DOCUMENT

EChO radiometric model description

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CHANGE RECORD

Issue 3	**Revision** 2		
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Add stellar SNR in Vis			
Add zodi contribution in noise definition			

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1 INTRODUCTION

1.1 Mission overview

EChO - the Exoplanet Characterisation Observatory – is a survey-type mission dedicated to the characterisation of exoplanetary atmospheres. Using the differential technique of transit and eclipse spectroscopy, EChO will obtain transmission and/or emission spectra of the atmospheres of a large and diverse sample of known exoplanets covering a wide range of masses, densities, equilibrium temperatures, orbital properties and host-star characteristics. The instantaneous spectral coverage of EChO is unique in its breadth, spanning the visible to thermal infrared through a series of contiguous spectrometer channels that provide continuous spectral coverage. This broad range opens up the possibility to study exoplanets with physical temperatures ranging from a few hundred to over a few thousand degrees Kelvin. Importantly, broad instantaneous spectral coverage that includes the visible waveband provides an essential means by which to monitor and subsequently correct for the effects of activity of the host star, which could otherwise introduce significant uncertainty into the final exoplanet spectrum and its interpretation.

EChO will observe the combined light from the exoplanet and its host star. The transit and eclipse spectroscopy method, whereby the signal from the star and planet are differentiated using knowledge of the planetary ephemerides, allows atmospheric signals from the planet at levels of at least 10-4 relative to the star to be measured. Photometric stability rather than angular resolution is therefore key, and in fact the most stringent requirement of EChO, driving many engineering design and operational aspects of the mission. For the brightest targets it will be possible to obtain high quality spectra in a single visit; for fainter targets the necessary signal-to-noise will be built up through repeated visits over the mission lifetime.

EChO will address the following fundamental questions:

- Why are exoplanets as they are?
- What are the causes for the observed diversity?
- Can their formation and evolution history be traced back from their current composition?

EChO will allow scientists to study exoplanets both as a population and as individuals. The mission will target super-Earths, Neptune-like, and Jupiter-like planets, in the very hot to temperate zones (planet temperatures of 300 K - 3000 K) of F to M-type host stars. The spectroscopic information (at resolving powers of ~ 300 below 5 um and ~30 above) on the atmospheres of the large, select sample of exoplanets that EChO will provide will allow the compositions, temperature (profile), size and variability to be determined at a level never previously attempted. These in turn, will be used to address a wide range of key scientific questions relative to exoplanets:

- What are they made of?

- Do they have an atmosphere?
- What is the energy budget?
- How were they formed?
- Did they migrate and if so how?
- How do they evolve?
- How are they affected by starlight, stellar winds and other time-dependent processes?
- Weather: how do conditions vary with time?

And:

- Do any of the planets observed have habitable conditions?

1.2 Scope of document

This document provides a detailed description of the ESA EChO radiometric model. This model will be used to validate the mission requirements detailed in [AD1] which have, in turn, been derived from the science requirements detailed in [AD4]. The model provides the means to calculate, for a given host star/exoplanet target:

- The SNR that can be achieved in a single primary transit
- The SNR that can be achieved in a single occultation
- The number of transit/occultation revisits necessary to achieve a specified SNR
- The total number of revisits that could be achieved during the proposed mission lifetime

The radiometric model will be used to establish whether proposed samples of known or model targets can be observed to the signal-to-noise ratio (SNR) called for in [AD4], with mission requirements given in [AD1] (i.e. within the mission lifetime, with the observation efficiency required and the minimum design requirements), and to confirm the minimum design requirements for the mission.

In section 2 we describe the EChO measurement principle , detailing the expressions and assumptions used to calculate the astronomical signal in section 3. Derivations of expressions to calculate the SNR of exoplanet spectra measured during transit and occulation/eclipse are given in section 4, along with the SNR on the stellar signal alone. Derivations of key astronomical parameters are provided in the appendix.

The details of the model in its current implementation are specific to a classical dispersive spectrometer design. If an alternative spectrometer design were to be proposed, for example an FTS, then an equivalent radiometric model would need to be developed that allows an objective comparison between the different spectrometer designs.

1.3 Symbols

All symbols used in this document are detailed in the following tables. Unless stated otherwise, subscripts i/j shall be replaced by s or p when used respectively for a star or a planet, d for the detector, and M1 or M2 for the primary and secondary mirrors.

Symbol	Name	Unit	Comment
A_{eff}	Telescope effective area	[m²]	Effective photon gathering area of telescope. Includes reduction from central hole and M2 obscuration, excludes M1 and M2 reflectivity.
A_i	Geometrical area of element i	[m²]	i = M1 or M2
A	Semi-major axis	[m]	
d_{i-j}	Distance between objects i and j	[m]	For the distance between the exoplanet and the star, it is approximated as the semi-major axis a
Dark	Dark current	[e⁻/pixel/s]	
E	Eccentricity of the exoplanet's orbit	NA	
F	Heat re-distribution factor	NA	Measurement of atmospheric circulation [RD1]
G	Exoplanet surface gravity	[m.s⁻²]	
H	Atmosphere scale height	[m]	Height of the exoplanet's atmosphere above which it is transparent to stellar flux
$I_i(\lambda,T)$	Irradiance of body i at (λ,T)	[W.m⁻²]	
L	Distance from observed system to telescope	[m]	Assumed as star-Sun distance
M_i	Mass of the body i	[kg]	
N	Number of pixels	NA	
$N_i(\lambda,\Delta\lambda)$	Number of electrons produced due to i	[e⁻]	Between $\lambda-\Delta\lambda/2$ and $\lambda+\Delta\lambda/2$
N_{bin}	Number of independent spectral bins in an EChO spectrum	NA	
P	Exoplanet period around its host star	[s]	
$P_i(\lambda,\Delta\lambda)$	Power emitted by the body i	[W]	Between $\lambda-\Delta\lambda/2$ and $\lambda+\Delta\lambda/2$
P	Pressure	[Pa]	p_0 is the pressure at "sea level"
QE	Quantum efficiency	[e⁻/photon]	Ratio of arriving photons to produced electrons at the detector level

R	Resolving power	NA	Ratio of a wave length over a wave range
R_i	Radius of the body i	[m]	
r_i	Reflectivity of the optical element i	NA	
r_a	Apogee	[m]	
r_p	Perigee	[m]	
SNR	Signal to noise ratio	NA	
T	Integration time	[s]	
T_i	Black body temperature of body i	[ºK]	
V_i	View factor from optical element i to detector	NA	i = M1 or M2
Z	Height	[m]	In the atmosphere of the exoplanet, z=0 at "sea level"
zodi(λ,$\Delta\lambda$)	Zodiacal background	[e^-]	Contains both local and exo-zodi. Between λ-$\Delta\lambda/2$ and $\lambda+\Delta\lambda/2$.
α_{geom}	geometric albedo	NA	Taken for the purpose of this model to be wavelength independent
α_{Bond}	Bond albedo	NA	Taken for the purpose of this model to equal the optical albedo
ε_i	Emissivity of the body i	NA	
λ	Wavelength	[m]	
σ_{Dark}	Dark current noise	[e^-/pixel/s]	$\sigma_{Dark} = \sqrt{(Dark)}$
σ_{RN}	Read out noise	[e^-/pixel]	
η	Instrument throughput	NA	Product of transmission/reflection of all optical elements on the optical path, excluding M1 and M2.
$\varphi(t/\tau)$	Exoplanet day-side view factor	NA	Describes exoplanet phases: from full "Moon" to new "Moon" via crescents
μ	Molecular weight	[kg/mol]	Describes the relative weight of different atmospheres
$T=T_{14}$	Transit duration and available transit time	[s]	Time interval between 1st and 4th contact eg. interval between time at which projection of the limb of exoplanet and star first and last coincide. Period of ingress and egress assumed to be negligible

Table 1: List of symbols

All necessary fundamentals constants are given in the following table:

Symbol	Name	Unit	Value	Comment
C	Speed of light	[m/s]	2.99792×10^{8}	In vacuum
G	Gravitational constant	[$m^3.kg^{-1}.s^{-2}$]	6.67300×10^{-11}	
H	Planck constant	[$m^2.kg.s^{-1}$]	6.62607×10^{-34}	
K	Boltzmann constant	[$m^2.kg.s^{-2}.K^{-1}$]	1.38065×10^{-23}	
Na	Avogadro number	[mol^{-1}]	6.02214×10^{23}	

Table 2: List of constants

Additionally, the following table provides conversion factors:

Unit	Value in ISU	Comment
1 R_J	7.1492×10^{7} m	Jupiter radius (equatorial)
1 M_J	1.8986×10^{27} kg	Jupiter mass
1 R_{SUN}	6.9551×10^{8} m	Solar radius
1 M_{SUN}	1.9889×10^{30} kg	Solar mass
1 R_{EARTH}	$6.3781\text{x}10^{6}$ m	Earth radius (equatorial)
1 M_{EARTH}	$5.9742\text{x}10^{24}$kg	Earth mass
1 au	1.49560×10^{11} m	Astronomical unit
1 pc	3.0857×10^{16} m	Parsec
1 erg	$1\text{x}10^{-7}$ J	

Table 3: List of conversion factors

1.4 Reference documentation

1.4.1 Applicable documents

[AD1] ECHO MRD (Mission Requirements Document), SRE-PA/2011.038/
[AD2] ECHO PDD (Payload Definition Document), SRE-PA/2011.039/
[AD3] ECHO radiometric model description (the present document), SRE-PA/2011.040/
[AD4] ECHO SciRD (Science Requirements Document), SRE-PA/2011.037/

1.4.2 Reference documents

[RD1] Seager, S., "Exoplanet Atmospheres: Physical Processes", Princeton University Press ISBN 978-0-691-14645-4
[RD2] Allard, F., Homeier, D., Freytag, B., "Models of Stars, Brown Dwarfs and Exoplanets" ,2011, arXiv:1112.3591
[RD3] Cohen, M., Wheaton, W. A., & Megeath, S. T. 2003, AJ, 126, 1090 and K_s 2MASS passband downloaded from http://www.ipac.caltech.edu/2mass/releases/allsky/doc/sec6_4a.html
[RD4] Cohen, M., Megeath, S. T., Hammersley, P. L., Martín-Luis, F., Stauffer, J. 2003, AJ, 125, 2645 and V-band passband from Bessell, M., & Murphy, S. 2012, PASP, 124, 140
[RD5] Mandel, , K., Agol, E., "Analytic light curves for planetary transit searches", 2002, ApJ, 580, L171

[RD6] Ollivier, M., Van Boekel, R., Deroo, P., Ribas, I., Snellen, I., Tinetti., G., Waldmann., I.P. Synthesis note on EChO Signal-to-Noise calculation June 2012
[RD7] Seager, S., Whitney, B.A., Sasselov, D.D., "Photometric Light Curves and Polarization of Close-in Extrasolar Giant Planets", 2000, ApJ, 540, 504

2 ECHO MEASUREMENT PRINCIPLE

2.1 ECHO measurement description

EChO will make spatially unresolved observations of an exoplanet at different points in its orbit around its host star The signal from the stellar host and exoplanet are collected simultaneously. The signal from the exoplanet is a very small fraction of the total, and can be isolated by making differential measurements of observations with, and without the exoplanet contribution. By making such differential measurements it is possible to determine the transmission, reflection and emission spectrum of the exoplanet atmosphere.

For the radiometric model we consider two cases: measurements made during/around occultation (marked as B in Figure 1) and during/around primary transit (marked as A in the same Figure).

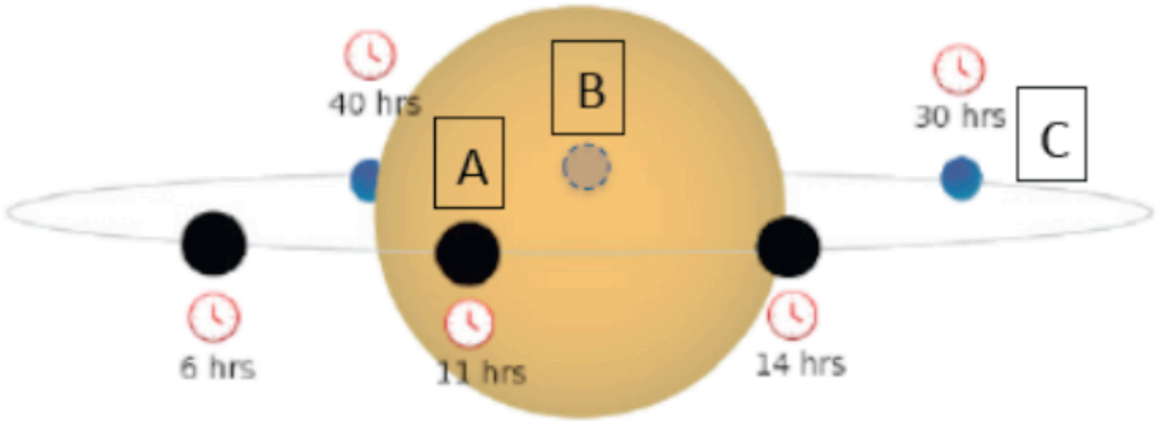

Figure 1: A schematic cartoon illustrating the orbit of an exoplanet around its host star. Event A **is referred to as a primary transit,** B **as an occultation and** C **as the orbital phase.**

In the case of occultation, observations are taken:

- Out-of-occultation (just before and just after), when the day-side of the exoplanet and its host star are in full view
- In-occultation, when the exoplanet is behind the host star and so only the star itself is visible

The exoplanet signature can be isolated by taking the difference between measurements taken in- and out-of-occultation. Contributions from exoplanet emission and reflection can be separated through their different spectral dependence TBC.

In the case of primary transit, the observations are taken:

- In-transit, during which the total signal observed is the sum of the fraction of the stellar light that is not blocked by the exoplanet disk, the emission from the night-side of the exoplanet and the light transmitted through the exoplanet atmosphere
- Out-of-transit (just before and just after), during which the signal comprises emission from the full stellar disk and a contribution (emission) from the night-side of the planet.

The exoplanet signal can be isolated by taking the ratio of the difference between the out-of-transit and in-transit measurements to the out-of-transit measurement.

Orbital phase measurements will also be made during the EChO mission (TBC), however these are not the subject of the current radiometric model.

2.2 Integration times

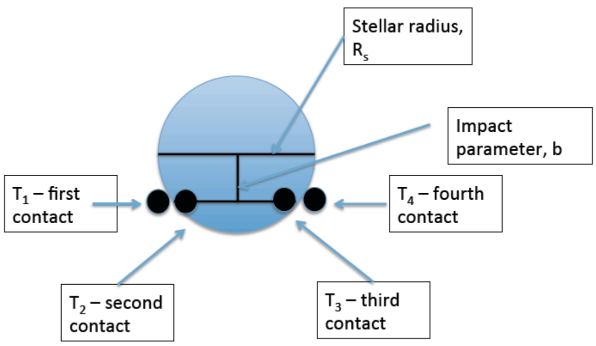

Figure 2: Point of first contact (beginning of ingress, T_1); Point of second contact (end of ingress, T_2); Point of third contact (beginning of egress, T_3); Point of fourth contact (end of egress, T_4)

The maximum integration time available during an individual eclipse/transit event is set by the duration of the eclipse, T. For the purpose of the radiometric model we assume that the time spent during ingress (T_1 -> T_2 = T_{12}) and egress (T_3 -> T_4 = T_{34}) (see Figure 2) is negligible compared to the total transit time, and so that the available transit time is given by $T=T_{23}=T_{14}$. For both primary transit and occultation measurements we assume that an equal time will be spent before/after and in-eclipse/transit (i.e. $1/2xT_{14}$ before and $1/2xT_{14}$ after). The total observing time required for a single eclipse/transit measurement is therefore $2xT_{14}$. Details of how transit times are calculated for individual systems are given in section 5.

The duration of a transit (~ hrs) is typically several orders of magnitude longer than the destructive readout time (≤ 10s), and so many destructive readouts will be taken during a single transit. The sum of all detector reset times after each destructive readout within a single transit/eclipse should be negligible compared to the transit/eclipse time to ensure that the transit duration is a good approximation to the total integration time.

2.3 Pixel and frame binning

The light within each spectral resolution element is dispersed over n pixels (channel dependent), and so the signal from all n pixels contained in each spectral band can be summed up.

Additionally, all destructive readouts taken within a 90 second interval (30 second goal) can also be summed up, where these time intervals are set by the science requirement on cadence given in [AD4].

The level to which both these binning procedures can be done on-board before data downlink (in order to reduce the amount of data that needs to be stored and downlinked) is TBC.

2.4 Spectral band binning

In the case that the spectrometer is designed with a resolving power exceeding the requirement, successive spectral resolution elements can also be binned together to the level of the required R (300 or 30).

The level to which this binning can be done on-board before data downlink (in order to reduce the amount of data that needs to be stored and downlinked) is TBC.

3 SCIENCE INPUTS

3.1 Stars

Stellar emission is modelled by spectral energy distributions (SEDs) that have been generated using the Phoenix atmosphere models [RD2] assuming a surface gravity/log g = 4.5 and solar metallicity. SEDs, specifically the radiation flux density at the surface of the star as a function of wavelength, are provided in a library of files supplied by the SST alongside this document for all stellar types in the mission reference sample[1]. The first issue of the library was made in late 2011. There, the mapping between spectral type and effective temperature for M-type stars was taken from Table A5 of Kenyon & Hartmann (1995, ApJ, 101, 117), connecting smoothly with Golimowski et al. (2004, AJ, 127, 3516) for the late-Ms, and using Baraffe et al. (1998, A&A, 337, 403) 1-Gyr isochrone for the radii. The effective temperatures and radii for FGK stars were taken from Allen's Astrophysical Quantities 4th edition (Cox, ed., 2000, Springer), which in turn comes from Schmidt-Kaler (1982, Landolt-Börnstein) and de Jager & Nieuwenhuijzen (1987, A&A, 177, 217). In the second issue of the SED library, Table A5 of Kenyon & Hartmann was used for all stars up to late Ms, with the late M-type stars taken from Golimowski et al. as before.

In all files, wavelength is given in [micron], and the flux, $S_s(\lambda)$, in units of [erg/s/cm²/A], equivalent to [10^1 W/m²/μm].

The spectral irradiance from the host star at the input of the EChO telescope is given by the product of the stellar flux density measured at its surface and the solid angle subtended by the star at the telescope:

$$E_s(\lambda) = S_s(\lambda).\left(\frac{R_s}{l}\right)^2 \quad \text{in [W/m}^2/\mu\text{m]}$$

3.2 Exoplanets

3.2.1 Emission

Emission from the exoplanet day-side is modelled as a black body. The wavelength-dependent surface flux density is given by:

$$S_p(\lambda, T) = \pi.\frac{2.h.c^2}{\lambda^5}.\frac{1}{e^{\frac{hc}{\lambda.k.T}} - 1} \quad \text{in [W/m}^2/\mu\text{m]}$$

[1] The SED library is available from ESA by request.

The spectral irradiance from the exoplanet day-side surface as measured at the EChO telescope is then the product of the blackbody emission and the solid angle subtended by the exoplanet at the telescope:

$$E_p^{emission}(\lambda,T) = S_p(\lambda,T).\left(\frac{R_p}{l}\right)^2 \qquad \text{in [W/m}^2\text{/µm]}$$

Note that due to the fact that the night-side temperature is much lower than that of the day-side, emission from the night-side of the planet is considered to be negligible.

3.2.2 Reflected signal

A fraction of the stellar light incident on the exoplanet is reflected, with the amount dependent on the wavelength-dependent geometric albedo. The reflected signal is typically a strong function of wavelength, and can be significant at visible wavelengths. The flux reflected from the host star by the exoplanet and incident at the EChO telescope is given by:

$$E_p^{reflection}(\lambda) = \alpha_{geom}.S_s(\lambda).\left(\frac{R_s}{l}\right)^2.\frac{{R_p}^2}{d_{s-p}^2} \qquad \text{in [W/m}^2\text{/µm]}$$

where α_{geom} is the geometric albedo. We use the derivation for the geometric albedo appropriate to Lambertian sphere, and so equal to $2\text{x}\alpha_{Bond}/3$ (e.g. [RD7]), assume that the value is wavelength-independent and that (near) occultation the phase term $\varphi=1$, i.e. equivalent to full illumination/ "full moon". The derivation of the distance between the exoplanet and star, $d_{s\text{-}p}$ is given in section 5.

3.2.3 Transmission

The fraction of the stellar light that passes through the exoplanetary atmosphere is determined by the ratio of the projected area of the atmosphere to that of the stellar disk, and is equal to (neglecting the term $o(\Delta z^2)$):

$$\left(\frac{2.R_p.\Delta z(\lambda)}{{R_s}^2}\right)$$

whilst the difference between the out-of and in-transit signal is given:

$$\left(\frac{R_p^2 + 2.R_p.\Delta z(\lambda)}{{R_s}^2}\right).S_s(\lambda,T).\frac{R_s^2}{l^2}$$

where $\Delta z(\lambda)$ is 5xH, and H the scale height of the atmosphere. Details of how the atmospheric scale height is evaluated are given in section 5.6.

4 SNR MODELLING

4.1 Astronomical signal

The spectral irradiances, i.e. power per unit wavelength per unit area incident on a surface (the EChO telescope entrance aperture), for the various components of the stellar and exoplanet signal can be converted into the number of electrons per spectral resolution element $N(\lambda, \Delta\lambda)$ at the output of the focal plane detectors using the following telescope and instrument design parameters:

$$N(\lambda, \Delta\lambda) = E(\lambda) \times t \times \Delta\lambda \times A_{eff} \times r_{M1} \times r_{M2} \times \eta \times \frac{\lambda}{h.c} \times QE \quad \text{in [e}^{-}\text{/spectral band]}$$

where t is the integration time in seconds.

This expression is only valid if the spectral width $\Delta\lambda$ of the resolution element is small enough, allowing the integral of the spectral irradiance within the resolution element to be approximated by the product of the spectral irradiance at the central wavelength and the width of the resolution element. The SEDs in the library are given at a higher resolution than that specified by the requirement in [AD4]: 1/1000th micron below 5.2 micron, and 5/1000th above, and so need to be averaged as appropriate to the resolving power called for.

Note:

$$\left[\frac{e^{-}}{spectral\ band}\right] = \left[\frac{W}{m^2 \times \mu m} \times s \times \frac{\mu m}{spectral\ band} \times m^2 \times \frac{photon}{J} \times \frac{e^{-}}{photon}\right]$$

The following notation will be used:

- $N_0(\lambda, \Delta\lambda)$ is the equivalent number of electrons per spectral resolution element generated from the stellar emission.
- $N_1(\lambda, \Delta\lambda)$ is the equivalent number of electrons per spectral resolution element generated from the exoplanet day-side emission.
- $N_2(\lambda, \Delta\lambda)$ is the equivalent number of electrons per spectral resolution element generated from the starlight reflected by the exoplanet.

4.2 Background signal

All electron-producing phenomena have to be accounted for in addition to the signal generated by the science target (star + exoplanet). These include:

- Zodiacal background (exo- and local- zodi), denoted zodi, in [e^{-}/s/pixel].

- PLM thermal background (telescope and instrument box), denoted $N_3(\lambda,\Delta\lambda) = N_{31}(\lambda,\Delta\lambda) + N_{32}(\lambda,\Delta\lambda)$ respectively.
- Detector dark current, denoted Dark, in [e$^-$/s/pixel]
- Stray light (note that at this point of the study the contribution from stray light is not known)

Since these background signals will be common to all measurements (in- and out- of eclipse/transit), they will be summed up and expressed as $N_{background}^{signal}(\lambda,\Delta\lambda)$.

In the case of occultation measurements, providing this background does not vary with time, this background signal will difference out in the same way as the stellar signal, providing this background does not vary with time. Its contribution to the photon noise (square root of the variance of the signals) adds, however, and so will have to be accounted for twice in the SNR.

In other cases (e.g. ratio of in- and out-of transit), this background contribution will have to be calibrated out, by using any (or a combination) of the following:

- Off-source pixels (i.e. pointing towards deep space). These would contain all the above mentioned signals.
- Dark pixels (with a fixed or an actuated shutter). These would only enable cancellation of the dark current.
- Accurate modelling of, and correction for, the thermal background through measurements of the telescope and instrument box(es) temperatures using thermocouples. Note that this would only allow nulling of the thermal background.

4.3 Noise contributions

Noise in the unresolved stellar/exoplanet measurement can be divided into three categories:

- Shot noise from stellar and background (as detailed in section 4.2) signals, including:
  - Target star and exoplanet
  - Zodiacal background
  - PLM thermal background
  - Dark current
  - Stray light
- Additional non-Poisson noise, such as detector read out noise, denoted σ_{RN}, in [e$^-$/s/pixel]
- Photometric variations, such as those due to stellar variability or pointing jitter etc.

All noise contributions shall be summed in RSS, with dependent noise sources added linearly (e.g. thermal effects etc.).

The total noise budget shall not exceed the following allocation:

$$Noise_{TOTAL} \leq \sqrt{(N_0(\lambda,\Delta\lambda) + zodi) \times (1+X) + N_{\min}} \quad \text{in [sqrt(e}^{-}\text{/spectral band)]}$$

Where X and $N_{\min}$ are factors to be determined. The term $(N_0(\lambda,\Delta\lambda) + zodi) \times X$ represents the part of the noise budget that is relative to the target being observed (e.g. photometric variations due to pointing jitter), while the term $N_{\min}$ represents the absolute noise floor that occurs whatever the target being observed (e.g. detector read out noise).

For example, in the case of the measurement out-of-eclipse (just before or just after the occultation), a noise budget (neglecting photometric variation terms) could be given by:

$$\sqrt{\sqrt{N_0(\lambda,\Delta\lambda)}^2 + \sqrt{N_1(\lambda,\Delta\lambda)}^2 + \sqrt{N_2(\lambda,\Delta\lambda)}^2 + \sqrt{N_3(\lambda,\Delta\lambda)}^2 + \sqrt{zodi}^2 + \sqrt{n.(t.Dark + \sigma_{RN}{}^2)}^2}$$
$$\approx \sqrt{N_0(\lambda,\Delta\lambda) + N_3(\lambda,\Delta\lambda) + zodi + n.(t.Dark + \sigma_{RN}{}^2)} \text{ in [sqrt(e}^{-}\text{/spectral band)]}$$

where the photon noise from the exoplanet contributions is assumed to be negligible compared to that of the star.

Complying with the noise requirement then means:

$$\sqrt{N_3(\lambda,\Delta\lambda) + n.(t.Dark + \sigma_{RN}{}^2)} \leq \sqrt{X \times (N_0(\lambda,\Delta\lambda) + zodi) + N_{\min}}$$

The total noise and its breakdown/budgeting into its individual components will depend on the specific instrument design (e.g. choice of detectors etc.). Detailed allocation of the noise budget into the individual noise terms is left to contractors and instrument consortia, to be done according to their instrument designs, but to include as a minimum:

- Thermal background noise from the telescope
- Thermal background noise from the instrument box
- Detector noises (dark current and read out noises)
- Stray light noise
- Photometric variations from the star
- Thermal background variations
- Photometric variations due to pointing jitter
- Margin commensurate with the study phase and level of maturity of the design

4.4 Occultation SNR

In the case of occultation, the exoplanet signal is derived from the difference of the in- and out- of occultation measurements:

$$[N_0(\lambda,\Delta\lambda)+N_1(\lambda,\Delta\lambda)+N_2(\lambda,\Delta\lambda)+N_{background}^{signal}(\lambda,\Delta\lambda)]-[N_0(\lambda,\Delta\lambda)+N_{background}^{signal}(\lambda,\Delta\lambda)]$$
$$=N_1(\lambda,\Delta\lambda)+N_2(\lambda,\Delta\lambda) \qquad \text{in [e}^-\text{/spectral band]}$$

The noise of both measurements adds up in this process, resulting in a total noise of:

$$\sqrt{2\times[(1+X)\times(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}]}$$

And the SNR is then:

$$SNR_{occultation}=\frac{N_1(\lambda,\Delta\lambda)+N_2(\lambda,\Delta\lambda)}{\sqrt{2\times[(1+X)\times(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}]}}$$

4.5 Primary transit SNR

In the case of primary transit, the signal due to the exoplanet is given by the difference between the out-of and in- transit measurements:

$$[N_0(\lambda,\Delta\lambda)+N_{background}^{signal}(\lambda,\Delta\lambda)]-[N_0(\lambda,\Delta\lambda)\times(1-\frac{\pi.(R_p+\Delta z(\lambda))^2}{\pi.R_S{}^2})+N_{background}^{signal}(\lambda,\Delta\lambda)]$$

$$=N_0(\lambda,\Delta\lambda)\times\frac{(R_p+\Delta z(\lambda))^2}{R_S{}^2} \qquad \text{in [e}^-\text{/spectral band]}$$

However, if simply making the difference between out-of and in- transit measurements, the signal in the formulae above contains two variables, i.e. the stellar signal $N_0(\lambda,\Delta\lambda)$ and the atmospheric scale height $\Delta z(\lambda)$. As such, one cannot isolate the information on the atmospheric scale without precise knowledge of the stellar signal. This can be done by additional processing, specifically taking the ratio of the difference between out-of and in-transit (i.e. expression above) and the out-of transit measurement (containing the stellar light only):

$$\frac{N_0(\lambda,\Delta\lambda)\times\frac{(R_p+\Delta z(\lambda))^2}{R_S{}^2}}{N_0(\lambda,\Delta\lambda)+N_{background}^{signal}(\lambda,\Delta\lambda)}$$

If the background signal is small and can be neglected (as in the visible, where contributions from the zodiacal light and the thermal background are small, and detectors typically have low dark currents) this then reduces to:

$$\frac{(R_p + \Delta z(\lambda))^2}{R_S{}^2}$$

As one moves to increasing wavelengths, the background signal can no longer be neglected, and needs to be subtracted out using one of the techniques mentioned at the end of section 4.2 (introducing additional noise by making another difference, as in section 4.4). In this way the exoplanet component can be isolated:

$$\frac{N_0(\lambda,\Delta\lambda)\times\frac{(R_p + \Delta z(\lambda))^2}{R_S{}^2}}{[N_0(\lambda,\Delta\lambda)+N_{background}^{signal}(\lambda,\Delta\lambda)]-N_{background}^{signal}(\lambda,\Delta\lambda)} = \frac{(R_p + \Delta z(\lambda))^2}{R_S{}^2}$$

where the only remaining unknown is the term $\Delta z(\lambda)$, which can easily be isolated from the other terms R_p and R_S through its wavelength dependence.

The noise associated with the process described above is then:

$$\frac{(R_p+\Delta z)^2}{R_S{}^2}\times\sqrt{\left[\frac{\sqrt{2\times[(1+X)\times(N_0+zodi)+N_{\min}]}}{N_0\times\frac{(R_p+\Delta z)^2}{R_S{}^2}}\right]^2+\left[\frac{\sqrt{N_0+2\times\left[zodi+X\times(N_0+zodi)+N_{\min}\right]}}{N_0}\right]^2}$$

Note:

$$\left(\frac{\Delta\left(\frac{A}{B}\right)}{\left(\frac{A}{B}\right)}\right)^2=\left(\frac{\Delta A}{A}\right)^2+\left(\frac{\Delta B}{B}\right)^2 \Leftrightarrow \Delta\left(\frac{A}{B}\right)=\frac{A}{B}\times\sqrt{\left(\frac{\Delta A}{A}\right)^2+\left(\frac{\Delta B}{B}\right)^2}$$

Where:

- A is the difference between the out-of and in- transit measurements
- and B is the difference between the out-of transit and the background signal measurements

$$A = [N_0(\lambda,\Delta\lambda) + N^{signal}_{background}(\lambda,\Delta\lambda)] - [N_0(\lambda,\Delta\lambda)\times(1-\frac{\pi.(R_p+\Delta z(\lambda))^2}{\pi.R_S{}^2}) + N^{signal}_{background}(\lambda,\Delta\lambda)] = N_0(\lambda,\Delta\lambda)\times\frac{(R_p+\Delta z(\lambda))^2}{R_S{}^2}$$

$$\Delta A = \sqrt{2\times[(1+X)\times(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}]}$$

$$B = [N_0(\lambda,\Delta\lambda) + N^{signal}_{background}(\lambda,\Delta\lambda)] - N^{signal}_{background}(\lambda,\Delta\lambda) = N_0(\lambda,\Delta\lambda)$$

$$\Delta B = \sqrt{N_0(\lambda,\Delta\lambda) + 2\times[zodi + X\times(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}]}$$

$N_0(\lambda,\Delta\lambda)$ only appears once in ΔB since the measurement of the background only is done without imaging the target star, e.g. through measurements made with off-source pixels as explained in section 4.2.

The resulting SNR is then:

$$SNR_{transit} = \frac{\dfrac{(R_p+\Delta z)^2}{R_S{}^2}}{\dfrac{(R_p+\Delta z)^2}{R_S{}^2}\times\sqrt{\left[\dfrac{\sqrt{2\times[(1+X)\times(N_0+zodi)+N_{\min}]}}{N_0\times\dfrac{(R_p+\Delta z)^2}{R_S{}^2}}\right]^2+\left[\dfrac{\sqrt{N_0+2\times[zodi+X\times(N_0+zodi)+N_{\min}]}}{N_0}\right]^2}}$$

$$= \frac{N_0\times\dfrac{(R_p+\Delta z)^2}{R_S{}^2}}{\sqrt{2\times[(1+X)\times(N_0+zodi)+N_{\min}]+[N_0+2\times[zodi+X\times(N_0+zodi)+N_{\min}]]\times\left(\dfrac{(R_p+\Delta z)^2}{R_S{}^2}\right)^2}}$$

In practice, the science quantity that is used in further analysis is only the fraction of the total signal that is transmitted through the exoplanetary atmosphere (the light blocked by the exoplanet's radius is a fixed term and can be subtracted) and thus the appropriate expression for the SNR is given by:

$$SNR_{transit} = \frac{N_0\times\dfrac{(2\times R_p\times\Delta z)}{R_S{}^2}}{\sqrt{2\times[(1+X)\times(N_0+zodi)+N_{\min}]+[N_0+2\times[zodi+X\times(N_0+zodi)+N_{\min}]]\times\left(\dfrac{(R_p+\Delta z)^2}{R_S{}^2}\right)^2}}$$

4.6 Generalisation of SNR expressions

In the previous subsections expressions for the SNR for the exoplanet signal measured both in occultation and transit have been derived using two simplifying assumptions: (a) equal integration time is spent in and out of transit (b) the exoplanet signal can be isolated from the difference between the in and out of transit observations. To first order, the exoplanet light curve is given by equation (8) in [RD5]. In the limit that equal time in- and out-of-transit is considered, and that all spectral bins in a spectrum are considered to be independent, then the expression for SNR for the transit and occultation cases above match that derived from equation (8).

In the more general case, the factor of 2 in the denominators is replaced by:

$$1+\frac{1}{Y}$$

$$\Rightarrow SNR_{occultation} = \frac{N_1(\lambda,\Delta\lambda)+N_2(\lambda,\Delta\lambda)}{\sqrt{\left(1+\frac{1}{Y}\right)\times[(1+X).(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}]}}$$

where Y in turn is given by $\frac{N_{Bin}.T_{out}}{T_{in}}$, with N_{bin} the number of (correlated) spectral channels, and T_{in} and T_{out} the time spent in and out of transit respectively. A very good knowledge of the real relative stellar spectrum is needed (at the level of 10^{-4}) in order to be able to take advantage of the correlation between neighbouring spectral bins, however, and this places stringent requirements on calibration which are not currently addressed. Unless otherwise stated, we therefore maintain a conservative approach and assume that N_{bin} = 1. In addition we assume that the time spent out-of-transit is equal to that spent in (ie. T_{14}), unless otherwise stated. Under these two conditions the expressions for the SNR given above hold.

4.7 Stellar SNR at visible wavelengths

An accurate measurement of the signal from the star in the visible is needed in order to correct the EChO spectrum for the effects of stellar variability. In this subsection we derive the SNR for the stellar signal alone. It should be noted that SNR can be evaluated over as short a time interval as a single read-out time. We consider the SNR that can be achieved in a single visit by considering the total time of the visit, 2 x T14.

4.7.1 During occultation

The stellar signal is given by $N_0(\lambda,\Delta\lambda)$ and the noise term by $\sqrt{(1+X).(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}}$. During the measurement of the stellar signal, both thermal backgrounds and zodi will also be measured. These are much smaller than the stellar signal and so their contribution to the total signal term can be neglected. The SNR during 1xT14 is given by:

$$SNR = \frac{N_0(\lambda,\Delta\lambda)}{\sqrt{(1+X).(N_0(\lambda,\Delta\lambda)+zodi)+N_{\min}}}$$

Note that the zodi term is not accounted for in the signal as it is negligible, however it is left in the noise term to remain consistent with the definition of the total noise as defined in Section 4.3.

Just before/after the occultation, there is an additional contribution to the signal (and to the noise) from the exoplanet (emission and reflection), however again these are negligible compared to the stellar signal. The expression for the SNR is therefore equivalent to the one above, and can be evaluated over a maximum of 2x0.5xT14.

As a result, the total SNR during an occultation visit is given by:

$$SNR = \frac{N_0(\lambda, \Delta\lambda)}{\sqrt{(1+X).(N_0(\lambda, \Delta\lambda) + zodi) + N_{\min}}}$$

Where the integration time to be considered is 2xT14.

4.7.2 During transit

During transit, the stellar signal is given by:

$$N_0(\lambda, \Delta\lambda) \times (1 - \frac{\pi.(R_p + \Delta z(\lambda))^2}{\pi.R_S{}^2})$$

Again assuming that the contribution from the zodi and backgrounds, as well as from the night-side emission of the exoplanet, can be neglected.

The noise can be approximated by:

$$\sqrt{(1+X).(N_0(\lambda, \Delta\lambda) + zodi) + N_{\min}}$$

The SNR is therefore given by:

$$SNR = \frac{N_0(\lambda, \Delta\lambda) \times (1 - \frac{\pi.(R_p + \Delta z(\lambda))^2}{\pi.R_s^2})}{\sqrt{(1+X).(N_0(\lambda, \Delta\lambda) + zodi) + N_{\min}}}$$

To be considered over a maximum integration time of 1xT14.

Just before/after the transit, assuming a negligible night side emission, the SNR is given by:

$$SNR = \frac{N_0(\lambda, \Delta\lambda)}{\sqrt{(1+X).(N_0(\lambda, \Delta\lambda) + zodi) + N_{\min}}}$$

With an integration time of 2x0.5xT14.

As a result, for the duration of the complete transit observation (2xT14), the SNR can be evaluated as a worst case:

$$SNR = \frac{N_0(\lambda, \Delta\lambda) \times (1 - \frac{\pi.(R_p + \Delta z(\lambda))^2}{\pi.R_s^2})}{\sqrt{(1+X).(N_0(\lambda, \Delta\lambda) + zodi) + N_{\min}}}$$

Which is slightly less than the stellar SNR that can be achieved during an occultation visit.

5 APPENDIX A: DERIVATION OF KEY ASTRONOMICAL PARAMETERS

5.1 Stellar Masses

Stellar masses have been calculated using the approximation $M_{star} = R_{star}$ where stellar mass and radius are in units of $[M_{SUN}]$ and $[R_{SUN}]$ respectively.

5.2 Stellar distances

Stellar distances given in [AD4] have been calculated using the appropriate stellar SED and the target apparent K-band magnitude using the definition of apparent magnitude:

$$m_K = -2.5\log\left(\frac{R_s^{\;2}.S_s(\Delta\lambda)}{l^2.S_0^K(\Delta\lambda)}\right) \qquad \text{(A1)}$$

where $S_0^K(\Delta\lambda)$ is the zero point flux for the standard K-band filter profile, and $\Delta\lambda$ the filter band pass given in [RD3], and $S_s(\Delta\lambda)$ the stellar flux density evaluated over the same bandwidth.

Note:

- The same equation holds in the V-band, with the filter band pass given in [RD4].
- $S_0^K(\Delta\lambda)$=1.122x10-7 erg/s/cm²
- $S_0^V(\Delta\lambda)$=3.327x10-6 erg/s/cm²

5.3 Semi-major axis of the exoplanet orbit

All orbits are assumed to be circular, with eccentricities of zero. Orbital radii have been calculated using the principle of energy balance which, when expressed in terms of effective temperatures, is given by:

$$T_P = T_S.\sqrt{\frac{R_s}{2.d_{s-p}}.\sqrt{f.(1-\alpha)}} \qquad \text{(A2)}$$

For known exoplanet/host star systems, the quantity that is to be derived/unknown is typically the equilibrium temperature of the exoplanet. In the case of the mission reference sample, standard temperatures have been adopted for the different exoplanet cases. The equation above is used instead to derive the orbit radius, and so semi-major axis of the exoplanet orbit:

$$a = d_{s-p} = \left(\frac{T_s}{T_p}\right)^2 . \frac{R_s}{2} . \sqrt{f.(1-\alpha)} \tag{A3}$$

where T_p and T_s are the effective temperatures of the exoplanet and star respectively and are given in [AD4], α the Bond albedo and f, the heat redistribution factor.

5.4 Exoplanet orbital period

Exoplanets periods are calculated using Kepler's third law:

$$P = 2\pi\sqrt{\frac{a^3}{G.M_s}} \Leftrightarrow a = \left(\left(\frac{P}{2\pi}\right)^2 .G.M_s\right)^{1/3} \tag{A4}$$

5.5 Exoplanet transit duration

Transit times are calculated by evaluating the fraction of the orbital period for which the exoplanet is in front of the star. An impact parameter, b, is used to account for the fact that the exoplanet may make not make an equatorial transit (see Figure 2). An average value of 0.5 (i.e. the midpoint of the equator and limb of the star) is adopted, with a resultant transit time from beginning of ingress to end of egress given by the time to cover orbital distance $2.R_s.\sqrt{1-b^2}$ at the mean orbital speed $(2.\pi.a)/P$:

$$T = T_{14} == \frac{2.R_s.\sqrt{1-b^2}}{2.\pi.a/P} = \sqrt{1-b^2}.\frac{R_s}{\pi.a}.P \tag{A5}$$

5.6 Height of exoplanet atmosphere

The height of the atmosphere of a planet is taken to be equivalent to the height above "sea-level" at which the pressure/density can be considered to be negligible. For the purpose of the radiometric model we take this to be the height above which less than 1% of the atmosphere exists. The pressure of an atmosphere at a height, z, as a fraction of that at "sea-level" is given by:

$$p(z) = p_0 \cdot e^{-z/H} \tag{A6}$$

If we consider an isothermal atmosphere, then the scale height – the height over which the pressure falls by $1/e$ - is given by:

$$H = \frac{kT_p\text{Na}}{\mu g} \tag{A7}$$

where g is the surface gravity:

$$g = \frac{G \cdot M_p}{R_p^{\ 2}} \tag{A8}$$

At a height of 5.H, one is above 99.5% of the atmosphere, and so the effective opaque height of the atmosphere is given by:

$$\Delta z(\lambda) = 5 \cdot H \tag{A9}$$

Note that the effective height will have a strong dependence on wavelength.