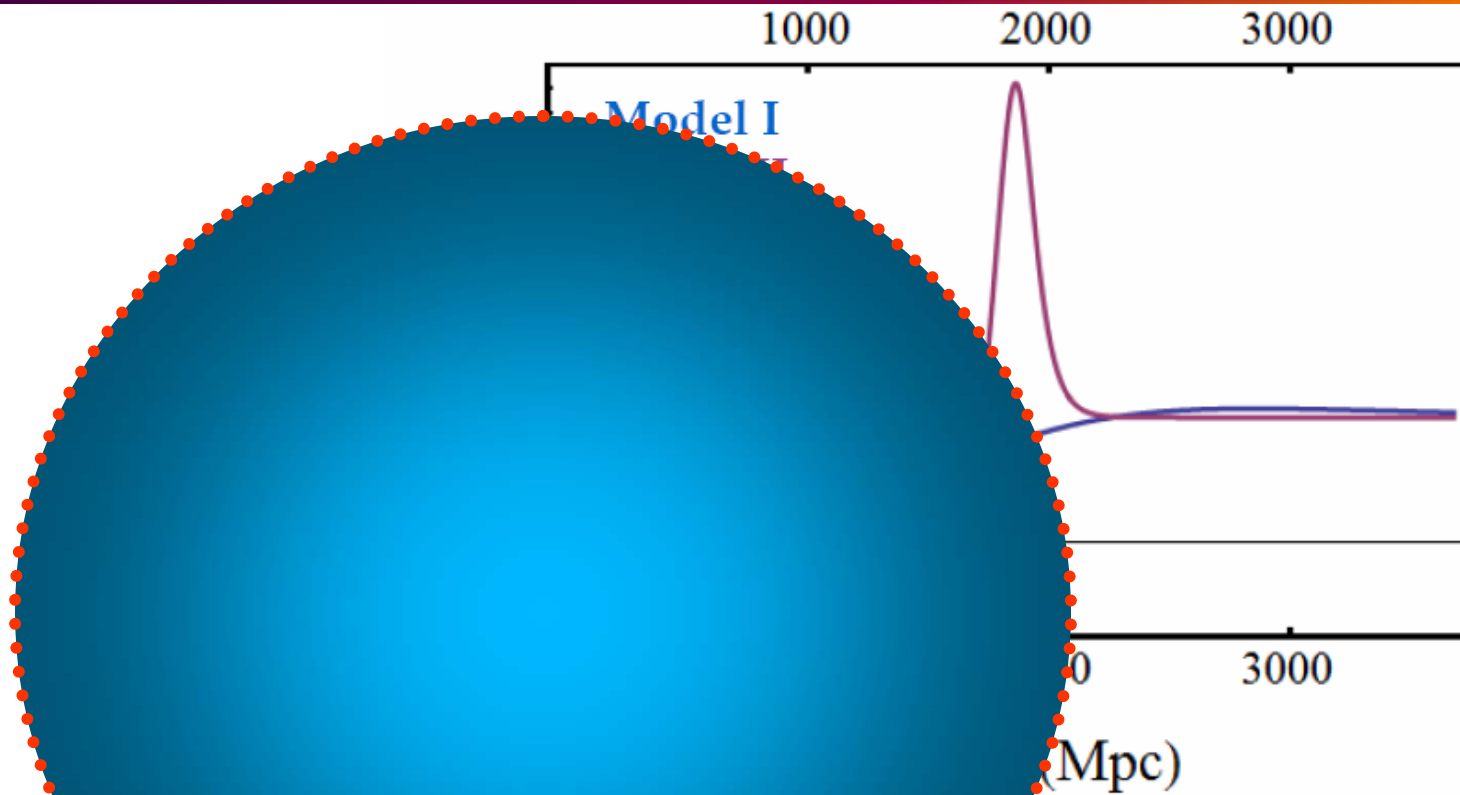
$$ds^2 = -dt^2 + \frac{[R'(t, r)]^2}{1 + \beta(r)} dr^2 + R^2(t, r) d\Omega^2$$

- Exact solution in a matter-dominated era

$$R = (\cosh \eta - 1) \frac{\alpha}{2\beta} + R_{lss} \left[\cosh \eta + \sqrt{\frac{\alpha + \beta R_{lss}}{\beta R_{lss}}} \sinh \eta \right]$$

$$\sqrt{\beta} t = (\sinh \eta - \eta) \frac{\alpha}{2\beta} + R_{lss} \left[\sinh \eta + \sqrt{\frac{\alpha + \beta R_{lss}}{\beta R_{lss}}} (\cosh \eta - 1) \right]$$

LTB models

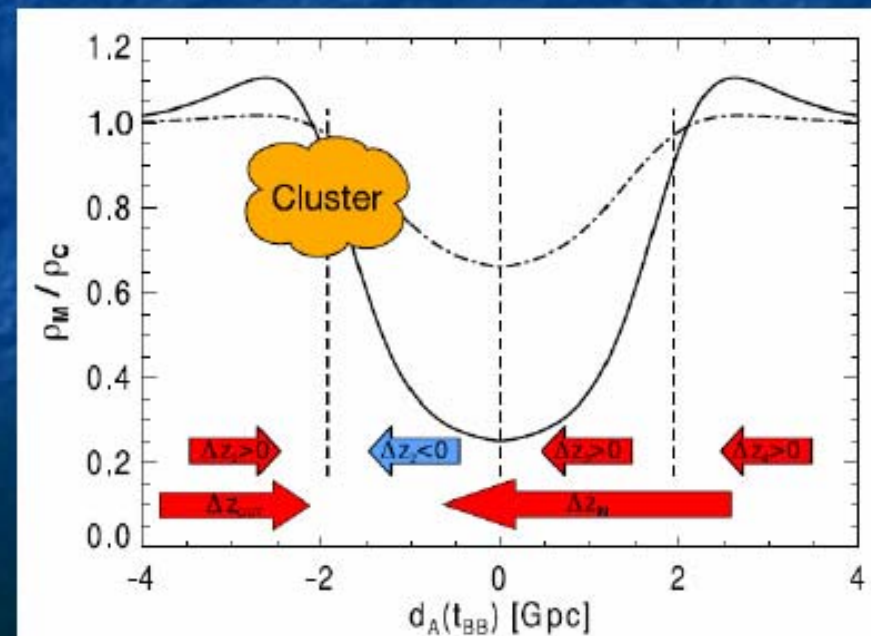


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Constraints on Void Models

- Large voids (> 1.5 Gpc) are in conflict with
 - CMB blackbody spectrum
 - *Caldwell & Stebbins: 0711.3459 (PRL)*
 - Kinematic Sunyaev-Zeldovich effect from large clusters
 - *García-Bellido & Haugbolle: 0807.1326 (JCAP)*

- Sharp transitions could be in conflict with SDSS LRG or SNe distribution (no excess at $z \approx .3$)



Estimating the Cosmic Parallax

- Calculating the Cosmic Parallax require solving the full LTB geodesic equations
- Simple, *non-consistent* estimate \rightarrow flat FRW universe with $H(t) \rightarrow H(t, r)$
- Assume 2 sources initially separated by ΔX & $\Delta\theta$.

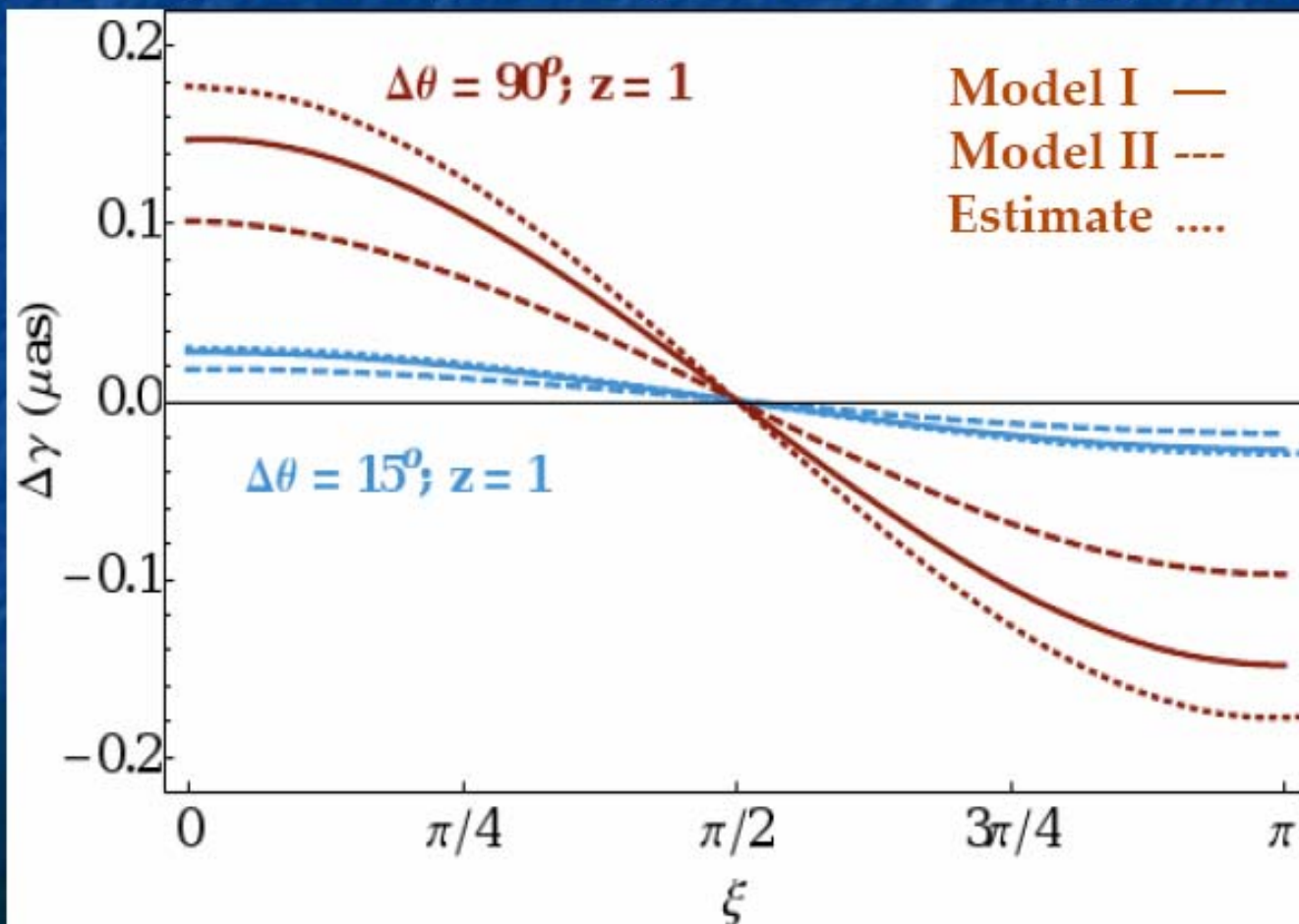
$$\Delta_t \gamma \simeq \Delta t \left(\overline{H}_{\text{obs}} - \overline{H}_X \right) \frac{X_{\text{obs}}}{X} \left(\cos \theta \Delta\theta + \sin \theta \frac{\Delta X}{X} \right)$$

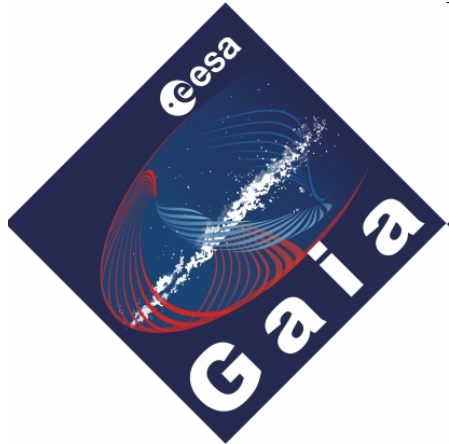
distance to the
void center

“physical” distance

Results

- Actual effect \rightarrow need to solve the LTB geodesic eqs.
- $\Delta_t \gamma$ in 10 yrs for a pair of quasars at $z=1$ (typical for Gaia)





Gaia: Complete, Faint, Accurate

	Hipparcos	Gaia
Magnitude limit	12	20 mag
Completeness	7.3 – 9.0	20 mag
Bright limit	0	6 mag
Number of objects	120 000	26 million to V = 15 250 million to V = 18 1000 million to V = 20
Effective distance	1 kpc	50 kpc
Galaxies	None	$10^6 - 10^7$
Accuracy	1 milliarcsec	7 μ arcsec at V = 10 10-25 μ arcsec at V = 15 300 μ arcsec at V = 20
Photometry	2-colour (B and V)	Low-res. spectra to V = 20
Radial velocity	None	15 km/s to V = 16-17
Observing	Pre-selected	Complete and unbiased

Cosmic Parallax with Gaia

- SNe → off-center distance $X_0 \leq 150$ Mpc. *Alnes & Armazguioui astro-ph/0607334*
- CMB dipole → off-center dist. $X_0 \leq 15$ Mpc. *astro-ph/0610331*

- Assuming:
 - $X_0 = 15$ Mpc (aggressive);
 - Astrometric precision of $30 \mu\text{as}$;
 - Nominal Gaia duration ($\Delta t = 5$ years)
- Gaia can detect the Cosmic Parallax at 1σ if # sources $\geq 450,000$ (conservative)

Noise and Systematics

- Most **obvious** source of noise → peculiar velocities

$$\Delta t \gamma_{\text{pec}} = \left(\frac{v_{\text{pec}}}{500 \frac{\text{km}}{\text{s}}} \right) \left(\frac{D_A}{1 \text{ Gpc}} \right)^{-1} \left(\frac{\Delta t}{10 \text{ years}} \right) \mu\text{as}$$

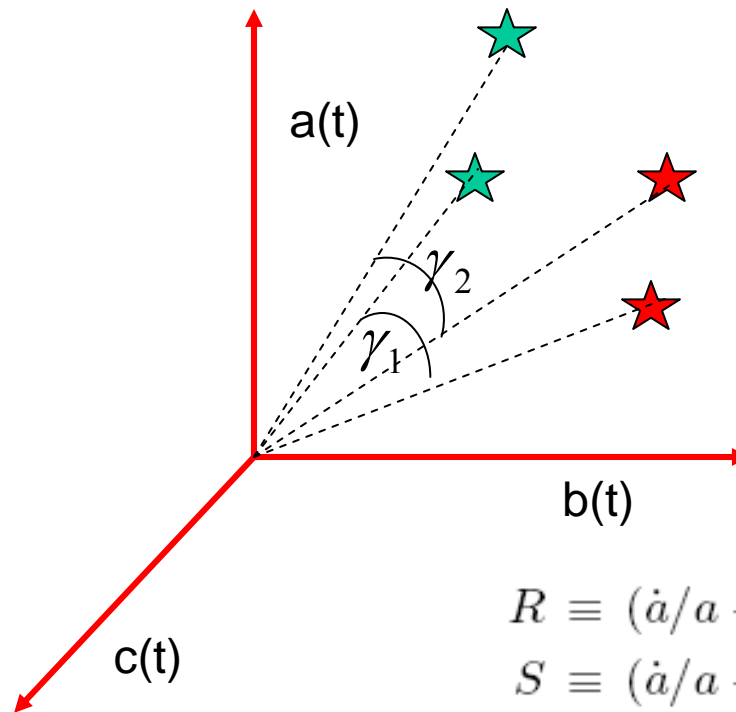
- Most **serious** source of noise → changing aberration due to acceleration of the solar system

Gaia predicts $\approx 4 \mu\text{as}$ effect, of which 90% could be subtracted → $0.4 \mu\text{as}$ spurious dipole

Kovalevsky 2003

Not only LTB

Bianchi I



$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y,$$
$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y.$$

Current limits on anisotropy

$$R = \frac{\Delta H}{H} \leq 10^{-4} \quad \text{at } z = 1000$$

$$\frac{\Delta H}{H} \leq 10^{-8} \quad \text{at } z = 0 \text{ in a } \Lambda\text{CDM universe}$$

$$\frac{\Delta H}{H} \leq ? \quad \text{at } z = 0 \text{ in anisotropic dark energy}$$

Anisotropic dark energy

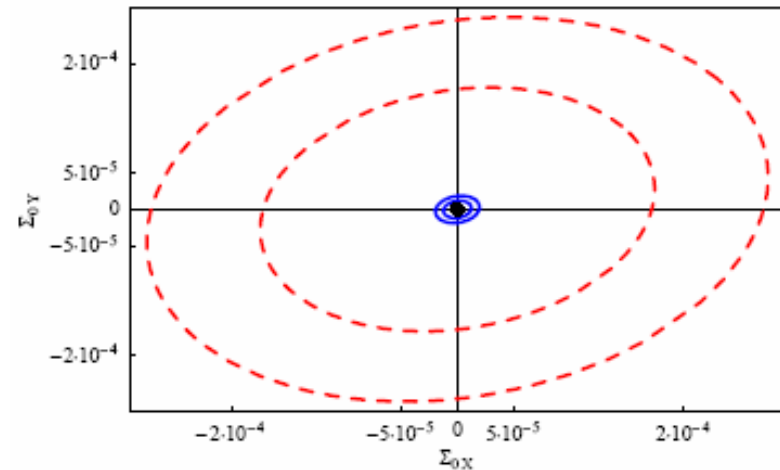
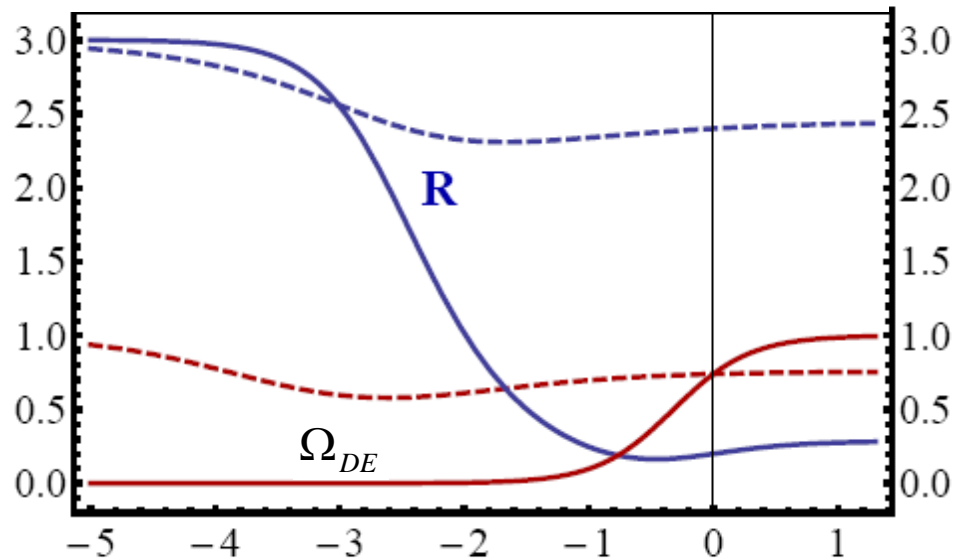
Mota & Koivisto 2008,
Barrow, Saha, Bruni, Rodrigues and many others..

$$T_{(\text{DE})\nu}^{\mu} = \text{diag}(-1, w, w + 3\delta, w + 3\gamma)\rho_{\text{DE}},$$

$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y,$$

$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y.$$

Experiment	N_s	σ_{acc}	Δt
Gaia	500,000	$10\mu\text{as}$	5yrs
Gaia+	1,000,000	$1\mu\text{as}$	10yrs



C. Quercellini, P. Cabella, L.A.,
M. Quartin, A. Balbi 2009

ESTEC 2009

$$R = \frac{\Delta H}{H} \leq 10^{-4} \quad , \quad \underline{\text{at any } z}$$

WERBUNG

DARK ENERGY

theory and observations

Contents

1 Overview	9
2 Expansion history of the universe	13
2.1 Friedmann equations	13
2.2 Hubble's law	16
2.3 Matter species in the universe	17
2.4 Cosmic distances	21
2.4.1 Comoving distance	21
2.4.2 Luminosity distance	22
2.4.3 Angular diameter distance	23
2.4.4 Degeneracy of the distance-redshift relation	24
2.5 The equation of state of dark energy	25
2.6 Problems	27
3 Correlation function and power spectrum	28
3.1 The correlation function	28
3.2 The n -point correlation function	30
3.3 The power spectrum	30
3.4 From the power spectrum to the moments	35
3.5 Problems	38
4 Basics of cosmological perturbation theory	39
4.1 Perturbing General Relativity	39
4.2 The Newtonian gauge	40
4.3 Single-fluid model	43
4.4 Scales larger than the horizon	46
4.5 Scales smaller than the Hubble radius	47
4.6 Two-fluid solutions	48
4.7 Velocity field	50
4.8 The redshift distortion	52
4.9 Baryons, photons, and neutrinos	55
4.10 The matter power spectrum	60
4.11 Perturbed photon propagation	64
4.11.1 The Sachs-Wolfe effect	66
4.11.2 Weak lensing	67
4.12 Problems	70
5 Observational evidence of dark energy	71
5.1 The age of the universe	71
5.2 Supernova observations	73
5.3 Cosmic Microwave Background	78
5.4 Baryon acoustic oscillations	84
5.5 Large-scale structure	87
5.6 Problems	90
6 Cosmological constant	91
6.1 Einstein equations with the cosmological constant	91
6.2 History of the cosmological constant	92
6.3 The fine tuning problem	93
6.4 The coincidence problem	94
6.5 Supersymmetric models	96
6.5.1 Vanishing cosmological constant under broken supersymmetry	97
6.5.2 de Sitter vacua in string theory	99
6.6 Cosmological constant and the anthropic principle	102
6.7 The decoupling of the cosmological constant from gravity	105
6.8 Problems	108
7 Dark energy as a modified form of matter I: Quintessence	109
7.1 Quintessence	110
7.2 Dynamical system approach	112
7.2.1 Exponential potential	113
7.2.2 Other potentials	116
7.2.3 Tracker solutions	117
7.3 Early dark energy	120
7.4 Quintessence potentials in particle physics	123
7.4.1 A fermion condensate model in Global supersymmetric theory	124
7.4.2 Supergravity models	124
7.4.3 Pseudo-Nambu-Goldstone models	127
7.4.4 Dilatonic quintessence	128
7.5 Reconstruction of quintessence from observations	130
7.5.1 Reconstructing the potential and the equation of state of quintessence	130
7.5.2 Statefinder	133
7.5.3 Quintessence in the (w_0, w_1) plane	135
7.6 Problems	137
8 Dark energy as a modified form of matter II	138
8.1 k-essence	138
8.1.1 k-essence models	138
8.1.2 Equation of state for k-essence	140
8.1.3 Stability conditions for k-essence	142
8.1.4 Cosmological dynamics for the dilatonic ghost condensate	143
8.1.5 k-essence and the coincidence problem	145
8.2 Phantom	148
8.3 Coupled dark energy	150
8.3.1 Coupled quintessence with an exponential potential	150
8.3.2 Decoupling the baryons	155
8.3.3 Parametrizing coupled dark energy	156
8.3.4 Mass varying neutrino scenario	159
8.3.5 Couplings to the electromagnetic field	160
8.4 Chameleon scalar fields	162
8.4.1 The field profile of the chameleon field	164
8.4.2 The violation of the equivalence principle	167
8.4.3 Solar system constraints	167
8.4.4 Constraints on the model with an inverse power-law potential	168
8.5 Dark energy models with scaling solutions	169
8.5.1 The condition for the existence of scaling solutions	170
8.5.2 Fixed points of the scaling Lagrangian	172
8.5.3 Two scaling regimes?	175
8.6 Unified models of dark energy and dark matter	177
8.6.1 Generalized Chaplygin gas model	178
8.6.2 k-essence as unified models of dark energy and dark matter	180
8.7 Future singularities	181
8.8 Problems	184
9 Dark energy as a modification of gravity	185
9.1 $f(R)$ gravity	185
9.1.1 $f(R)$ gravity in the metric formalism	186
9.1.2 Cosmological dynamics of $f(R)$ dark energy models in the metric formalism	187
9.1.3 Local gravity constraints on $f(R)$ gravity models in the metric formalism	193
9.1.4 The viability of $f(R)$ gravity models in the Palatini formalism	199
9.2 Scalar-tensor theories	202
9.2.1 Cosmological dynamics	204
9.2.2 Local gravity constraints	209
9.3 Gauss-Bonnet dark energy models	210
9.4 Braneworld models of dark energy	216
9.5 Problems	222
10 Cosmic acceleration without dark energy	223
10.1 Void models	223
10.2 Backreaction	228
10.3 Problems	231
11 Dark energy and linear cosmological perturbations	232
11.1 Perturbations in a general dark energy cosmology	232
11.2 Perturbations of a scalar field	239
11.3 From dark energy to dark force	241
11.4 A massive dark energy field	243
11.5 Sound speed of a scalar field	244
11.6 Perturbations in modified gravity models	246
11.6.1 $f(R)$ gravity	246
11.6.2 Scalar-tensor gravity	252
11.6.3 DGP braneworld model	256
11.6.4 Reconstruction of the metric at first order	258
11.7 Problems	260
12 Non-linear cosmological perturbations	261
12.1 Second-order perturbations	261
12.2 The bispectrum and the higher-order correction to the power spectrum	264
12.3 Spherical collapse	269
12.4 The mass function of collapsed objects	271
12.5 Dark energy N -body simulations	273
12.6 Problems	275
13 Statistical methods in cosmology	276
13.1 The likelihood function	276
13.2 Model selection	281
13.3 Fisher matrix	283
13.4 The Fisher matrix for the power spectrum	290
13.5 Principal component analysis	292

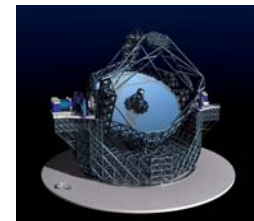
L. A. and S. Tsujikawa
 Cambridge University Press
 mid 2010

Breaking the degeneracies

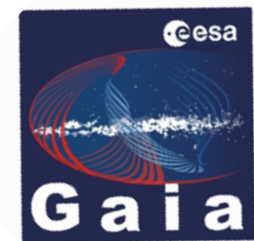
- The task of understanding the nature of Dark energy is plagued by several cosmic degeneracies
- We need to combine weak lensing and clustering to reconstruct the metric at first order
- We need to use real-time observables to distinguish between real and apparent acceleration
- Together with the Cosmic Parallax we can reconstruct the **full 3D picture of cosmic kinematics !**



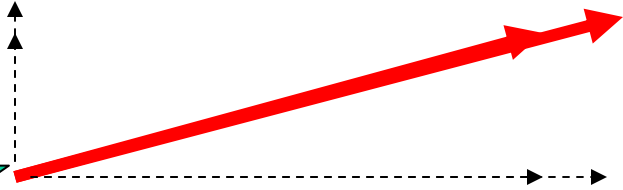
Euclid



EELT



Gaia



	radial	transverse
global		
local		

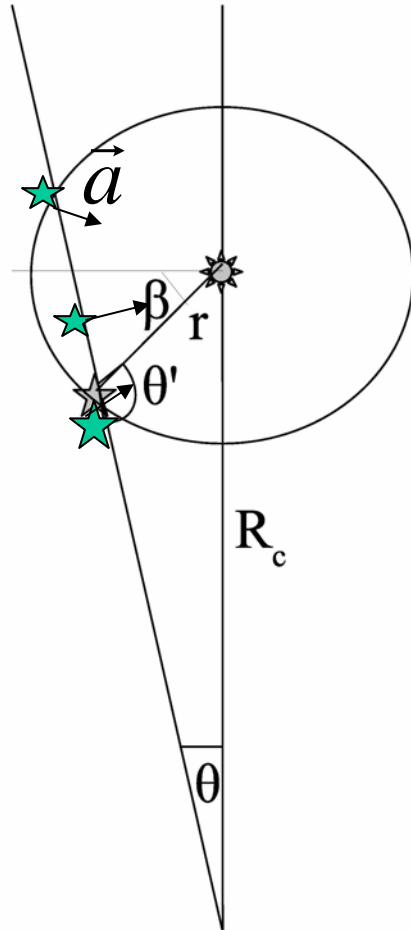
Real-time Cosmology

	radial	transverse
global	Expansion rate	anisotropy
local	Gravity at galactic scales: eg Newton vs Modif. Grav.	

ToDo

figura redshiftdrift void
figura cover book

Peculiar Acceleration



$$a_{pec} = \sin \beta \frac{GM(r)}{r^2}$$

The PA is a direct probe of the gravitational potential: it does not assume virialization or hydrostatic equilibrium.

Peculiar Acceleration

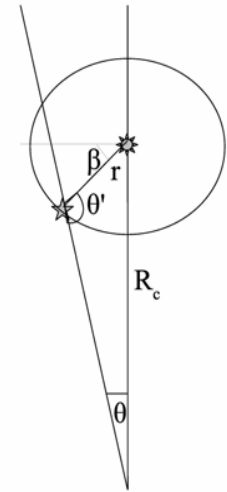
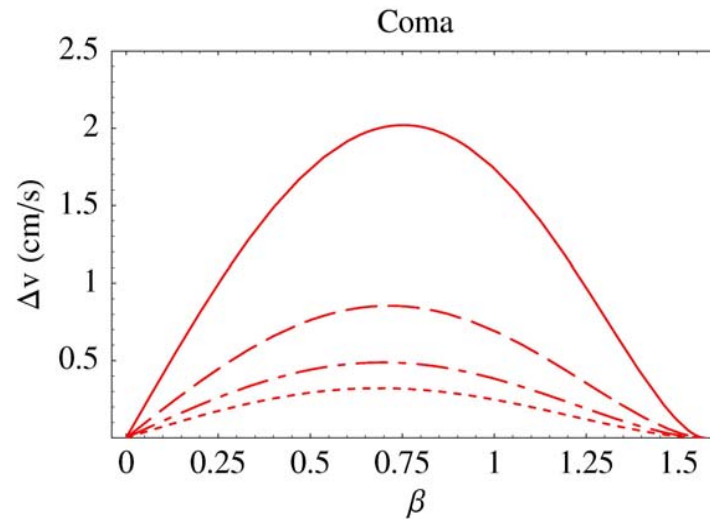
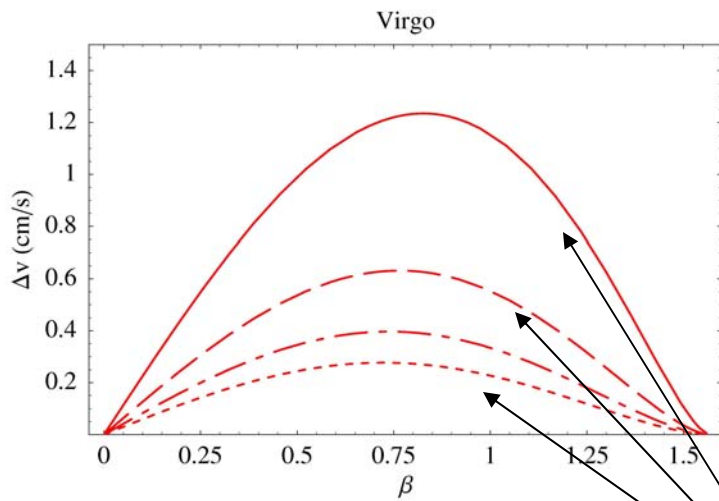
$$\rho_{NFW}(r) = \frac{\delta_c \rho_c}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}, \quad r_s = r_v / c$$

$$s(\beta) = \Delta v = \left(2 \frac{cm}{sec}\right) \sin \beta \frac{\Delta t}{10 yr} \frac{M_v}{10^{14} M_\odot} \left(\frac{r_s}{0.5 Mpc}\right)^{-2} C \left(\frac{\log(1 + \frac{r}{r_s})}{(r/r_s)^2} - \frac{1}{\frac{r}{r_s} (1 + \frac{r}{r_s})} \right)_{r=R_c \theta / \cos \beta}$$

	Mass	r_s
Andromeda	10^{11}	20 kpc
Virgo	1.2×10^{15}	0.55 Mpc
Coma	1.2×10^{15}	0.29 Mpc

Peculiar Acceleration

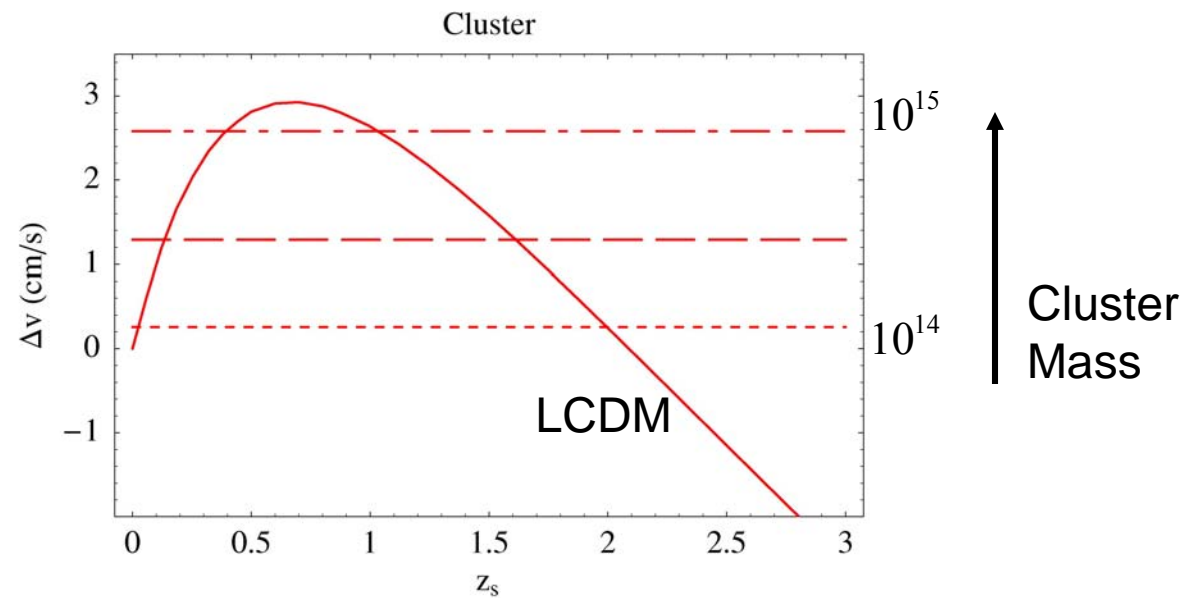
$$s(\beta) = \Delta v = \left(2 \frac{\text{cm}}{\text{sec}} \right) \sin \beta \frac{\Delta T}{10 \text{yr}} \frac{M_v}{10^{14} M_\odot} \left(\frac{r_s}{0.5 \text{Mpc}} \right)^{-2} C \left(\frac{\log(1 + \frac{r}{r_s})}{(r/r_s)^2} - \frac{1}{\frac{r}{r_s} (1 + \frac{r}{r_s})} \right)$$



different
lines of sight

L.A., A. Balbi, C. Quercellini,
astro-ph arXiv/0708.1132
Phys.Lett.B660:81,2008

PA versus Sandage effect



Peculiar acceleration in the Galaxy

- Can we use the peculiar acceleration to discriminate among **competing gravity theories**?
- Steps:
 - model the galaxy as a disc+CDM halo and derive the peculiar acceleration signal
 - model the galaxy as a disc in modified gravity (MOND)
 - analyse the different morphology of the signal in the Milky way

Spiral galaxy: Newton

test particle outside the disc, where the presence/absence of a CDM halo is more influent.

- Disc: Kuzmin potential,

$$\phi_K = -\frac{MG}{[R^2 + (|z|+h)^2]^{3/2}} \quad a_K = -\frac{MG}{[R^2 + (|z|+h)^2]^2}$$

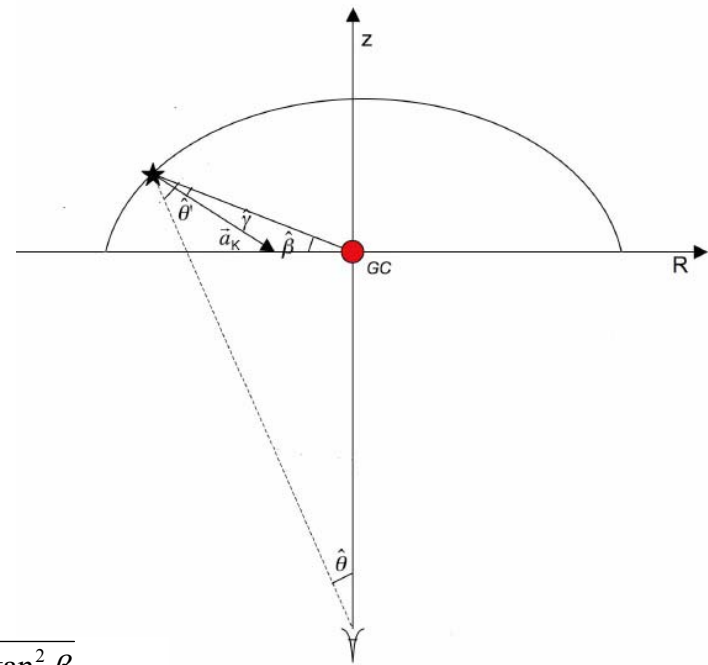
- CDM halo: logarithmic

$$\phi_L = \frac{1}{2}v_o^2 \log\left(R_c^2 + R^2 + \frac{z^2}{q^2}\right)$$

- The total line of sight acceleration:

$$a_{s,K} = \frac{MG}{R_g^2 \theta^2 \left[1 + \left(\frac{h}{R_g \theta}\right)^2\right]} \sin(\beta \mp \gamma)$$

$$a_{s,L} = \frac{v_o^2 R_g \theta \sqrt{1 + \frac{\tan^2 \beta}{q^2}}}{R_c^2 + R_g^2 \theta^2 \left(1 + \frac{\tan^2 \beta}{q^2}\right)} \sin \beta$$



C. Quercellini, L.A., A. Balbi 2008
arXiv:0807.3237

$$\Delta v = (a_{s,K} + a_{s,L}) \Delta t$$

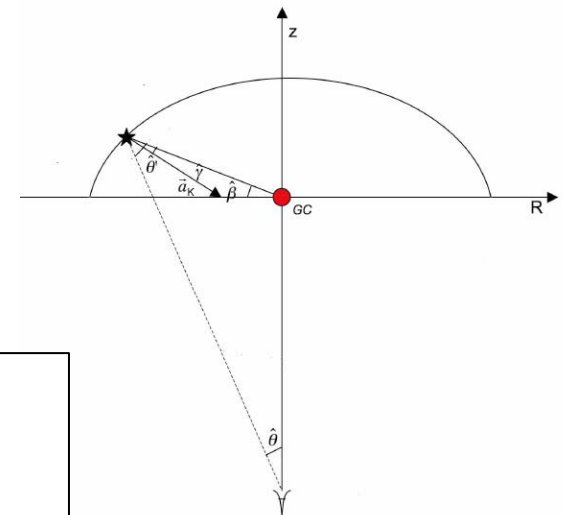
Spiral galaxy: MOND

- Beckenstein-Milgrom modified Poisson equation

$$\nabla \left[\mu \left(\frac{|\nabla \psi|}{a_0} \right) \nabla \psi \right] = 4 \pi G \rho$$

- peculiar acceleration in MOND:

$$a_{s,M} = \frac{MG \left(1 + \sqrt{1 + \frac{4a_0^2}{M^2 G^2} R_g^4 \theta^4 \left[1 + \left(|\tan \beta| + \frac{h}{R_g \theta} \right)^2 \right]^2} \right)^{1/2}}{\sqrt{2} R_g^2 \theta^2 \left[1 + \left(|\tan \beta| + \frac{h}{R_g \theta} \right) \right]} \cdot \sin(\beta \mp \gamma)$$



Most accelerated globular clusters

TABLE III: The ten globular clusters having the highest difference in signal between the CDM halo configuration and MOND. Astronomical data are from [12]

Name	v (cm/s) ^a	RA (hours) ^b	dec (degrees)	l (degrees)	b (degrees)	R_{Sun} (kpc)	R_{gc} (kpc) ^c	x (kpc)	y (kpc)	z (kpc)
Pal 1	1.18	03 33 23.0	+79 34 50	130.07	19.03	10.9	17.0	-6.6	7.9	3.6
NGC 2298	1.18	06 48 59.2	-36 00 19	245.63	-16.01	10.7	15.7	-4.3	-9.4	-3.0
NGC 1851	1.17	05 14 06.3	-40 02 50	244.51	-35.04	12.1	16.7	-4.3	-8.9	-6.9
NGC 1904 (M 79)	1.16	05 24 10.6	-24 31 27	227.23	-29.35	12.9	18.8	-7.6	-8.3	-6.3
NGC 288	1.15	00 52 47.5	-26 35 24	152.28	-89.38	8.8	12.0	-0.1	0.0	-8.8
NGC 5272 (M 3)	1.13	13 42 11.2	+28 22 32	42.21	78.71	10.4	12.2	1.5	1.4	10.2
NGC 5904 (M 5)	1.11	15 18 33.8	+02 04 58	3.86	46.80	7.5	6.2	5.1	0.3	5.4
NGC 1261	1.09	03 12 15.3	-55 13 01	270.54	-52.13	16.4	18.2	0.1	-10.1	-12.9
NGC 362	1.08	01 03 14.3	-70 50 54	301.53	-46.25	8.5	9.4	3.1	-5.0	-6.2
NGC 7099 (M 30)	1.07	21 40 22.0	-23 10 45	27.18	-46.83	8.0	7.1	4.9	2.5	-5.9

$$\Delta v \approx 1.5 \text{ cm / sec / yr}$$

Newton vs. MOND

9

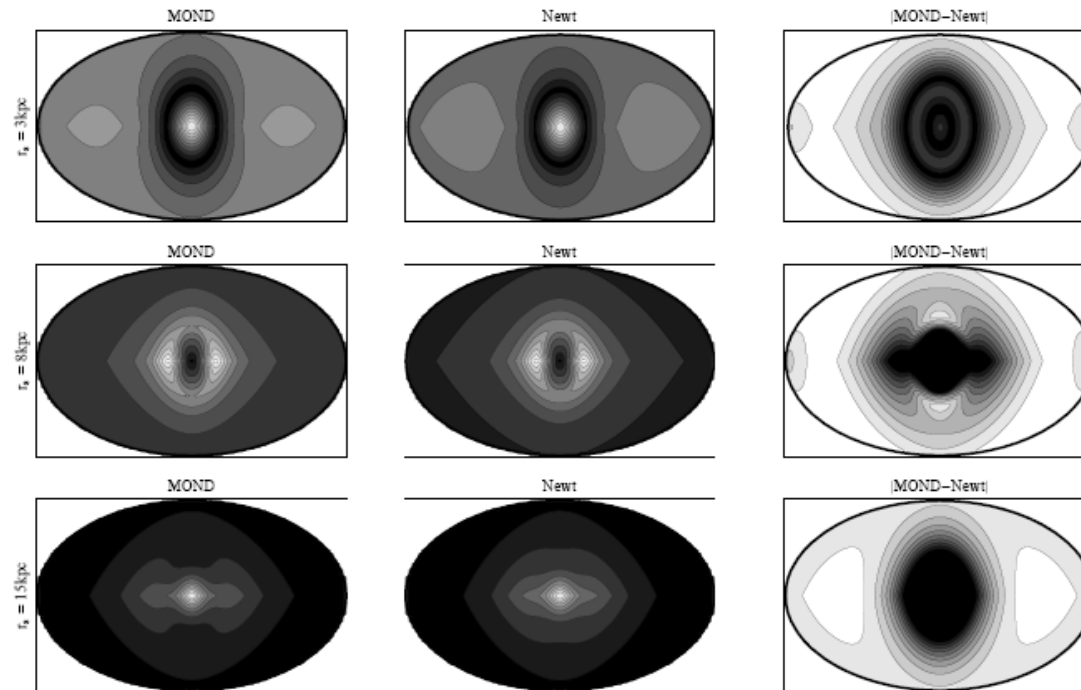


FIG. 10: Comparison in the velocity shift signal (for $T=15$ years) as seen in the sky from our position in the Milky Way, for the MOND, CDM halo configuration, and their difference (left to right) and for three distances from the Sun (3, 8 and 15 kpc, top to bottom). The sky signal is plotted in galactic coordinates and in Mollweide projection.

Memo for the future



- The Sandage-Loeb effect is a direct measure of the expansion/acceleration of the Universe
- The Cosmic Parallax is a direct test of anisotropy
- The Peculiar/Proper Acceleration is a direct measure of the gravitational potential
- **Full 3D picture of cosmic and local kinematics !**
- A sensitivity of 1 cm/sec could be achieved with the next generation of ELTs
- A sensitivity of 1-10 μ arcsec will be achieved by planned astrometric missions like GAIA, SIM, Jasmine
- New observables for DE, cosmology and gravity theories !